AN OPTIMIZATION TECHNIQUE TO DETERMINE RED OAK SURFACE AND INTERNAL MOISTURE TRANSFER COEFFICIENTS DURING DRYING

William T. Simpson

Research Forest
Products Technologist

and

Jen Y. Liu

Research General Engineer
USDA Forest Service[1]
Forest Products Laboratory
One Gifford Pinchot Drive
Madison, WI 53705-2398

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ABSTRACT

Lumber drying involves moisture transfer from the interior of the board to the surface, then from the surface to the surrounding air. These mechanisms can be characterized by internal and surface moisture transfer coefficients. This paper describes a least squares method, used in conjunction with standard mathematics of diffusion analysis, to determine these coefficients, resulting in good agreement between calculated and experimental drying curves.

Keywords: Kiln drying, diffusion, modeling.

INTRODUCTION

Lumber drying involves two types of moisture transfer. Moisture must move from the interior of the wood to the surface, then from the wood surface to the surrounding air. Either of these two types of transfer may control drying. In initial drying of a permeable species, the wood surface is wet and is supplied with moisture from the interior at a rate greater than, or nearly as great as, the evaporation rate from the surface. In this case, the surface evaporation rate, characterized by the surface transfer coefficient, controls or is a major factor in the rate of drying. As drying progresses, the supply of moisture from the interior can no longer keep pace with the rate of surface evaporation. Then, the internal transfer, characterized by the internal moisture transfer coefficient, controls the rate of drying.

These two types of moisture transfer can be dealt with mathematically for the purpose of developing equations to predict drying rate of individual boards, and ultimately into simulations that can predict the drying rate of full-width lumber stacks. One mathematical framework that might be used is the mathematics of diffusion. The mathematics of diffusion is usually thought to be restricted to drying below the fiber saturation point (fsp), where only water vapor and bound water are involved. This restricts its applicability in lumber drying, where initial moisture content is usually greater than the fsp; free water is also involved. However, there is precedent and some rationale for using the framework of the mathemat-

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ics of diffusion for lumber drying from above the fsp.

This paper describes a method to determine the surface and internal moisture transfer coefficients of red oak and evaluates the ability of the mathematics of diffusion to characterize drying from above the fsp.

BACKGROUND

Moisture transfer coefficients

The mathematics of diffusion, under isothermal conditions, is based on Fick's second law (Crank 1975):

$$\partial m/\partial t = \partial (D\partial m/\partial x)/\partial x \qquad (1)$$

where m is moisture content; t is time; x is the thickness direction; and D is the diffusion or internal transfer coefficient (cm²/s). The boundary condition that describes surface evaporation is

$$\partial m/\partial x = (S/D)(\mathrm{m}_e - m_s) \qquad (2)$$

where S is the surface emission coefficient (cm²/s); m_e is the equilibrium moisture content (EMC) the surface will eventually attain; and m_s is the surface moisture content at any time. Finite difference equations for solving Eq. (1) with the Eq. (2) boundary condition and a D that is either constant or varies with moisture content are given in Appendix A.

Several authors have developed methods to determine the surface emission coefficient. Choong and Skaar (1969, 1972) developed an approximate method based on drying data for two different thicknesses. Rosen (1978) applied this method and showed that S increased with air velocity. Avramidis and Siau (1987) investigated the effect of thickness, moisture content, and temperature on S. Liu (1989) modified the method of Choong and Skaar so that drying data from only one thickness was necssary. In all these evaluations, the calculations were based on drying times to the point where half the total moisture content change had occurred. Chen et al. (1995) developed an optimization method that considers data for all drying times. It searches for the optimum D and S pair that minimizes the sum of squares of deviations between experimental and calculated data. The solution they used for Eq. (1) assumed a D that was constant with moisture content.

Application of diffusion analysis above fsp

Isothermal movement of moisture in wood below the fsp (water vapor and bound water) is by diffusion—the random movement of water molecules in response to a moisture gradient. Movement is from regions of high to that of low moisture concentration. Movement of moisture in wood above the fsp (free water) is thought to be by capillary action. Movement depends on the relative sizes of the orifices leading into cell lumens and the presence or absence of air bubbles in the water in the cell lumens (Siau 1984). Capillary action allows for the possibility of cells with small orifices deep in the interior of the wood to lose their lumen moisture before cells closer to the surface lose their moisture, that is, counter to the overall moisture gradient.

Lumber drying is a combination of water vapor, bound water, and free water movement. A few woods may exhibit a constant rate drying period during initial drying. Most woods immediately exhibit a falling moisture rate drying period, which means that the surface fibers dry very quickly to below the fsp. Diffusion through these surface fibers may then be the controlling factor in drying, with capillary flow of little or no importance.

We can make two rationales for applying the mathematics of diffusion to drying from above the fsp. One is based on the acceptance of the mathematics at face value, which merely requires the assumption that the drying rate is proportional to the moisture gradient. No assumptions are made of the mechanism of drying, and we refer to an internal moisture transfer coefficient instead of a diffusion coefficient. The other rationale is that drying is controlled by and therefore can be characterized by diffusion through the surface fibers that are below the fsp rather than by capillary movement in the interior where moisture content is above the fsp.

A number of references in the literature relate to application of diffusion analysis to wood drying from above the fsp. Tuttle (1925) applied a solution to Eq. (1) with constant D to successfully describe the drying of Sitka spruce from an average moisture content of 50 to 8%. Hougen et al. (1940) stated that diffusion analysis is successful for calculating the average moisture content of wood at any time during drying from above the fsp, but that it can not accurately calculate moisture content gradients. They postulated that the movement of free water by capillarity is so restricted by the small openings and pits interconnecting wood cells that the forces of capillarity play only a minor role in movement of free water across the fibers. They believed that free water remains stored in cell lumens until the moisture content in the cell walls drops below the fsp by evaporation and diffusion. Free water is then absorbed by the cell walls and becomes bound water. Hart and Darwin (1971) also proposed this mechanism as part of the explanation for excessively slow drying rates in white oak. Bateman et al. (1939), Stamm (1964), and Hart (1964) have all stated that diffusion controls drying throughout the entire range of moisture content. In his drying simulation, Hart (1983) utilized diffusion coefficients to describe drying over the entire range of moisture content. Others that have applied diffusion analysis for drying from above the fsp are Nadler et al. (1985), Cunningham et al. (1989), Kouali and Vergnaud (1991), Mounji et al. (1991), and Chen et al. (1996). Hunter (1995) extended the concept of equilibrium moisture content up to complete saturation. He considered the driving force for water above the fsp to be capillary pressure, which can be related to relative humidity through the Kelvin equation. One result of Hunter's concept is that the diffusion coefficient is constant above the fsp.

EXPERIMENTAL

Freshly sawn boards from the northern red oak group were obtained from Wisconsin. The boards were processed into surfaced specimens 102 by 305 by 29 mm. Specimens were wrapped in two layers of plastic bags and stored in a freezer until ready for use. Before each test run, the edges and ends of each specimen were painted with two coats of a heavily pigmented aluminum paste-exterior varnish mixture.

Drying was done in a wind tunnel that was attached to a generator unit for temperature and relative humidity control. The wind tunnel was equipped with a blower whose speed could be adjusted for variable air velocity. A perforated plate was installed just upstream from the specimens to help create uniform air flow.

Experimental drying conditions were 43°C at 84% relative humidity (16.2% EMC). These conditions represent early kiln schedule drying conditions for red oak. There were two air velocities: 1.5 and 5.1 m/s. Three replicate runs of six specimens each were conducted for each air velocity. Specimens were weighed periodically during drying: at short time intervals early in drying when drying rate was high and at increasing time intervals as drying progressed and drying rate decreased.

ANALYTICAL

The basic analytical procedure was to determine the finite difference solution to Eq. (1) that results in the closest match between experimental and calculated moisture content–time data points. Solution of the finite difference formulas requires both D and S values (Appendix A). In a previous study, Simpson (1993) showed that the diffusion coefficient for red oak can be represented by

$$D = A\exp(-5280/T)\exp(BM/100) \quad (3)$$

where T is temperature in Kelvin; M is percent moisture content; and A and B are experimentally determined coefficients. The increase in D with moisture content expressed by Eq. (3) was used only up to 30% moisture content, and above that was kept constant at the 30% value. There are precedent and rationale for doing this. Nadler et al. (1985), Mounji et al.

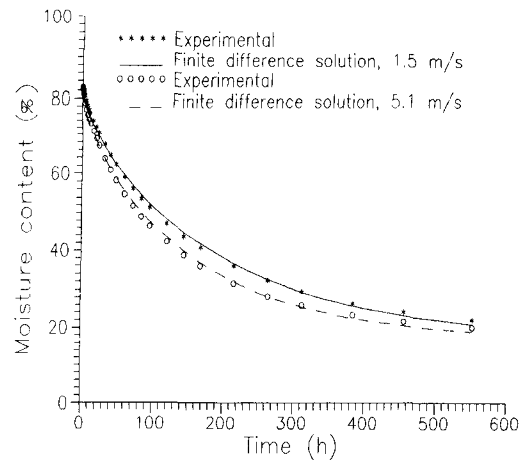

FIG. 1. Experimental and finite difference drying curves for 29-mm-thick red oak.

(1991), and Chen et al. (1996) applied a constant D above the fsp. Hunter's (1995) concept of extending EMC to saturation results in a constant D above fsp, and Yokota (1959) experimentally found that D increased with moisture content up to about the fsp, then leveled to a constant value.

The values for A, B, and S in the finite difference solutions were determined by a technique where the optimum values were the ones that minimized the sum of squares of the differences between calculated and experimental moisture content values at various times during drying. The sums of squares were minimized by a technique that sequentially adjusted each of the three coefficients until the sum of squares was minimized. This procedure was iterated until changes in the third significant digit of the coefficients resulted in no further reduction in the sum of squares. Initial estimates of the coefficients are required to initiate the technique and were available for A and B by Simpson (1993) and for S by Avramidis and Siau (1987). The calculated average moisture content values were determined by numerical integration of the finite difference solutions, which gave the moisture content gradient at various times during drying. The experimental moisture content values were the averages of the 18 individual specimens at the same times during drying. However, the experimental drying times were not the same for all specimens and needed to be interpolated to a common set of times for comparisons. A convenient way to do this interpolation was to apply nonlinear regression to fit an empirical equation to the experimental moisture content–time curves and calculate moisture content at the desired times. The method for doing this is given in Appendix B. The average initial moisture content was 82.5%, and this was taken as the common starting point for all comparisons.

TABLE 1. *Values of* A, B *(Eq. 3), and* S *(Eq. 2) that minimize the sum of squared deviations between experimental and calculated moisture content values.*

Coefficient	1.5 m/s	5.1 m/s
A (cm^2/s)	12.9	14.8
B	2.32	2.48
S (cm/s)	0.927×10^{-5}	1.51×10^{-5}

RESULTS AND DISCUSSION

Experimental

The moisture content–time curves are shown in Fig 1. It includes experimental values as well as average values calculated by numerical integration of the finite difference solutions based on the values of A, B, and S that minimized the sum of the squared deviations between experimental and calculated values. These values are shown in Table 1. The values of A and B should be independent of air velocity, but they are not the same for 1.5 and 5.1 m/s. A varies by about 15% and B about 7%. The reason for this difference can probably be explained by the fact that different specimens were used in the two air velocity tests. Considering the variability of wood, differences of 7 and 14% do not seem unreasonable.

The value of S was greater at the higher air velocity, which is what we would expect—that the surface transfer coefficient increases with air velocity. Rosen (1978), using different analysis methods, also found that S increased

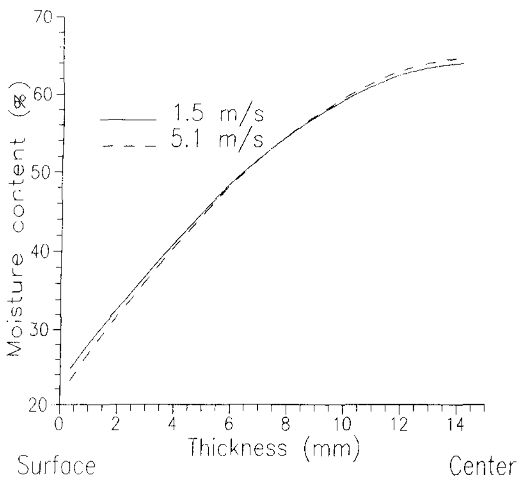

FIG. 2. Effect of air velocity on calculated moisture content gradient at 50 percent average moisture content.

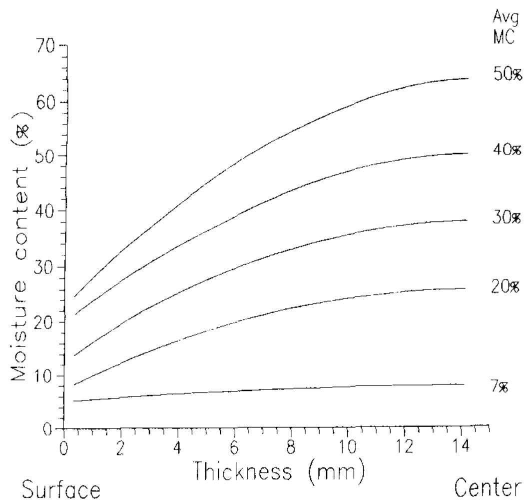

FIG. 3. Calculated moisture content gradients at 1.5 m/s air velocity.

with air velocity. He worked with black walnut in adsorption from 6% moisture content to 97% relative humidity at 25°C. For air velocities of 1 and 3.8 m/s, the resulting values of S were 0.38 and 0.65×10^{-5} cm/s. Avramidis and Siau (1987) determined S for western white pine in adsorption from oven-dry to various relative humidities and temperatures. At 45°C, 80% relative humidity, and an air velocity of 2.5 m/s, their result for S was approximately 1.1×10^{-5} cm/s. The values determined by Avramidis and Siau compare favorably with the results of our study, i.e., $S = 0.927 \times 10^{-5}$ cm/s at 1.5 m/s and $S = 1.5 \times 10^{-5}$ cm/s at 5.1 m/s. The values determined by Rosen are less than those determined in our study, but the reason for the difference is not known. Avramidis and Siau found that S decreased as temperature decreased. They also found that S decreased with increasing surface EMC (m_e in Eq. (2)). Only one surface EMC was employed in our study; therefore, we are not able to compare results.

Figure 1 shows that the mathematics of diffusion can be used successfully to predict average moisture content as a function of time even when drying is from above the fsp. Figures 2 and 3 show moisture content gradients predicted by the finite difference solutions. Figure 2 is the effect of air velocity on the gradient when the average moisture content is 50%. The solution predicts that the higher air velocity results in a slightly steeper moisture gradient. Figure 3 shows moisture content gradients at average moisture content values of 50, 40, 30, 20, and 7% for the air velocity of 1.5 m/s.

CONCLUSIONS

In this study, the mathematics of diffusion were successfully used as a framework to determine the surface and internal moisture transfer coefficients for red oak. This was done for initial moisture content above the fiber saturation point, and finite difference formulas and least squares techniques were used to find the optimum values of the coefficients.

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APPENDIX A: FINITE DIFFERENCE SOLUTIONS

The finite difference solutions to Eqs. (1) and (2) consist of three parts. The indexing system is $m_{i,j}$, where i is x (thickness) increment, and j is t (time) increment:

1. Surface moisture content m_o (Crank 1975)

$$m_{o,j+1} = m_{o,j} + 2R(D_{o+0.5}(m_{1,j} - m_{o,j}) - \\ - S\Delta x(m_{o,j} - m_e))$$

2. Interior moisture content m_i

$$m_{i,j+1} = m_{i,j} + R(D_{i+0.5,j}(m_{i+1,j} - m_{i,j}) - \\ - D_{i-0.5,j}(m_{i,j} - m_{i-1,j}))$$

3. Center moisture content m_n

$$m_{n,j+1} = m_{n,j} + 2RD_{n-0.5,j}(m_{n-1,j} - m_{n,j})$$

where $R = \Delta t/(\Delta x)^2$

In the preceding parts 1 to 3, D is calculated from Eq. (3) in the text using the appropriate moisture content.

The criterion for accurate and stable results is based on the Euler method of solution (Myers 1971), which requires that

$$D\Delta t/(\Delta x)^2 < 0.5$$

In this solution, the half-thickness was divided into 21 increments, so $\Delta x = 0.29$ mm and $\Delta t = 30$ s. Thus, $D\Delta t/(\Delta x^2) < 0.5$, assuming a maximum value for D of 4×10^{-6} cm²/s.

APPENDIX B: EMPIRICAL EQUATION FOR NONLINEAR REGRESSION FIT OF MOISTURE CONTENT–TIME DATA

The empirical equation is based on the generalized logistic equation (DeWitt 1943), which is

$$y = K/(1 + b\exp(F(x)))$$

where K and b are coefficients and $F(x)$ is some function of x.

Wilke (1944) applied a specific form of the logistic equation to drying data and found that it represented the data well. The form Wilke used can be written as

$$E = 1/(1 + a\exp(b\ln t + c(\ln t)^3))$$

where

E = $(m-m_e)/(m_i-m_e)$,
m = moisture content at any time $t(h)$,
m_e = equilibrium moisture content,
m_i = initial moisture content, and
a,b,c = coefficients determined by nonlinear regression.

Examination of Wilke's equation suggests that addition of a squared term for $\ln t$ could make it more powerful in its curve-fitting ability. This form is

$$E = 1/(1+a\exp(b\ln t + c(\ln t)^2 + d(\ln t)^3))$$

The regression coefficients were the following:

Coefficient	1.5 m/s	5.1 m/s
a	0.00188	0.00927
b	2.42	1.62
c	−0.410	−0.258
d	0.0386	0.0300