A NEW MODEL TO PREDICT THE LOAD-SLIP RELATIONSHIP OF BOLTED CONNECTIONS IN TIMBER

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ABSTRACT

The development of a new approach to predicting the load-displacement interaction of bolted wood connections is described.

The European Yield Theory for bolted wood connections has gained wide acceptance in recent years attributed to its closed form, simplicity, and accuracy. But the model does not relate capacity or any other loading state to joint slip. A method to determine deflection related properties of bolted joints in timber with a single equation would be an important contribution to the field of timber engineering. The model would simplify the analysis of large structures including schools, gymnasiums, bridges, and light-industrial buildings where wood could be utilized because of its economy, high-strength-to-weight and stiffness-to-weight ratios.

Based on yield characteristics predicted by the European Yield Theory, the joint was abstracted where the dowel rotates about the plastic hinge under a warped force-deformation plane. Through subsequent simple integration along the dowel length, a closed form solution could be developed. The analysis of a two-member connection assembled with a single bolt indicated that the model closely predicts the load-displacement relationship.

Keywords: Bolted connections, model, load-slip, timber.

PROBLEM OVERVIEW

Single bolted joints in wood under static loading have attracted much attention in the past and the subject has been extensively researched, largely because it constitutes the simplest loading condition and joint configuration (see McLain and Thangjitham 1983). Many studies provided empirical formulations that attempt to predict the behavior of the entire connection (e.g., Trayer 1932; Antonides et al. 1980; Gromala 1985; Humphrey and Ostman 1989; Wilkinson 1993). But purely empirical models contribute little to the overall comprehension of the complicated interactions of the many parameters and are frequently cumbersome to use. While experimental testing is necessary to provide information needed to fully comprehend connection response, it is not practical to test all of the possible geometries, materials, and construction techniques that may be used in wood structures. Hence, it is necessary to model subassemblies and move toward modeling complete structures. These models should be validated with limited tests (e.g., Falk and Moody 1989).

Modeling is a complementary part to experimental analysis. It not only aids in understanding, but also allows for experimental studies to be more specialized. Numerical modeling, such as the finite element technique, has become powerful enough to provide approximate solutions with reasonable accuracy of complex problems, such as bolted joints in

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Nomenclature.

Symbol	Description	Unit
α	Angle of bolt rotation	deg
β	Adjustment factor	
σ_{max}	Bending stress or bending yield strength	N/mm^2
b	Distance plastic hinge and shear plane as determined by EYT	mm
d	Fastener diameter	mm
k	Initial stiffness per unit length of dowel with diameter d	N/mm^2
k_b	Initial slope of moment function	N-mm/deg
p	Reaction force of foundation per unit length of dowel with diameter d	N/mm
r	$d/2$	mm
t, L	Member thickness	mm
x	Displacement	mm
F	Force acting in shear plane to overcome bending moment of dowel	N
F_e	Embedment strength	N/mm^2
$F_{resultant}$	Force resultant used to derive maximum moment	N
M_1	Slope of the asymptote of moment function	N-mm/deg
M_0	y-intercept of moment function	N-mm
M_b	Bending moment of fastener	N-mm
M_{el}	Elastic bending moment	N-mm
M_{pl}	Plastic bending moment	N-mm
M_{sp}	Bending moment in shear plane	N-mm
P	Total reaction force of foundation	N
P_1	Slope of the asymptote per unit length of dowel with diamter d	N/mm^2
P_0	y-intercept per unit length of dowel with diameter d	N/mm
P_y	Maximum joint load	N
P_{joint}	Total joint load	N
R	Displacement	mm
R_b	Force resultant	N
S	Total joint displacement (slip)	mm
X	Displacement	mm
Z	Distance	mm

timber (see Patton-Mallory 1996). However, major deficiencies of the finite element approach to modeling wood assemblies include its limited use for practitioners and its reliance on the full quantification of all basic material properties. Attributed to the anisotropic properties of wood and its natural variability, the constitutive relationship is only approximated, and not all basic elastic properties and their interaction that are reported in the literature are backed by comprehensive statistical analysis. Some properties, such as Poisson ratios, are difficult to measure.

Analytical closed-form models based on a mechanics approach contribute to understanding the factors influencing connection behavior and are useful for designers, since they can easily be solved with contemporary spreadsheet software. Despite an extensive amount of published work, few, if any, analytical, closed-form models that predict the load-slip interaction up to capacity (maximum load) of bolted wood connections exist to date.

A closed-form solution that has gained increasing acceptance in recent years is the European Yield Theory (EYT) (Johansen 1949; AF&PA 2000). The model laid the foundation for a sound engineering approach to the design of wood connections. It was adopted by building codes and in design specifications in Australia, Canada, Europe, New Zealand, and the United States. The model predicts lateral strength and yield mode of a connection containing a single dowel-type fastener, such as a bolt or a nail, based on the bending resistance of the fastener and the crushing strength of

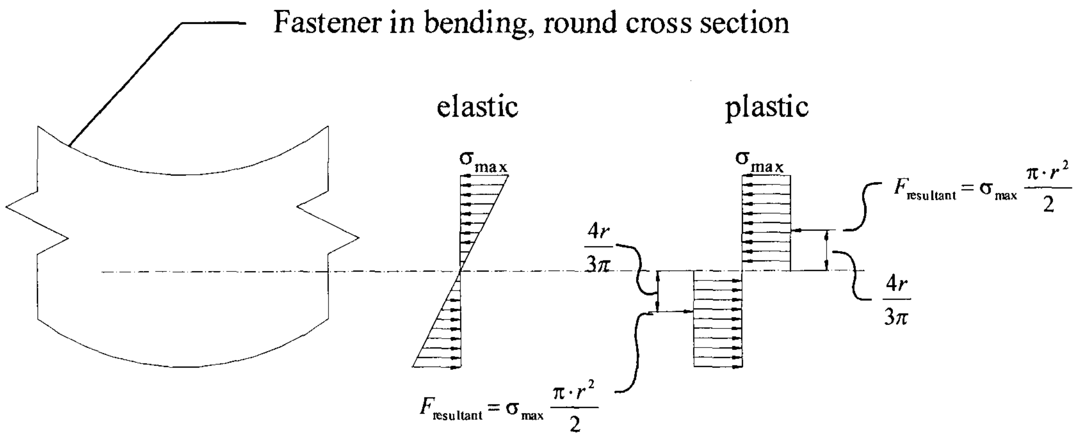

FIG. 1. Illustration of elastic and plastic bending capacity of the fastener.

wood. Compared with other analytical formulations, the EYT entails closed-form and rather simple equations. Considering the simplicity and underlying assumptions, it is surprisingly accurate. Despite its popularity, the EYT is somewhat incomplete as it does not predict deformations attributed to a given loading state. Therefore, displacement-related properties, such as stiffness, ductility, or energy dissipation, cannot be determined using the yield model.

To ensure continued high reliability of wood construction, improved knowledge of the expected response of the structure is required, which opens the need for design procedures to be based on engineering theory. In view of improving the understanding of bolted connections, and helping wood remain a competitive construction material, any closed-form analytical model that furnishes designers with accurate information on load-slip interaction would be beneficial.

This paper presents the results of an ongoing study whose principal objective is to formulate a closed-form analytical model that, while simple in application, is capable of predicting the load-slip deformation of a single bolt connection with wood members. At this stage only joints in single shear (two members connected by a bolt), yielding in Mode IV (development of two plastic hinges as described by the National Design Specifications (NDS) (AF&PA 1997)), are considered. The project utilized data obtained by Gutshall (1994) and Brinkman (1996) to validate the model.

KEY PARAMETERS INFLUENCING CONNECTION STRENGTH

In wood structures, connection strength is essentially a function of fastener bending yield strength, embedment strength, and fastener aspect ratio.

Fastener bending yield strength

Slender fasteners (i.e., fasteners with high aspect ratios (length/diameter)) typically bend when loaded in shear beyond the proportional limit. The fastener's plastic bending stress is referred to as the bending yield strength. It substantially influences joint capacity containing slender fasteners; whereas it has no effect on joints with rigid dowels, since the wood yields and fails before inelastic bending of the bolt takes place. Therefore, connection strength and yield mode are directly related to wood crushing and fastener bending yield strength. The idea was first introduced by Johansen (1949), who linked the elastic bending capacity of the fastener to joint strength (Eq. 1). In 1957, Meyer employed the full-plastic bending capacity to determine joint strength (Eq. 2 and Fig. 1).

$$M_{el} = \sigma_{\max,\text{elastic}} \frac{\pi \cdot d^3}{32}$$

Elastic bending moment (1)

$$M_{pl} = \sigma_{\max,\text{plastic}} \frac{2\pi \cdot r^2}{2} \frac{4r}{3\pi} = \sigma_{\max} \frac{d^3}{6}$$

Plastic bending moment (2)

Fastener yield strength is determined experimentally. In the United States, the fastener is bent during three-point loading and the load at a deflection equal to 5% fastener diameter offset is converted into bending yield strength by rearranging Eq. (2) and solving for $\sigma_{\max}$. The main disadvantage of the 5% diameter offset approach is that it uses neither proportional limit nor capacity as reference points. Moreover, attributed to joint settlement effects as loading commences, the fitting of an initial stiffness line is frequently ambiguous and judgmental. A capacity-based approach, on the other hand, exercised with appropriate adjustment factors to account for variability, may lead to optimum material utilization.

Dowel-bearing strength (embedment strength)

Dowel-bearing strength is a material property, determined experimentally, that describes a limit-state stress in the wood around a pin-loaded hole in compression. Dowel-bearing strength is considerably influenced by loading direction (parallel or perpendicular to grain) not only because of a different modulus of elasticity of wood when compressed parallel or perpendicular to grain, but also on account of a different failure mode. The connections discussed in this paper are considered to be loaded parallel to the grain, which is true for most bolted joints applied in timber construction.

Fastener aspect ratio

In connection design, fastener aspect ratio is defined as member thickness (L) divided by fastener diameter (d). Fastener aspect ratio determines to what degree fastener yield strength and embedment strength influence connection behavior. It controls the yield mode of the connection. Joints containing low aspect ratio fasteners tend to exhibit brittle failure because of wood splitting, whereas slender fasteners bend and develop plastic hinges. At decreasing aspect ratios, more wood yielding and less fastener bending are involved.

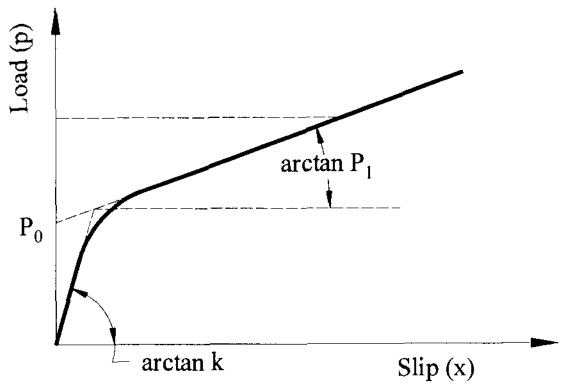

FIG. 2. Foundation model (Foschi 1974).

Friction

Bolted connections, with nuts drawn tight, develop significant friction between the adjacent surfaces, which increases the capacity of the joint. Nonetheless, due to the rheological characteristics of wood, the compression force achieved by tightening the nuts is quickly lost. In addition, possible moisture related dimensional changes of the wood members turn surface friction between members into a rather variable property. As a result, researchers have tested connections with bolts drawn fingertight or left loose (Trayer 1932; Johansen 1949). Considerable friction may, however, develop past the proportional limit when the bolts bend and draw the members against each other.

LOAD-DEFORMATION INTERACTION

A dowel-type fastener embedded in wood and laterally loaded within the elastic range is similar to a beam on an elastic foundation. Studies of beams on elastic foundations have been reported in the literature for more than a century. Early interest in this subject was

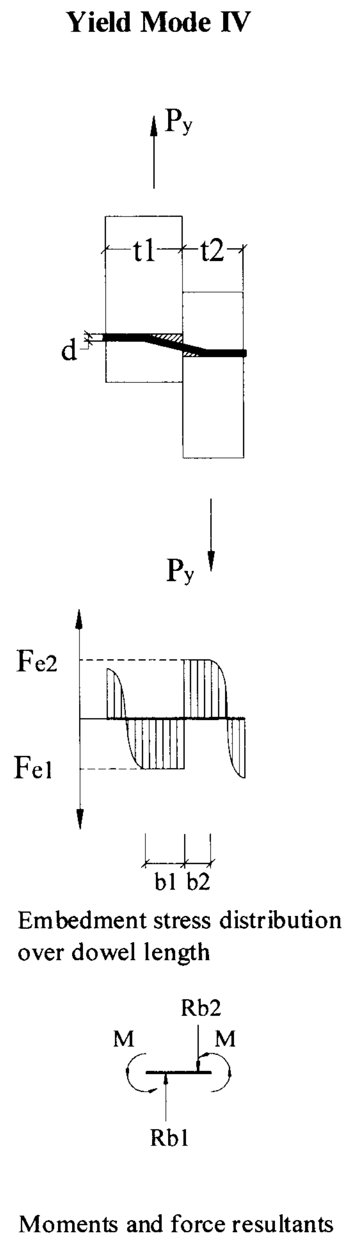

FIG. 3. Mechanics of Yield Mode IV.

sparked by the railroad industry that studied rail response. General analyses have focused on beams on a linear-elastic Winkler-type foundation (Hetényi 1946). In a Winkler-type foundation, the reaction forces are proportional to beam deflection at any point and there is no transfer of shear forces. The application of the theory of beams on elastic foundation to connections in wood has been studied by several researchers. Early work employed the Winkler foundation model in an attempt to fit a linear-elastic load-slip relationship (Kuenzi 1955; Noren 1962; Wilkinson 1971, 1972). But wood is not linear-elastic in compression and the application of the Winkler foundation model gives moderately accurate predictions at best. Furthermore, modern design methodologies are shifting from using linear-elastic approximations to nonlinear elastic-plastic analyses to assure more efficient material utilization.

Foschi (1974) used the approach of a beam on a nonlinear-elastic foundation to derive a finite element model capable of predicting the load-slip function of laterally loaded nails in wood. The characteristics of the foundation are expressed in Fig. 2 and the following equation:

$$p = (P_0 + P_1 x)(1 - e^{(-k \cdot x)/P_0}) \qquad (3)$$

For perfect yield (wood crushing at constant load), $P_1 = 0$, and Eq. (3) provides the load-slip relationship reported seven decades ago by Teichmann and Borkmann (1930 and 1931). The constants k, P_1 and P_0 may be determined from nonlinear least-squares fitting of experimental data obtained by embedment tests. Foschi's model has been used by many researchers (Dolan 1989; White 1995; Blass 1994; Frenette 1997) as input for comprehensive finite element analyses and empirical joint models.

THE EUROPEAN YIELD MODEL

The EYT utilizes a beam on a *plastic* foundation approach by assuming that, at capacity, wood crushing underneath the fastener is so advanced that the reaction force is uniformly distributed along the fastener and constant relative to beam deflection. This assumption greatly simplifies the problem of predicting capacity for connections containing slender fasteners (high aspect ratios). The value of the EYT would be significantly increased, how-