FE ANALYSIS OF CREEP AND HYGROEXPANSION RESPONSE OF A CORRUGATED FIBERBOARAD TO A MOISTURE FLOW: A TRANSIENT NONLINEAR ANALYSIS

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ABSTRACT

This paper presents a model using finite element method to study the response of a typical commercial corrugated fiberboard due to an induced moisture function at one side of the fiberboard. The model predicts how the moisture diffusion will permeate through the fiberboard's layers (medium and liners) providing information on moisture content at any given point throughout the structure. The hygroexpansion response and the creep response were predicted through the development of a finite element model capable of capturing the behavior of the fiberboard. Comparing the results generated from the model with actual experimental results validates the accuracy of the computational model. The model predicts the deformation response due to combined hygroexpansion and creep as the relative humidity rises from 38% RH to 86%. The parameters studied and calibrated include: the coefficient of moisture diffusion of the liner and the medium boards, the coefficient of moisture expansion, and the constants in the creep constitutive law. The results generated from the finite element model showed excellent agreement with the experimental results for a short column corrugated fiberboard and a board model representing a container box side-panel dimension. The results were generated in a cyclic relative humidity condition. A successful development of a reliable computational model holds the promise for analyzing collapse mechanism of container boxes in the service field under real weather condition data without dependency on expensive time-consuming experimental investigations. This is of great benefit to the shipping industry and the public.

Keywords: Corrugated fiberboard, finite element analysis (FEA), moisture diffusion, creep, liner, medium, coefficient of moisture conductivity, and coefficient of moisture expansion.

INTRODUCTION

Container boxes made of corrugated fiberboard are the most common form of packaging used in the shipping industry. The boxes can collapse when humidity rises. Collapsing occurs often, and is very costly. Corrugated fiberboard is very susceptible to changes in relative humidity and moisture conditions. In high humidity conditions, the boxes will collapse in warehouses due to hygroexpansion and creep deformation. The extent of collapse will be magnified further in cyclic day/night humidity conditions. Setting experimental investigation to study the behavior of corrugated fiberboard boxes at this large-scale and under uncontrolled environmental conditions is unrealistic and can be very costly. However, if a computational model is established and can accurately predict the behavior of corrugated fiberboard under such conditions, and is adaptable to changing parameters of real weather data, such a model has potential to be very valuable. This can be of extreme value to the shipping industry, the corrugating manufacturers, the general public, and most importantly from a conservation and utilization point of view, to the forests and forest products agencies.

The specific objectives and scope of this paper are: to identify the important physical parameters for the moisture flow phenomenon as they relate to the corrugated fiberboard, to calibrate the parameters generated by the finite element analysis with available experimental results, to develop a proper creep law and related constants to predict creep behavior of fiberboard, to predict the moisture distribution response of the fiberboard due to the increase in relative humidity conditions in the field, to establish the deformation response of the corrugated fiberboard due to the coupling effect of creep and hygroexpansion phenomenon, and to establish a reliable computational model that will address the creep phenomenon and the hygroexpansion phenomenon to be used to predict the behavior of corrugated boxes in a service environment due to change in real weather conditions.

The developed model was calibrated with controlled experimental results to establish reliability. The development of a 3D FE nonlinear model allows the prediction of moisture flow throughout corrugated fiberboard. The 3D FE model predicts moisture diffusion through layers of the fiberboard (medium and facings), deformation response, creep response, and loss of stiffness. The FE results were compared with experimental results conducted at the Forest Product Laboratory in Madison, WI, of a board subjected to a swept-sine humidity function to study its response to varying humidity functions. The board modeled in this analysis consists of two facings made from liner material separated by a corrugated medium. The liner and medium were assigned orthotropic stress-strain relations; bilinear stress-strain curves generated from experimental data under variable moisture contents, namely 50% and 90% RH. The liner and medium were modeled as 8-noded shell elements, which allow for curved medium geometry. The major orthogonal directions were the machine direction (MD), the cross machine direction (CD), and the out-of-plane z-direction. The layers were assumed perfectly bonded at juncture lines. The moisture flow was considered in one direction across the thickness of the fiberboard. The fiberboard was subjected to transient moisture analysis coupled with an edgewise-compressive loading. The transient analysis was dependent on the following material constants: coefficient of moisture diffusivity and coefficient of moisture expansion. The FE model provided results showing moisture diffusion flow, creep deformation response in the fiberboard under the action of a static compressive loading, and periodic moisture from actual experimental data.

MATERIALS AND METHODS

The material properties for the liner and the medium are described as a bilinear curve as shown in Fig. 1. The curves are based on best-fit bilinear relationships generated from the stress-

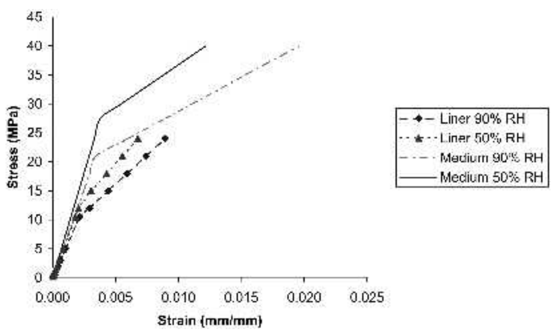

FIG. 1. Stress–strain curve for liner and medium.

strain data obtained from experimental measurements of the liner and the medium in 50% RH and 90% RH environments. Paper specimens (30 mm × 50 mm × 0.31 mm) were conditioned and tested in humidity-controlled environments to generate the stress-strain relationship. The developed finite element model used the material properties of liner and medium board for varying relative humidity conditions. When the relative humidity conditions were between the upper and lower RH values, the model performed linear interpolation to adjust for intermediate conditions. The finite element analyses in this paper had the following features: The geometry of the corrugated fiberboard was generated based on accurate commercial fiberboard, compressive loading was applied, and simply supported boundary conditions were assumed; the moisture diffusion distribution throughout the corrugated fiberboard was computed based on the cyclic humidity schedule outlined in the experimental investigation (Urbanik and Lee 1995), followed by hygroexpansion moisture deformation transient nonlinear analyses at each step of the RH cycle. Finally the creep response was calculated and coupled with the hygroexpansion response to produce the overall structural behavior. The first structure analyzed represented the size of an edge-crush corrugated fiberboard, and then the structure size was increased to represent the dimensions of a side-panel of a corrugated container box. The experimental work performed by Urbanik and Lee (1995) concluded "conventional RH test schedules that rely on fixed cycle periods do not adequately reflect the significance of moisture diffusion and hygroexpansion rate phenomena on the creep of corrugated containers and general wood products." The experimental investigation was conducted in humidity-controlled environments under the influence of swept-sine schedule of varying periods. The specimens were made of corrugated fiberboard of a C-fluted commercial construction under the action of compressive loading. The hygroexpansion and the creep response were recorded for the duration of the experiments. The finite element model in this paper was a representation of the experimental set-up and was designed to predict the experimental results. Having a reliable finite element model provides the basis for further analyses beyond the experimental limitations.

The finite element model

The FE model was developed to represent an actual C-fluted geometry of a corrugated fiberboard. The model in this analysis consisted of liners, medium, and interface joints. The liner and the medium were modeled as 8-node shell elements, and the interface joints were modeled by 8-node layered shell elements. Figure 2 shows a detailed representation of the FE geometry (Rahman 1997). The model and analyses were generated using the commercial finite element computer program ANSYS version 6.1.

A 2-in. by 2-in. corrugated fiberboard was subjected to a steady-state moisture flow due to a rise in relative humidity from 50% to 90%. The finite element model analyzed was based on the experimental study conducted by Urbanik (1995) The experiment was performed on a short column subjected to a single-frequency humidity function, as shown in Fig. 3.

Determination of coefficients, moisture conductivity, and expansion

A derivation of two essential properties, namely the coefficient of moisture conductivity

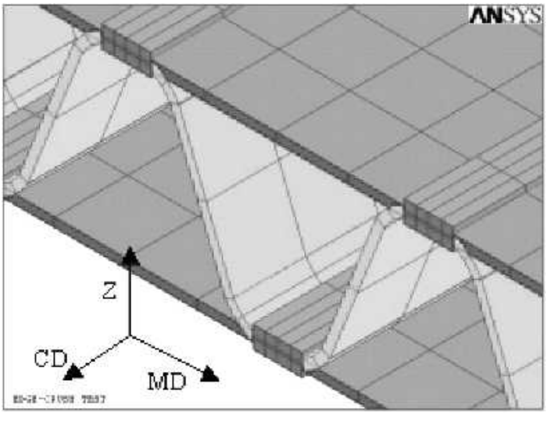

FIG. 2. FE geometry of the corrugated fiberboard.

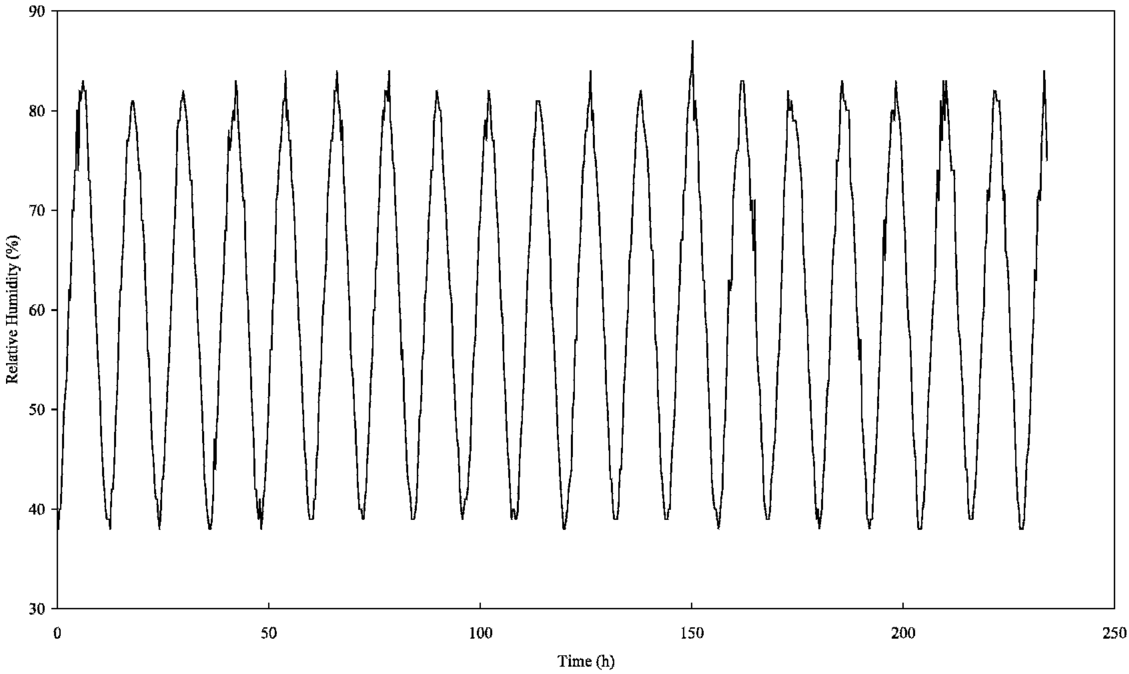

FIG. 3. Actual experimental relative humidity data vs. time used as input (Urbanik 1995).

D_w and the coefficient of moisture expansion α for the liner and the medium materials, was needed to make the finite element analysis possible. Rahman et al. 2002 reported the material properties and moisture coefficients of the components. Table 1 summarizes the values used. These constants were not available in the literature for these materials. Therefore, FEA was used first to derive these constants. The research work by Margot Sehlstedt-Persson (2001) was expanded to obtain the coefficient of moisture diffusion per unit mass for a solid material such as paper using the following equation:

$$D_w = \frac{100x}{A\rho\Delta M}\frac{m}{t} \qquad [m^2/s] \qquad (1)$$

TABLE 1. *Moisture-related properties of corrugated fiberboard components. Radhakrishnan et al. (2000), Rahman et al. 2002.*

D_w	7×10^{-6} m²/s
α (Liner)	5.0×10^{-5}/C
α (Medium)	2.5×10^{-5}/C

where: D_w is the coefficient of moisture diffusivity
α is the coefficient of moisture expansion.

where, D_w = vapor diffusion coefficient [m/s²], m = mass of water transported in time t, [kg], x = sample thickness [m], t = time [s], A = area [m²], ρ = basic density (dry weight, raw volume) [kg/m³], and ΔM = moisture content difference [%]. The above equation was used for the calculation of the diffusion coefficient for the liner and medium board. A basis weight of 0.203×10^{-3} g/m² was used for the liner, and a basis weight of 0.126×10^{-3} g/m² was used for the medium. The above equation utilizes the results data generated from the water vapor steady-state moisture flow measured using diffusion cup experiments as detailed in the work of Sehlstedt-Persson (2001) and Radhakrishnan et al. (2000). The m/t term in the equation is the slope of the flow curve of the mass of water transported as a function of time during the steady-state condition in the diffusion cup experiment.

These constants were used as input in an FE model. The determination of the coefficients of moisture expansion α for liner and medium boards was obtained by constructing a 12-in. by

12-in. plate and giving the material properties of liner and medium one at a time. This was detailed in the study by Rahman et al. 2002. The plate was subjected to uniformly compressive loading. The actual stress-strain diagram for the material was used as an input at 50% RH, and the α coefficient was varied until the stress-strain diagram at 90% RH was reached. The plate was subjected to moisture stress analysis coupled with mechanical stress analysis. In Rahman's study, the coefficient of moisture expansion α caused the fiberboard to expand when the relative humidity conditions changed from 50% RH to 90% RH, resulting in further edge pressure on the fiberboard, thus reducing the board's stiffness and producing a weakening effect. This mechanism was utilized to calibrate the predicted α value to the experimental measurements of the stress-stain curve at 90% RH. In Radhakrishnan et al. (2000), the coefficient of moisture diffusivity D_w was studied for a paperboard stack in a steady-state moisture transport experiment. The experiments were conducted using a Plexiglas diffusion cup at approximately 23°C. Under steady-state conditions, the total water vapor flux through the stack was measured in order to calculate the coefficient of moisture diffusivity. Table 2 in the study summarizes the values obtained for D_w at different %RH. A value of 7.1×10^{-6} m^2/s is very close to the value arrived at by means of Eq. (1) in this finite element analysis for comparable relative humidify conditions.

Moisture transient analysis

The transient analysis was conducted first in order to calculate the relative humidity values throughout the fiberboard in all the layers as a function of time for a duration of 150 h (6.25 days), as shown in Figs. 3 and 4. The second stage of the analysis was to calculate the fiberboard deformation response (hygroexpansion) due to the change in relative humidity values. This analysis provided the deformation vs. time at different relative humidity values. The maximum deformation occurred at the highest level of RH (86%) and the minimum deformation occurred at the lower RH value (38%). The FEA deformation response of the structure is shown in Fig. 5. The amplitude of deformation is about 0.1 mm. This is close to the amplitude value measured in the experiment. This hygroexpansion response is superimposed on the creep response to predict the overall response of the fiberboard.

TABLE 2. *Paper creep constants for short column model.*

Constant number	38% RH	86% RH
C_1	5.11×10^{-30}	6.24×10^{-30}
C_2	4	4
C_3	−0.5882	−0.5882
C_4	−3.85	−3.65

Nonlinear creep analysis

This FE research generates the creep model that describes the behavior of a typical paper material. Urbanik 2002 presented a creep model for paper; however, the equations generated do not conform to the forms of the creep equations available in the finite element program used to perform creep analysis. The creep response is dependent on many factors including: material properties, the rate of loading, the time duration, and the relative humidity conditions. The primary creep equation for the specimen described follows the following form (ANSYS 2002):

$$\varepsilon = \frac{C_1}{C_3 + 1} \sigma^{C_2} e^{-C_4} t^{C_3+1} \tag{2}$$

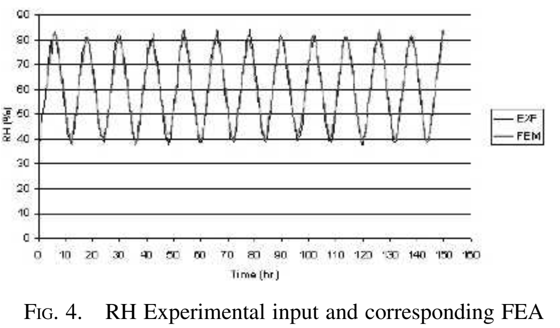

FIG. 4. RH Experimental input and corresponding FEA RH output from moisture diffusion analysis: RH variation was between 38% and 86%.

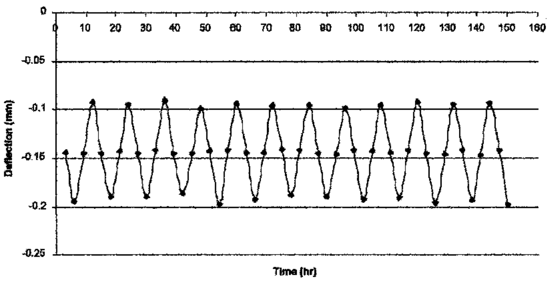

FIG. 5. FEA hygroexpansion response due to RH schedule.

where: C_1, C_2, C_3, C_4 are constants dependent on the material properties of the paper. σ is the compressive stress acting on the edge of the specimen, and *t* is the time in seconds.

The constants in the creep equations are dependent on the above factors. Changes in the relative humidity are reflected in the values of the constant C_4. Equation (3) gives the relationship between the constant C_4 as and the %RH within the range of %RH considered (38%–86%).

Constant C_4 is affected by the relative humidity. The following equation was derived to relate the value of the constant C_4 to the relative humidity by the following equation:

$$C_4 = \left(\frac{RH = 38}{48}\right)(0.2) - 3.85 \qquad (3)$$

The short column finite element model considered was a 2-in. by 2-in. corrugated fiberboard, while the experimental fiberboard was 2 in. wide and 1.5 in. high (a standard edge-crush test specimen). Therefore, a linear defection adjustment for the height was used in order to produce the deflection creep response shown in Fig. 6.

Board hygroexpansion response

An expansion of the 2-in. by 2-in. short column model into the corrugated fiberboard size (12 in. by 12 in.), representing the sideboard of a container box, was possible. The moisture diffusion law was used in the moisture transient analysis to predict the relative humidity values in the interior layers. Figure 7 shows a typical output of the finite element deformation prediction in mm of the fiberboard under compressive

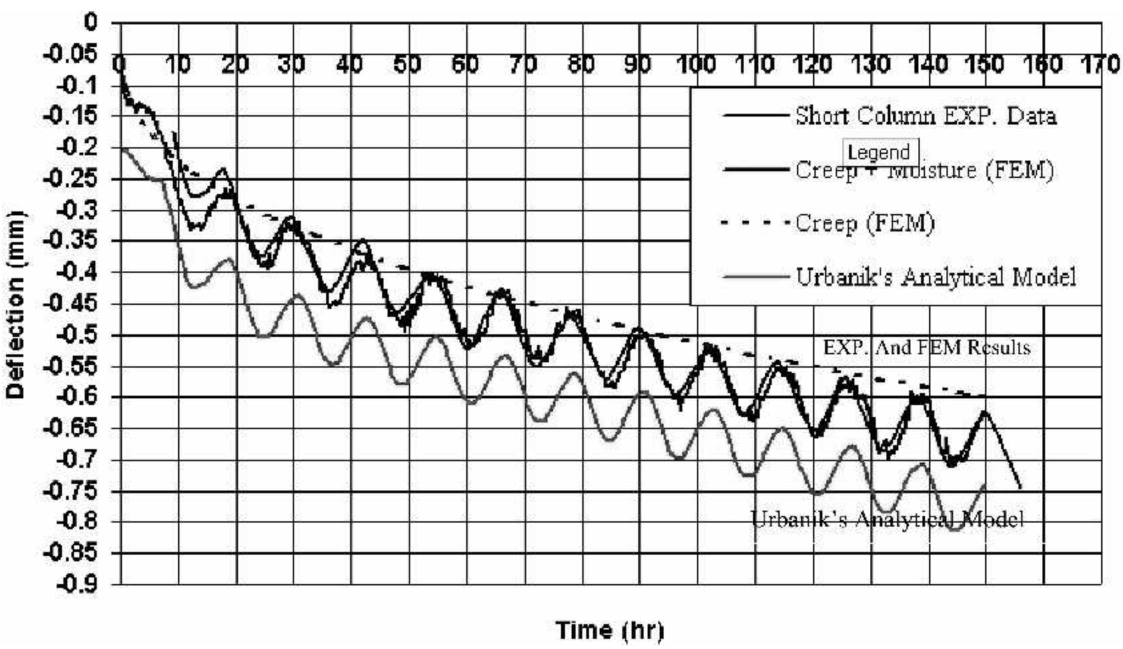

FIG. 6. Corrugated short column deflection due to creep and humidity expansion.

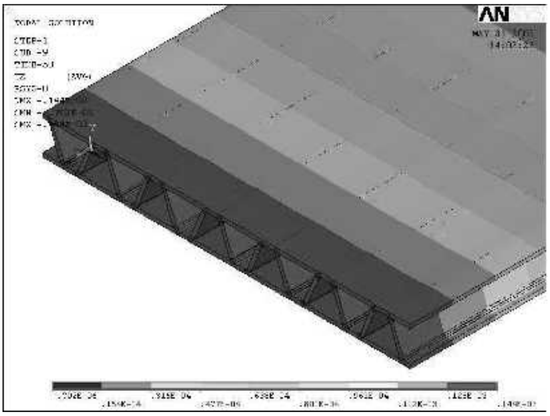

FIG. 7. Moisture deformation due to 86% RH.

loading at the relative humidity condition of 86% RH. The deformation response is predicted for all the humidity conditions ranging from 38% RH to 86% RH. The finite element moisture distribution prediction was compared with the experimental results as shown in Fig. 8.

One calibration was required in the computational model before the results could be compared with the experimental results. This is a logical and justified calibration from a physical point of view. The experimental investigation clearly shows that, for humidity frequency period P = 12 h., the hygroexpansion amplitude was maintained at a certain value; however, when P changed to 24 h., the amplitude dropped by a factor of 2 and when P reached 4 h., the amplitude increased by a factor of 2. The computational model was not designed to capture this physical phenomenon, which is related to time-dependent swelling in the material; therefore, the α value needed to be adjusted in the computational model to account for this change. The FE model results were compared to two independent experiments: one for hygroexpansion prediction presented in Fig. 9, and a second for a combined effect of creep and hygroexpansion shown in Fig. 10. If proper adjustment was performed to the constants of moisture expansion constant α, remarkable agreement was achieved for any variation in the humidity cycle. The humidity frequency period *P* was a major factor in affecting the amplitude of hygroexpansion deformation in the fiberboard as reported in Fig. 7 by Urbanik and Lee 1995 and shown here in Fig. 9. As the frequency increased, the deformation amplitude decreased. While the relationship was nonlinear initially, it can be assumed to be linear after a frequency of 1 cycle/day.

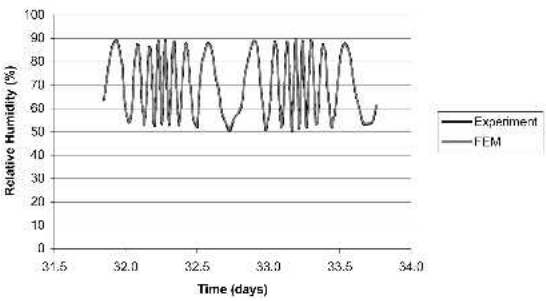

FIG. 8. Relative humidity schedule as measured from experimental data and generated by FEA for a board.

The constant α was calibrated for the short column analysis as shown in Table 1, corresponding to a constant humidity frequency period $P = 12$ h. In Fig. 9, the α value was used for the low frequency and was reduced by a factor of 2 for the high frequency, for the reason mentioned above. In Fig.10 where the humidity frequency period was low and close to $P = 24$ h. the α value was increased by a factor of 2 and the high frequency period $P = 4$ h. was reduced by a factor of 2. This was logical from a physical point of view; the longer time it takes to cycle the humidity, the higher the hygroexpansion amplitude response because the material has ample time to expand fully. For a high humidity frequency, the deformation response was decreased because the material was not allowed enough time to absorb the moisture; therefore, the deformation response was reduced.

Board creep and hygroexpansion

The creep equation above was valid for the expanded board size because it describes the paper material's response in a corrugated fiberboard. The creep equation constants are shown in Table 3 for the 86% RH level. The creep response was superimposed on the hygroexpan-

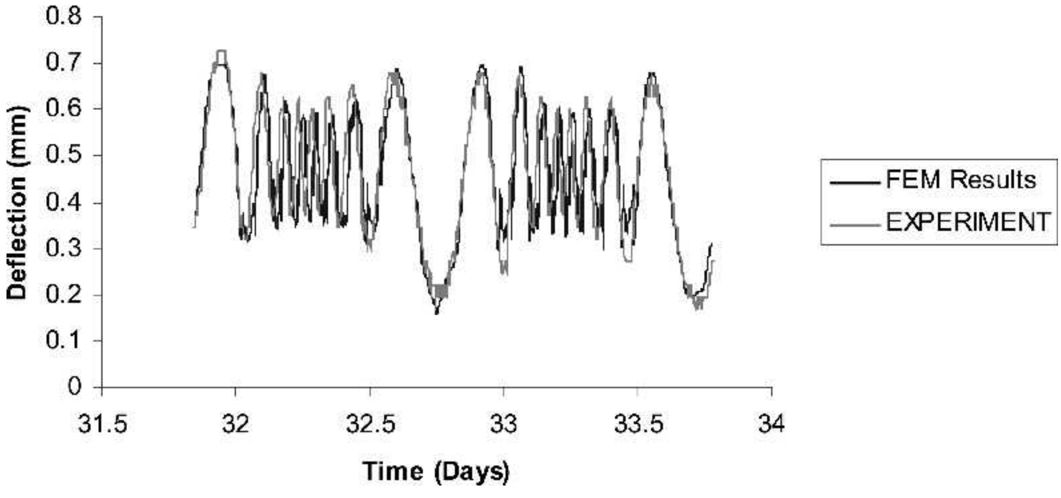

FIG. 9. The deformation response vs. time predicted by the FE model and measured by the experiments for a corrugated board.

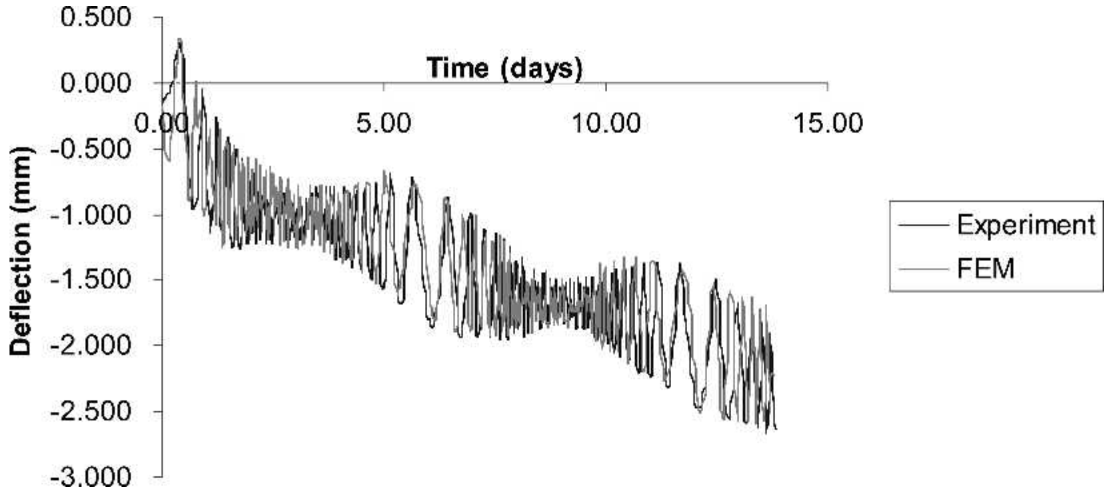

FIG. 10. Creep and hygroexpansion deformation (mm) response as a function of time (days).

TABLE 3. *Paper creep constants for corrugated board (86% RH).*

C_1	3.26×10^{-30}
C_2	4
C_3	−0.5882
C_4	−4.3

sion response for a 12-in. by 12-in. board, and the results are shown in Fig. 10.

For calibration purposes, a full size board (12-in. by 12-in.) model was studied for the moisture transient analyses and hygroexpansion response for comparison with a scaled 2-in. by 2-in. model. The results were the same, suggesting that this type of analysis was not size-sensitive. The deformation response was computed at a single relative humidity value. Performing the analyses of the large board for the full humidity schedule was beyond the computational capability of this research and was not practical since each analysis needs approximately 2 days on a Pentium III PC machine. A typical humidity schedule can consist of thousands of data points.

However on the smaller model, repeated analyses can be programmed to scan through all the humidity points with relative ease without compromising accuracy. Figure 11 shows the full board size model. The moisture diffusion and creep analysis on such a model can be performed without much difficulty. However, when performing the nonlinear transient analyses for hygroexpansion, major numerical difficulty arises stemming from computational crunching time. Luckily, the results for such analyses were not size-dependent on the corrugated fiberboard model, which enabled the results to be performed on a smaller size model without compromising accuracy.

DISCUSSION AND CONCLUSIONS

Figure 6, for a constant frequency period, and Fig. 10, for a variable frequency period, show the total deformation response from the finite element analysis compared to the experimental data for the combined effect of humidity expansion and creep deformation on the fiberboard. Very close correlation was observed between the FE results and the experimental results. The adjustments of the constants in Eq. (2) allowed for calibrating the FEA for different paper types and moisture conditions. The transient moisture analysis, which was a dynamic nonlinear procedure, was time-consuming, and for a large model can present computational limitations. The nonlinear creep analysis, however, was more rapid and can easily be performed for a variety of paper constants.

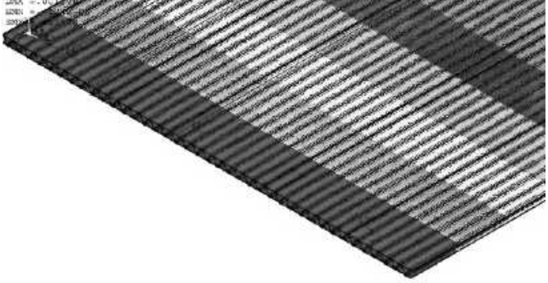

FIG. 11. Full-size FE model of a corrugated (12-in. by 12-in.) for moisture transit analysis.

The development of an FE model to study the moisture expansion and the creep response by establishing a viable creep model for a paper material opens the possibility of considering actual weather data as an input to the model. This can predict the response of a corrugated container box in the field. The 3D FE developed here can predict the behavior of a typical corrugated fiberboard with remarkable accuracy for any variation in the humidity cycle. The model accounts for the combined effect of hygroexpansion and creep. This provides the promise of analyzing the response of a container box to real weather data under service conditions. The amplitude of the hygroexpansion was dependent on the relative humidity frequency. The higher the frequency, the lower the expansion response.

In summary, the following conclusions can be drawn:

1. The FE model and analysis can replicate the hygroexpansion experimental response and show that the amplitude of expansion varies with RH frequency as in the actual experiments.
2. The FE analysis can be calibrated for creep calculation based on known experimental constants for the fiberboard material, which provide the four constants needed in the constitutive creep law. These constants are reported in Tables 2 and 3 for the fiberboard analyzed in this research. Researchers can prepare a library of tables for different types of fiberboard and components to cover a wide range of commercial materials to be used as input in the computational model.
3. To have a comprehensive model, we will need to calibrate the paper data (constants) as an input to be obtained at various RH values to determine the constants of the creep law; then the computation of the response of the corrugated structure will be possible as the paper laws change as a function of instantaneous moisture conditions and stress. However, this requires a more specialized nonlinear code to be incorporated into the ANSYS

code or independent of it. In the ANSYS creep solution, routine limitations exist in the requirement that the creep law constants do not update automatically corresponding to changing real conditions of the paper components.

4. The sequence of the ANSYS solution routine performs first, the transient moisture transfer (analogous to heat transfer), then static moisture stress and strain expansion analysis was done, and finally the static creep analysis was superimposed onto the previous response. The ANSYS creep routine was limited to static analysis and does not support transient dynamic analysis, which makes performing the three types of analysis in one ANSYS session not possible.
5. The high nonlinearity nature of the problem: being nonlinear in term of transient analysis, nonlinear in creep laws, and nonlinear in material properties, provides a real challenge in this type of analysis. Creep laws in ANSYS are lacking transient analysis capability for changing moisture conditions of paper material.
6. With an improvement on the adaptability of the creep laws in ANSYS, this analysis can be used to investigate the response of real weather phenomena.

The analysis for using diffusion laws yields accurate hygroexpansion response for a complicated RH input; therefore, with better creep laws in a commercial software such as ANSYS, we can better predict how corrugated fiberboard will creep to real weather data inputs.

ACKNOWLEDGMENT

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