

# A NEW MODEL TO PREDICT THE LOAD-SLIP RELATIONSHIP OF BOLTED CONNECTIONS IN TIMBER

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## ABSTRACT

The development of a new approach to predicting the load-displacement interaction of bolted wood connections is described.

The European Yield Theory for bolted wood connections has gained wide acceptance in recent years attributed to its closed form, simplicity, and accuracy. But the model does not relate capacity or any other loading state to joint slip. A method to determine deflection related properties of bolted joints in timber with a single equation would be an important contribution to the field of timber engineering. The model would simplify the analysis of large structures including schools, gymnasiums, bridges, and light-industrial buildings where wood could be utilized because of its economy, high-strength-to-weight and stiffness-to-weight ratios.

Based on yield characteristics predicted by the European Yield Theory, the joint was abstracted where the dowel rotates about the plastic hinge under a warped force-deformation plane. Through subsequent simple integration along the dowel length, a closed form solution could be developed. The analysis of a two-member connection assembled with a single bolt indicated that the model closely predicts the load-displacement relationship.

**Keywords:** Bolted connections, model, load-slip, timber.

## PROBLEM OVERVIEW

Single bolted joints in wood under static loading have attracted much attention in the past and the subject has been extensively researched, largely because it constitutes the simplest loading condition and joint configuration (see McLain and Thangjitham 1983). Many studies provided empirical formulations that attempt to predict the behavior of the entire connection (e.g., Trayer 1932; Antonides et al. 1980; Gromala 1985; Humphrey and Ostman 1989; Wilkinson 1993). But purely empirical models contribute little to the overall comprehension of the complicated interactions of the many parameters and are frequently cumbersome to use. While experimen-

tal testing is necessary to provide information needed to fully comprehend connection response, it is not practical to test all of the possible geometries, materials, and construction techniques that may be used in wood structures. Hence, it is necessary to model subassemblies and move toward modeling complete structures. These models should be validated with limited tests (e.g., Falk and Moody 1989).

Modeling is a complementary part to experimental analysis. It not only aids in understanding, but also allows for experimental studies to be more specialized. Numerical modeling, such as the finite element technique, has become powerful enough to provide approximate solutions with reasonable accuracy of complex problems, such as bolted joints in

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## Nomenclature.

Symbol	Description	Unit
$\alpha$	Angle of bolt rotation	deg
$\beta$	Adjustment factor	
$\sigma_{\max}$	Bending stress or bending yield strength	N/mm <sup>2</sup>
$b$	Distance plastic hinge and shear plane as determined by EYT	mm
$d$	Fastener diameter	mm
$k$	Initial stiffness per unit length of dowel with diameter $d$	N/mm <sup>2</sup>
$k_b$	Initial slope of moment function	N-mm/deg
$p$	Reaction force of foundation per unit length of dowel with diameter $d$	N/mm
$r$	$d/2$	mm
$t, L$	Member thickness	mm
$x$	Displacement	mm
$F$	Force acting in shear plane to overcome bending moment of dowel	N
$F_e$	Embedment strength	N/mm <sup>2</sup>
$F_{\text{resultant}}$	Force resultant used to derive maximum moment	N
$M_1$	Slope of the asymptote of moment function	N-mm/deg
$M_0$	y-intercept of moment function	N-mm
$M_b$	Bending moment of fastener	N-mm
$M_{el}$	Elastic bending moment	N-mm
$M_{pl}$	Plastic bending moment	N-mm
$M_{sp}$	Bending moment in shear plane	N-mm
$P$	Total reaction force of foundation	N
$P_1$	Slope of the asymptote per unit length of dowel with diameter $d$	N/mm <sup>2</sup>
$P_0$	y-intercept per unit length of dowel with diameter $d$	N/mm
$P_v$	Maximum joint load	N
$P_{\text{joint}}$	Total joint load	N
$R$	Displacement	mm
$R_b$	Force resultant	N
$S$	Total joint displacement (slip)	mm
$X$	Displacement	mm
$Z$	Distance	mm

timber (see Patton-Mallory 1996). However, major deficiencies of the finite element approach to modeling wood assemblies include its limited use for practitioners and its reliance on the full quantification of all basic material properties. Attributed to the anisotropic properties of wood and its natural variability, the constitutive relationship is only approximated, and not all basic elastic properties and their interaction that are reported in the literature are backed by comprehensive statistical analysis. Some properties, such as Poisson ratios, are difficult to measure.

Analytical closed-form models based on a mechanics approach contribute to understanding the factors influencing connection behavior and are useful for designers, since they can easily be solved with contemporary spread-

sheet software. Despite an extensive amount of published work, few, if any, analytical, closed-form models that predict the load-slip interaction up to capacity (maximum load) of bolted wood connections exist to date.

A closed-form solution that has gained increasing acceptance in recent years is the European Yield Theory (EYT) (Johansen 1949; AF&PA 2000). The model laid the foundation for a sound engineering approach to the design of wood connections. It was adopted by building codes and in design specifications in Australia, Canada, Europe, New Zealand, and the United States. The model predicts lateral strength and yield mode of a connection containing a single dowel-type fastener, such as a bolt or a nail, based on the bending resistance of the fastener and the crushing strength of

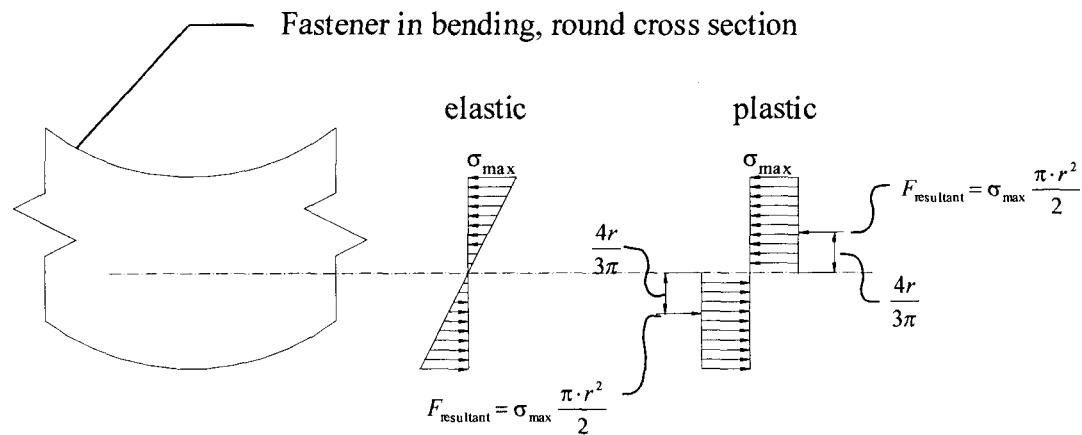


FIG. 1. Illustration of elastic and plastic bending capacity of the fastener.

wood. Compared with other analytical formulations, the EYT entails closed-form and rather simple equations. Considering the simplicity and underlying assumptions, it is surprisingly accurate. Despite its popularity, the EYT is somewhat incomplete as it does not predict deformations attributed to a given loading state. Therefore, displacement-related properties, such as stiffness, ductility, or energy dissipation, cannot be determined using the yield model.

To ensure continued high reliability of wood construction, improved knowledge of the expected response of the structure is required, which opens the need for design procedures to be based on engineering theory. In view of improving the understanding of bolted connections, and helping wood remain a competitive construction material, any closed-form analytical model that furnishes designers with accurate information on load-slip interaction would be beneficial.

This paper presents the results of an ongoing study whose principal objective is to formulate a closed-form analytical model that, while simple in application, is capable of predicting the load-slip deformation of a single bolt connection with wood members. At this stage only joints in single shear (two members connected by a bolt), yielding in Mode IV (development of two plastic hinges as described by the National Design Specifications (NDS)

(AF&PA 1997)), are considered. The project utilized data obtained by Gutshall (1994) and Brinkman (1996) to validate the model.

#### KEY PARAMETERS INFLUENCING CONNECTION STRENGTH

In wood structures, connection strength is essentially a function of fastener bending yield strength, embedment strength, and fastener aspect ratio.

##### *Fastener bending yield strength*

Slender fasteners (i.e., fasteners with high aspect ratios (length/diameter)) typically bend when loaded in shear beyond the proportional limit. The fastener's plastic bending stress is referred to as the bending yield strength. It substantially influences joint capacity containing slender fasteners; whereas it has no effect on joints with rigid dowels, since the wood yields and fails before inelastic bending of the bolt takes place. Therefore, connection strength and yield mode are directly related to wood crushing and fastener bending yield strength. The idea was first introduced by Johansen (1949), who linked the elastic bending capacity of the fastener to joint strength (Eq. 1). In 1957, Meyer employed the full-plastic bending capacity to determine joint strength (Eq. 2 and Fig. 1).

$$M_{el} = \sigma_{\max, \text{elastic}} \frac{\pi \cdot d^3}{32}$$

Elastic bending moment (1)

$$M_{pl} = \sigma_{\max, \text{plastic}} \frac{2\pi \cdot r^2}{2} \frac{4r}{3\pi} = \sigma_{\max} \frac{d^3}{6}$$

Plastic bending moment (2)

Fastener yield strength is determined experimentally. In the United States, the fastener is bent during three-point loading and the load at a deflection equal to 5% fastener diameter offset is converted into bending yield strength by rearranging Eq. (2) and solving for  $\sigma_{\max}$ . The main disadvantage of the 5% diameter offset approach is that it uses neither proportional limit nor capacity as reference points. Moreover, attributed to joint settlement effects as loading commences, the fitting of an initial stiffness line is frequently ambiguous and judgmental. A capacity-based approach, on the other hand, exercised with appropriate adjustment factors to account for variability, may lead to optimum material utilization.

#### *Dowel-bearing strength (embedment strength)*

Dowel-bearing strength is a material property, determined experimentally, that describes a limit-state stress in the wood around a pin-loaded hole in compression. Dowel-bearing strength is considerably influenced by loading direction (parallel or perpendicular to grain) not only because of a different modulus of elasticity of wood when compressed parallel or perpendicular to grain, but also on account of a different failure mode. The connections discussed in this paper are considered to be loaded parallel to the grain, which is true for most bolted joints applied in timber construction.

#### *Fastener aspect ratio*

In connection design, fastener aspect ratio is defined as member thickness (L) divided by fastener diameter (d). Fastener aspect ratio determines to what degree fastener yield strength

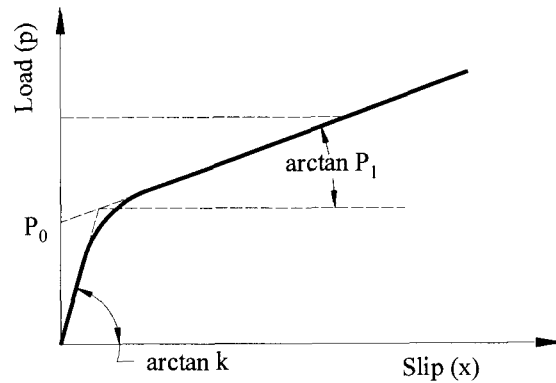


FIG. 2. Foundation model (Foschi 1974).

and embedment strength influence connection behavior. It controls the yield mode of the connection. Joints containing low aspect ratio fasteners tend to exhibit brittle failure because of wood splitting, whereas slender fasteners bend and develop plastic hinges. At decreasing aspect ratios, more wood yielding and less fastener bending are involved.

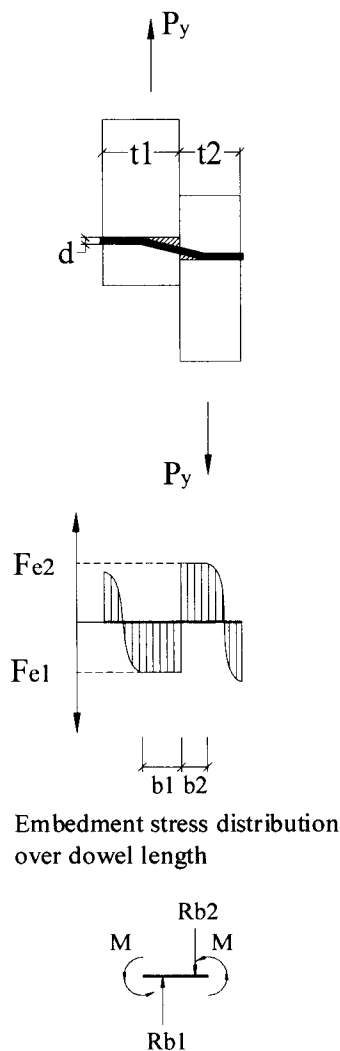
#### *Friction*

Bolted connections, with nuts drawn tight, develop significant friction between the adjacent surfaces, which increases the capacity of the joint. Nonetheless, due to the rheological characteristics of wood, the compression force achieved by tightening the nuts is quickly lost. In addition, possible moisture related dimensional changes of the wood members turn surface friction between members into a rather variable property. As a result, researchers have tested connections with bolts drawn fingertight or left loose (Trayer 1932; Johansen 1949). Considerable friction may, however, develop past the proportional limit when the bolts bend and draw the members against each other.

#### LOAD-DEFORMATION INTERACTION

A dowel-type fastener embedded in wood and laterally loaded within the elastic range is similar to a beam on an elastic foundation. Studies of beams on elastic foundations have been reported in the literature for more than a century. Early interest in this subject was

## Yield Mode IV



## Moments and force resultants

FIG. 3. Mechanics of Yield Mode IV.

sparked by the railroad industry that studied rail response. General analyses have focused on beams on a linear-elastic Winkler-type foundation (Hetényi 1946). In a Winkler-type foundation, the reaction forces are proportional to beam deflection at any point and there is no transfer of shear forces. The application of

the theory of beams on elastic foundation to connections in wood has been studied by several researchers. Early work employed the Winkler foundation model in an attempt to fit a linear-elastic load-slip relationship (Kuenzi 1955; Noren 1962; Wilkinson 1971, 1972). But wood is not linear-elastic in compression and the application of the Winkler foundation model gives moderately accurate predictions at best. Furthermore, modern design methodologies are shifting from using linear-elastic approximations to nonlinear elastic-plastic analyses to assure more efficient material utilization.

Foschi (1974) used the approach of a beam on a nonlinear-elastic foundation to derive a finite element model capable of predicting the load-slip function of laterally loaded nails in wood. The characteristics of the foundation are expressed in Fig. 2 and the following equation:

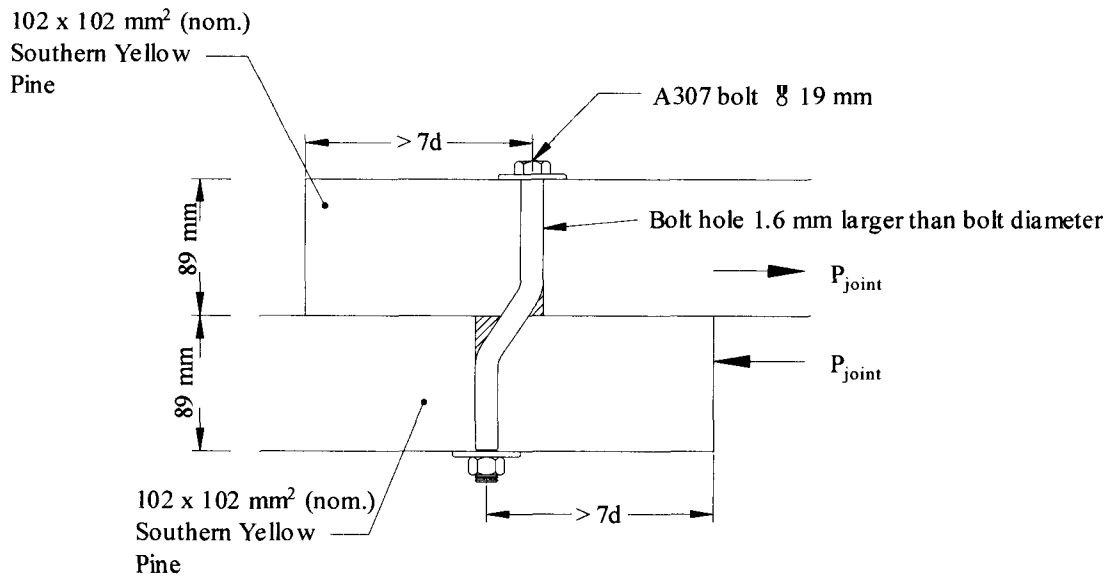
$$p = (P_0 + P_1 x)(1 - e^{(-k \cdot x)/P_0}) \quad (3)$$

For perfect yield (wood crushing at constant load),  $P_1 = 0$ , and Eq. (3) provides the load-slip relationship reported seven decades ago by Teichmann and Borkmann (1930 and 1931). The constants  $k$ ,  $P_1$  and  $P_0$  may be determined from nonlinear least-squares fitting of experimental data obtained by embedment tests. Foschi's model has been used by many researchers (Dolan 1989; White 1995; Blass 1994; Frenette 1997) as input for comprehensive finite element analyses and empirical joint models.

## THE EUROPEAN YIELD MODEL

The EYT utilizes a beam on a *plastic* foundation approach by assuming that, at capacity, wood crushing underneath the fastener is so advanced that the reaction force is uniformly distributed along the fastener and constant relative to beam deflection. This assumption greatly simplifies the problem of predicting capacity for connections containing slender fasteners (high aspect ratios). The value of the EYT would be significantly increased, how-

## Mode IV Yield



## Free Body Diagram of Half Joint

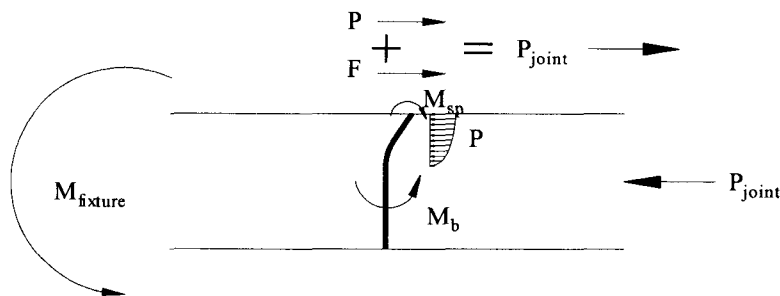


FIG. 4. Analyzed and tested connection with 4×4/4×4 side members (southern yellow pine (*Pinus Spp*)), with load applied parallel to grain and end distance in excess of 7 times bolt diameter.<sup>1</sup>

ever, if capacity or any other loading state could be related to connection slip and connection behavior prior to maximum load could be accounted for. Aune and Patton-Mallory

<sup>1</sup> Note that the moment that is caused by an offset in  $P_{joint}$  is small compared to the shear force and is compensated for by the test fixture ( $M_{fixture}$ ) during testing or joints at the end of the members in construction, respectively.

(1986), McLain and Tangjitham (1983), and Soltis et al. (1986) established the applicability of the EYT to nails and bolts in wood, leading to an adoption of the yield equations in the 1991 edition of the National Design Specification (NDS) (AF&PA 1997). Unlike the model presented in this paper, the displacement models that were developed as part of

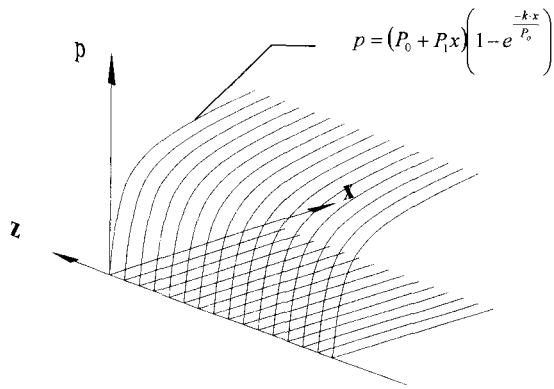


FIG. 5. Reaction force-deformation function of the wood foundation surrounding a dowel of diameter  $d$  plotted along the length of the dowel.

these research projects were empirical in nature and were based on regression analysis of test data.

Depending on joint geometry, the fastener may form a plastic hinge in each member or it may simply rotate if its bending strength is sufficiently high to crush the surrounding wood along its length. The yield model in use today analytically identifies four basic yield modes for fasteners in single shear (see Appendix). The derivation of Mode IV is reviewed below in more detail, because it forms the foundation of the solution derived in this work. The accompanying equations are based on a publication by Hilson (1995) and do not include any safety factors, unlike the formulations published in the NDS.

Maximum load  $P_y$  resisted by the joint may be expressed as a function of embedment strength  $F_e$  (of member 1 or 2), the distance  $b$  between plastic hinge and shear plane, and fastener diameter  $d$  (Fig. 3). In view of the underlying assumptions, the sections outside the plastic hinges are in equilibrium and do not contribute to resisting joint load. It follows that  $P_y$  may be expressed as

$$P_y = F_{e1}b_1d = F_{e2}b_2d \quad (4)$$

The maximum moment  $M_{pl}$  occurs at the plastic hinge where the shear force is zero. Referring to Fig. 3, the moment may be stated as

$$2M_{pl} = -R_{b1}\frac{b_1}{2} + R_{b2}\left(b_1 + \frac{b_2}{2}\right) \quad \text{or} \quad (5)$$

$$2M_{pl} = -F_{e1}d\frac{b_1^2}{2} + F_{e2}db_2\left(b_1 + \frac{b_2}{2}\right) \quad (6)$$

Solving Eq. (4) for  $b_2$ , introducing the result in Eq. (6), rearranging and solving for  $b_1$  yields

$$b_1 = \frac{2\sqrt{M_{pl}}}{\sqrt{F_{e1}d\left(1 + \frac{F_{e1}}{F_{e2}}\right)}} \quad (7)$$

Finally, substituting  $b_1$  back into Eq. (4) results in

$$P_y = 2F_{e1}d \sqrt{\frac{M_{pl}}{F_{e1}d\left(1 + \frac{F_{e1}}{F_{e2}}\right)}} \quad (8)$$

Apart from the actual equation that describes connection capacity, perhaps one of the most important equations to notice is Eq. (7), the equation that determines  $b_1$ ; a similar equation may be formulated for  $b_2$ . The EYT provides the location of maximum moment in the fastener, which is also the location of the two plastic hinges for this particular yield mode.

#### DISCUSSION

No closed-form analytical model that predicts the load-displacement relationship of bolted joints in timber up to maximum load could be found. But such a model would be of use to the field of timber engineering. Apart from tedious numerical modeling or costly empirical studies, the designer of timber structures may use the EYT to determine joint capacity but is left with no alternative to quantify displacement related properties such as stiffness. The EYT relates embedment behavior of the wood foundation to fastener properties. More important, it quantifies how the fastener deforms. This information forms the starting point for the solution derived hereinafter.

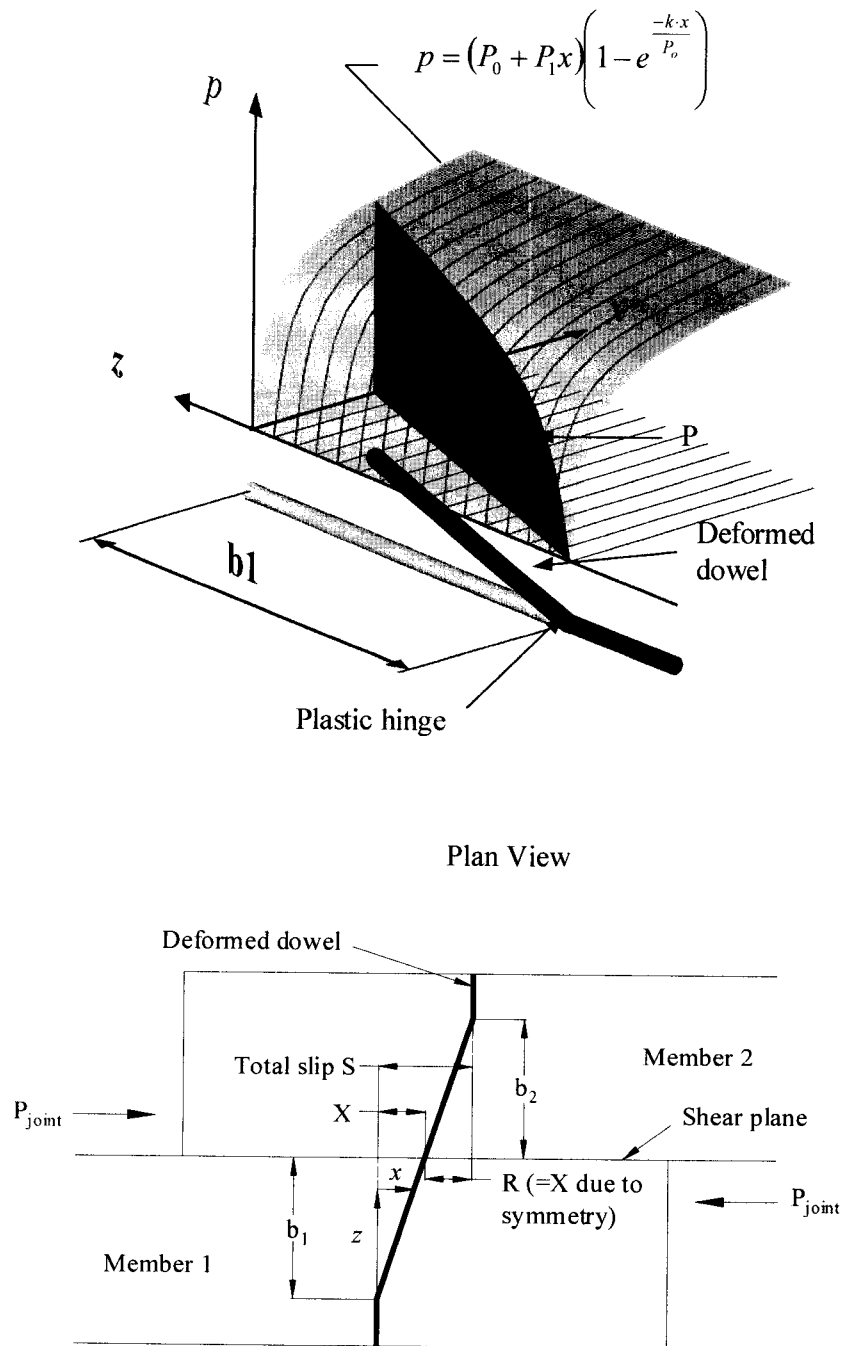


FIG. 6. Foundation reaction force on deformed dowel.



## Moment rotation function fitted to three-point bending of bolt

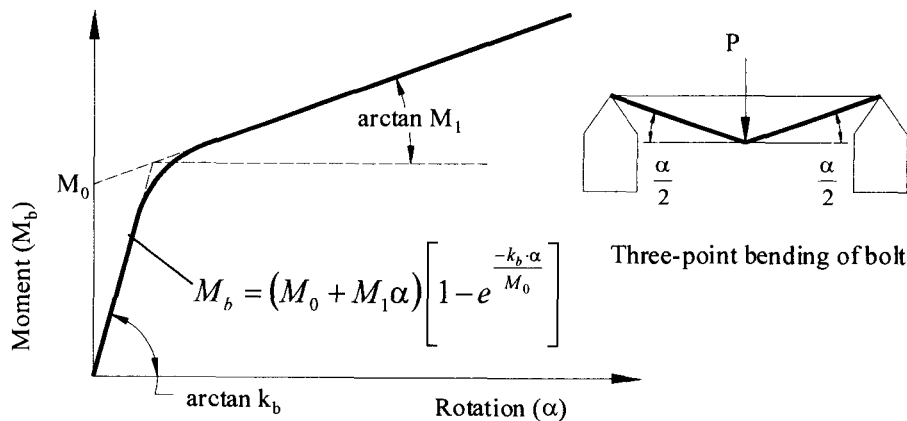


FIG. 7. Three-point bending test of bolt to determine bending moment  $M$  versus rotation  $\alpha$ .

## MODEL DESCRIPTION

A single-shear connection, with side members of equal thickness and equal embedment strength, is analyzed to derive a robust load-slip model based on mechanics (Fig. 4). The model is not limited to this connection type but it simplifies the analysis due to symmetry. This joint was also tested by Gutshall (1994) whose data were available to validate the model predictions that follow.

A practical closed-form analytical formulation predicting the intricate load-deformation interaction can be developed if simplifying assumptions enter the analysis, which serve to abstract the matter studied.

First, assume that the foundation (wood surrounding the fastener) is incapable of transferring shear forces. That is, the foundation can be visualized as an array of an infinite number of independent springs. Further, assume that the force-deformation per unit length of the foundation can be described by Eq. (3). If this is true, then the reaction force-deformation function of a dowel with diameter  $d$ , plotted in three-dimensional space as a function of length ( $z$ -direction), deformation ( $x$ -direction), and reaction per unit length ( $p$ -direction) resembles a warped plane as depicted in Fig. 5.

Next, assume that the dowel is infinitely stiff along its length except at the infinitesimal small region  $\Delta Z$ , which is the region where the plastic hinge forms as described by the EYT. To conceptualize the load-deformation interaction of dowel and foundation, the dowel is placed in the  $z$ -direction under the reaction force-deformation plane of the foundation (Fig. 6). Then, the force necessary to rotate the dowel about the plastic hinge and to press it into the foundation is equal to the shaded area  $P$  (Fig. 6), plus the force it takes to overcome the bending moment of the dowel at the location of the plastic hinge. Observe that  $P$  acts in the  $x$ - $z$ -plane parallel to the  $x$ -axis.

Given these assumptions, the problem reduces to a geometrical problem, where the projected area needs to be expressed as a function of  $X$ . Further, the bending resistance of the dowel must be introduced. Notice that total connection slip is equal to  $R + X$  or, in this case, two times  $X$  due to symmetry (Fig. 6).

Let the origin of the  $z$ -axis coincide with the location of the plastic hinge. The distance between the plastic hinge and shear plane equals  $b_1$ , which may be determined by Eq. (7). Suppose the dowel is forced to rotate a

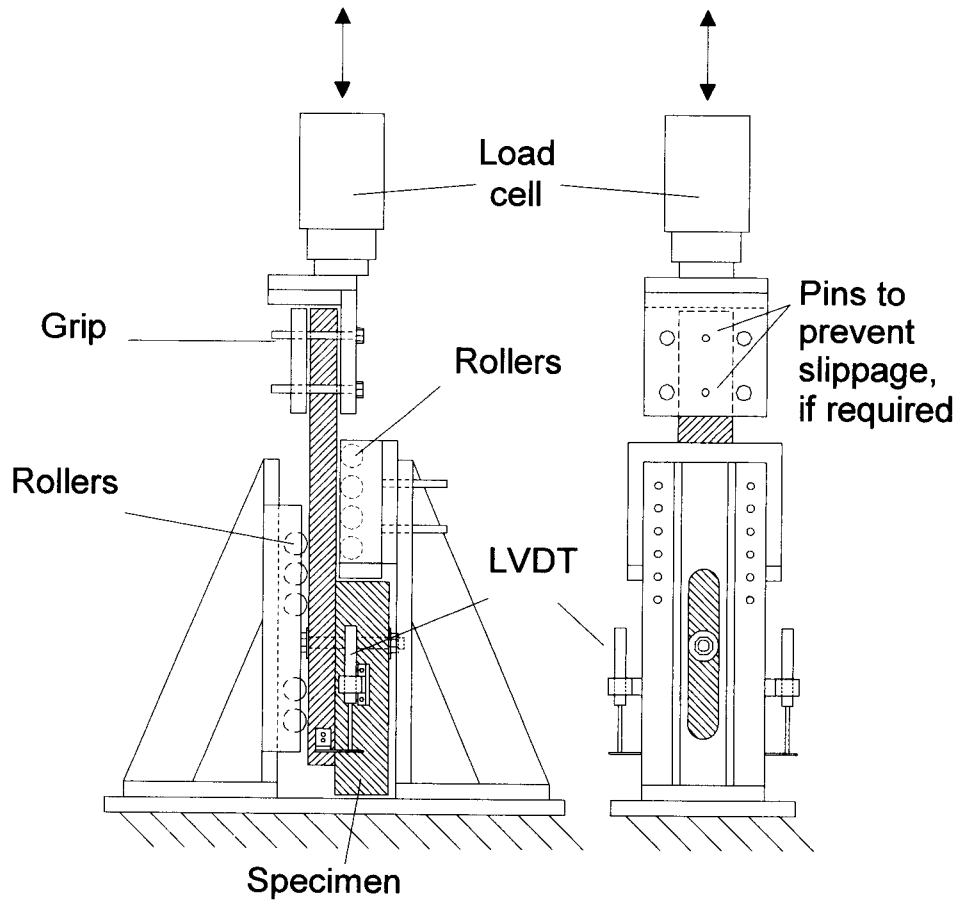


FIG. 8. Test set-up utilized by Gutshall (1994) to obtain load deflection data of a single bolt joint loaded in shear.

distance  $X$ . In that case, the reaction force  $p$  over the infinitesimal small length  $\Delta Z$  of the foundation at any distance  $Z$  from the plastic hinge measured along the  $z$ -axis, may be obtained by simply expressing the distance  $x$  traveled by the dowel at any  $Z$  in terms of  $X$  and  $Z$ .

$$x = \frac{X}{b_1} \cdot Z \quad (9)$$

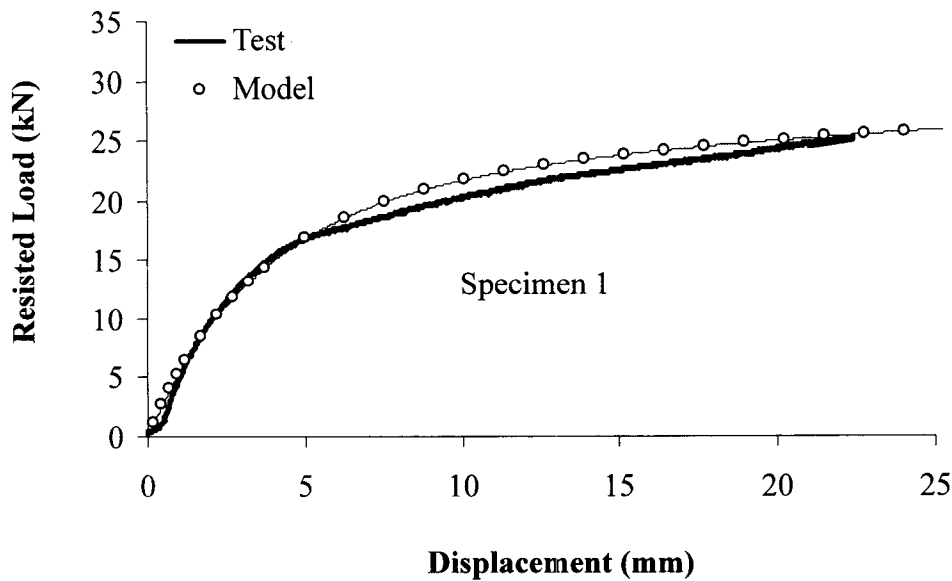
Introducing Eq. (9) into Eq. (3) provides, as a function of  $Z$  and  $X$ , the expression of the curve circumscribing  $P$ , or more specifically, the load  $p$  at  $\Delta Z$  (for  $\Delta Z \rightarrow 0$ ) acting in  $x$ -direction. Hence, by stating that

$$p = \left( P_0 + P_1 \frac{X}{b_1} \cdot Z \right) (1 - e^{(-k \cdot X \cdot Z)/P_0 \cdot b_1}) \quad (10)$$

the total reaction force of the foundation acting in  $x$ -direction may be obtained by integrating Eq. (10) over  $Z$  between the limits 0 and  $b_1$ . Thus, for all  $X > 0$

$$\begin{aligned} P &= \int_0^{b_1} \left( P_0 + P_1 \frac{X}{b_1} \cdot Z \right) (1 - e^{(-k \cdot X \cdot Z)/P_0 \cdot b_1}) dZ \\ &= \frac{b_1}{2k^2} \left[ P_1 k^2 X - \frac{2P_0^2 (e^{(k \cdot X)/P_0} - 1)(k + P_1)}{X \cdot e^{(k \cdot X)/P_0}} \right. \\ &\quad \left. + 2kP_0 \left( k + \frac{P_1}{e^{(k \cdot X)/P_0}} \right) \right] \quad (11) \end{aligned}$$

In view of generalizing the results to account for nonsymmetrical connections,  $X$  may be expressed, with reference to Fig. 6, as



Embedment Data (same for both members)			Bolt Data		Units
$b_1$	33.5	mm	$d$	19.1	mm
$P_0$	618.5	N/mm	$M_0$	602661	N-mm
$P_1$	6.68	N/mm <sup>2</sup>	$M_1$	2926	N-mm/deg
$k$	386.01	N/mm <sup>2</sup>	$k_b$	239415	N-mm/deg
$F_e$	33.64	N/mm <sup>2</sup>			

FIG. 9. Predicted and observed load-deflection relation of Specimen 1.

$$X = S - R = \frac{S}{1 + \frac{b_2}{b_1}} \quad (12)$$

Note, for symmetrical joints,  $b_2 = b_1$  and  $X = S/2$ . Introducing Eq. (12) into Eq. (11) provides  $P$  as a function of total connection slip.

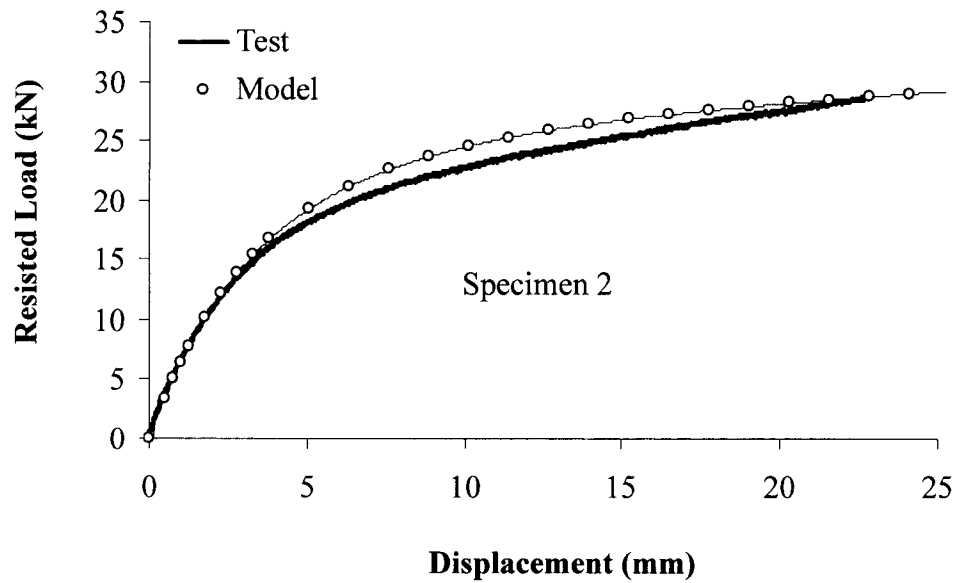
The next step is to incorporate the elastic and plastic bending moment  $M_b$  of the fastener at the plastic hinge. The idea is to fit Eq. (3) by means of nonlinear least squares fitting to the moment-rotation plot,  $M_b$  versus  $\alpha$ , of a bolt in three-point bending. This test is relatively easy to implement and is outlined in ASTM F1575 (1997) (Fig. 7).

Let the deflected shape of the bolt be approximated by a triangle (Fig. 7). The force in the shear plane that is necessary to bend the

bolt about the location of the plastic hinge may then be described by

$$F = \frac{1}{b_1} \left( M_0 + M_1 \arctan \left( \frac{X}{b_1} \right) \right) \times \left\{ 1 - \exp \left[ \frac{-k_b \cdot \arctan \left( \frac{X}{b_1} \right)}{M_0} \right] \right\} \quad (13)$$

At this point, it is important to draw attention to the consequences of the assumptions. By assuming infinite stiffness of the dowel and rotation over only a single point, a significant overestimation of joint stiffness results. Moreover, the correlation of stiffness between two points along the dowel as well as the foundation is neglected. The best way to rectify the



Embedment Data (same for both members)			Units	Bolt Data		Units
$b_l$	30.0	mm		$d$	19.1	mm
$P_0$	775.3	N/mm		$M_0$	602661	N-mm
$P_l$	8.91	N/mm <sup>2</sup>		$M_l$	2926	N-mm/deg
$k$	470.23	N/mm <sup>2</sup>		$k_b$	239415	N-mm/deg
$F_e$	41.91	N/mm <sup>2</sup>				

FIG. 10. Predicted and observed load-deflection relation of Specimen 2.

problem is to use an adjustment factor that incorporates material properties of the dowel to account for various bending stiffness assumptions. Thus, if  $\beta$  represents the adjustment factor the solution may be written as

$$P_{\text{joint}} = (P + F)\beta \quad (14)$$

where,  $P$  and  $F$  are described by Eqs. (11) and (13), respectively.

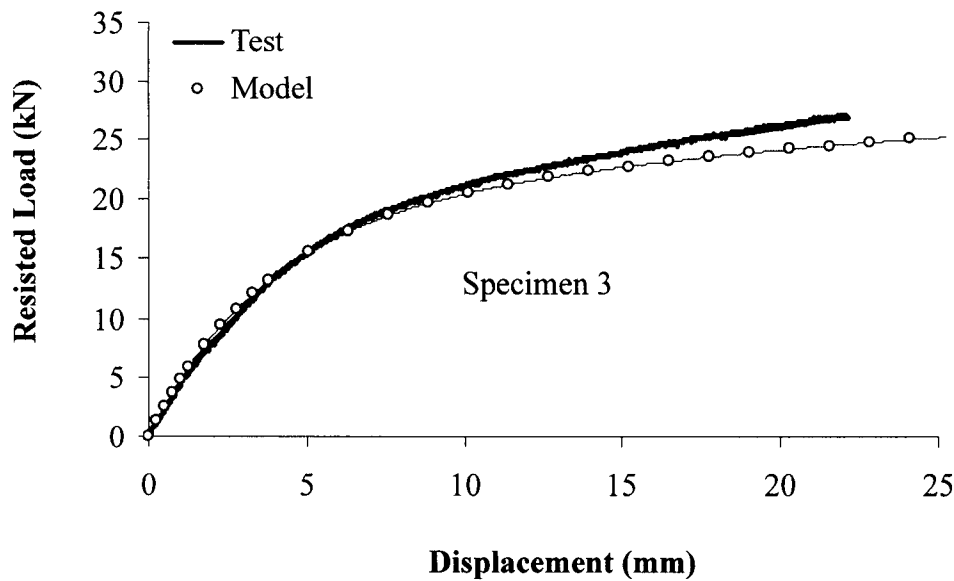
Initial analysis using the theory of beams on elastic foundations suggests that  $\beta$  is constant even beyond the elastic range, and is in the order of  $\frac{2}{3}$  for the specific connection configuration studied<sup>2</sup> (Fig. 4).

<sup>2</sup> The approach was to consider a beam on elastic Winkler foundation loaded by a concentrated force at its end.

#### RESULTS AND IMPLICATIONS

Equation (14) was validated against test data of 5 specimens obtained by Gutshall (1994) and Brinkman (1996) (Figs. 4 and 8). Gutshall tested nailed and bolted connections of several configurations subjected to displacement-controlled monotonic loading (rate: 0.1 in./min). He limited the maximum displacement to one inch because if a connection were to slip that amount, the load resisted by the connection would be transferred to other locations given the redundancy of structural systems (Gutshall 1994).

Brinkman (1996) conducted embedment tests on Gutshall's specimens following the procedures outlined in ASTM D5764 (ASTM 1995). Parameters  $k$ ,  $P_l$ , and  $P_0$ , as described



Embedment Data (same for both members)			Units	Bolt Data		Units
$b_1$	33.8	mm		$d$	19.1	mm
$P_0$	590.5	N/mm		$M_0$	602661	N-mm
$P_1$	9.25	N/mm <sup>2</sup>		$M_1$	2926	N-mm/deg
$k$	281.00	N/mm <sup>2</sup>		$k_b$	239415	N-mm/deg
$F_e$	32.82	N/mm <sup>2</sup>				

FIG. 11. Predicted and observed load-deflection relation of Specimen 3.

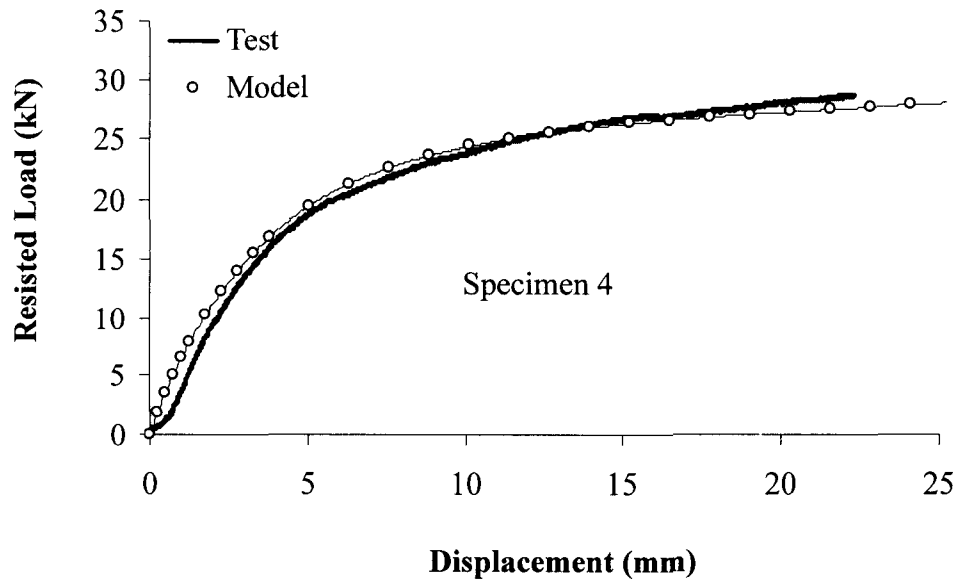
by Eq. (3), were determined manually from each data plot.

Bending moment data for typical A307 bolts ( $\varnothing$  0.75 inch) were obtained from three-point bending tests according to ASTM F1575 (1997).

Figures 9 through 13 reveal predictions and actual data for the five specimens analyzed. Overall, with a  $\beta$  of  $\frac{2}{3}$ , the accuracy of fit to test data is excellent. There appears to be good agreement between predictions made by the model and actual data within the elastic region (before appreciable plastic deformation commences), although, attributed in part to oversized holes, some specimens exhibited settlement and realignment that is evident by a concave-shaped load-deflection plot at low loads. Also, based on the results, it may be

induced that, for all practical purposes,  $\beta$  is indeed a constant for this particular joint type even beyond the elastic range. This suggests that an adjustment factor may be developed that is based on material properties of the bolt using the theory of a beam on elastic foundation.

At large displacements, the model tends to somewhat underpredict joint resistance, which may be due to increased friction between the members caused by a tensile force in the deflected bolt pulling the members to each other. In addition, the model neglects that a component of the tensile force acts in loading direction opposite to the applied force. Other reasons for deviations between model and actual data may include localized splitting of the surrounding wood at larger



Embedment Data (same for both members)			Bolt Data		
		Units			Units
$b_1$	30.7	mm	$d$	19.1	mm
$P_0$	757.8	N/mm	$M_0$	602661	N-mm
$P_1$	0.00	N/mm <sup>2</sup>	$M_1$	2926	N-mm/deg
$k$	537.60	N/mm <sup>2</sup>	$k_b$	239415	N-mm/deg
$F_e$	39.99	N/mm <sup>2</sup>			

FIG. 12. Predicted and observed load-deflection relation of Specimen 4.

displacements and radical geometry changes that could result in translation of the plastic hinges. Ongoing research is investigating the sensitivity of the model to these factors.

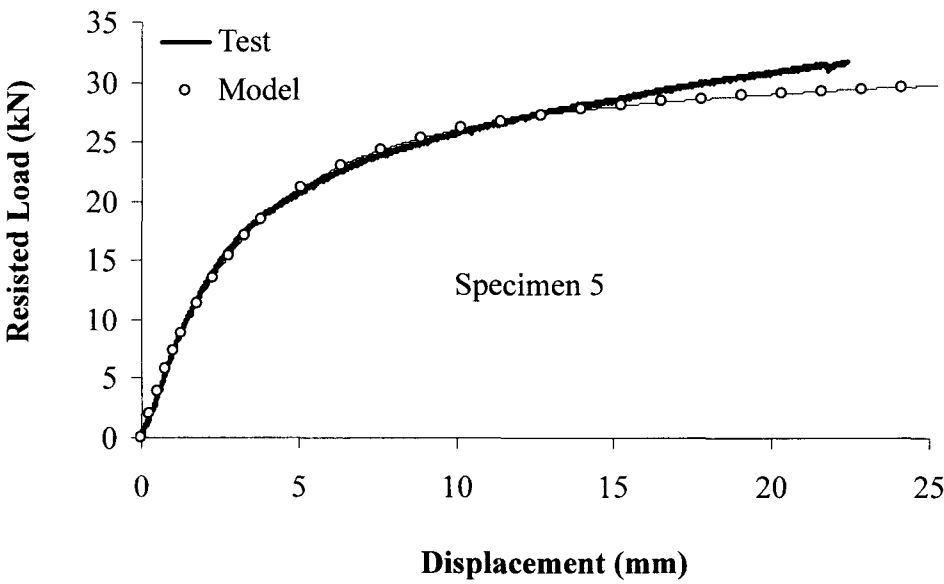
Equal embedment behavior of both members for a given specimen was assumed (refer to the embedment data listed under each load deflection plot of Figs. 9 through 13). This was made possible because Gutshall used matched and clear samples. That is, both side members were cut from the same board, which was clear of defects such as checks and knots. However, even if this assumption cannot be made, and the joint is not symmetric, Eq. (14) still applies because the load in both members is equal but opposite. In that case,  $b_2$  must be determined by the EYT in a similar fashion as  $b_1$  and input in Eq. (12).

In an effort to extend the solution, a similar

approach may be utilized to derive a closed-form solution for other yield modes such as Mode II (Fig. 14), and to use a hybrid of the formulation derived above and Mode II to develop a solution for Mode III. Additional work will focus on generalizing the solution and providing supplementary test results using a range of configurations.

#### SUMMARY AND CONCLUSIONS

A closed-form solution was derived that predicts the load-slip interaction of a single-shear bolted joint in timber exhibiting two plastic hinges at yield. The model provides a new, simpler and practical method to analyze timber structures. The approach gives fundamental information that may allow for more efficient and reliable designs in the field of



Embedment Data (same for both members)			Units	Bolt Data		Units
$b_1$	28.7	mm		$d$	19.1	mm
$P_0$	845.0	N/mm		$M_0$	602661	N-mm
$P_1$	0.00	N/mm <sup>2</sup>		$M_1$	2926	N-mm/deg
$k$	632.48	N/mm <sup>2</sup>		$k_b$	239415	N-mm/deg
$F_e$	45.39	N/mm <sup>2</sup>				

FIG. 13. Predicted and observed load-deflection relation of Specimen 5.

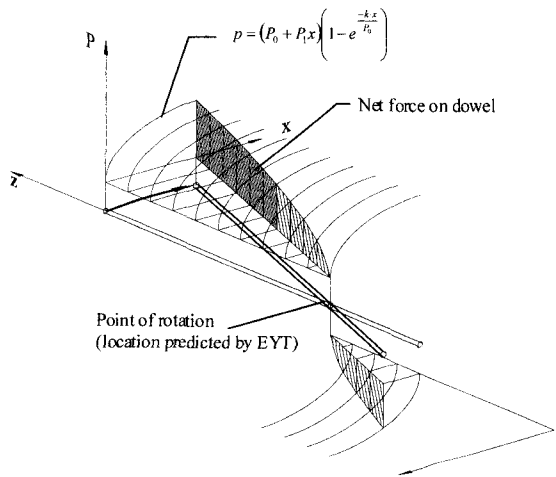


FIG. 14. Proposed approach for Yield Mode II.

timber engineering. The model is based on and is intended to augment the European Yield Theory, which is widely utilized in the academic and industrial community to determine joint capacity.

In conclusion, the formulation gives excellent predictions of actual joint data and the approach may be generalized and be applicable to a wide range of joint configurations.

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