TWO-DIMENSIONAL GEOMETRIC THEORY FOR MAXIMIZING LUMBER YIELD FROM LOGS[1]

Yage Zheng

Associate Professor
Department of Wood Science and Technology
Nanjing Forestry University
Nanjing, China

Francis G. Wagner and *Philip H. Steele*

Associate Professor and Research Scientist
Mississippi Forest Products Utilization Laboratory
Mississippi State, MS 39762

and

Zhendong Ji

Associate Professor
Wuhan University of Water Transportation Engineering
Wuhan, China

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ABSTRACT

A two-dimensional geometric theory for maximizing lumber yield from logs was developed. Centered cant sawing solutions for both circular and elliptical shaped logs were derived. Sawline placement for maximum yield is dependent upon the diameter of round logs or upon the cross-sectional axis of elliptical logs. The width of the face of the cant is equal to 0.707 times the diameter or parallel-axis of the log. Slab thickness is equal to 0.147 times the diameter or perpendicular-axis of the log. It assumes circular and elliptical log shapes and provides a method that may substantially reduce computation time when applied to computerized log breakdown decisions.

Keywords: Log breakdown, log sawing algorithm, cant sawing.

INTRODUCTION AND BACKGROUND

Research has shown that many factors can affect the yield of lumber from logs. Such factors include log size, log form, saw kerf, sawing variation, rough green lumber size, product mix, and log breakdown decisions [Hallock and Lewis 1976; Steele 1984; and Steele and Wagner (in press)]. The modification of these factors to improve lumber yield has long been the goal of researchers, equipment manufacturers, and sawmill managers alike. Two of these factors, log form and log breakdown decisions, are addressed in this paper by the introduction of a two-dimensional geometric theory for maximizing lumber yield.

The Best Opening Face (BOF) concept for optimizing log breakdown decisions was first introduced by Hallock and Lewis in 1971. The BOF system is used extensively today in computerized decision-making and process control sawing equipment. It is a complex three-dimensional simulation model that assumes logs

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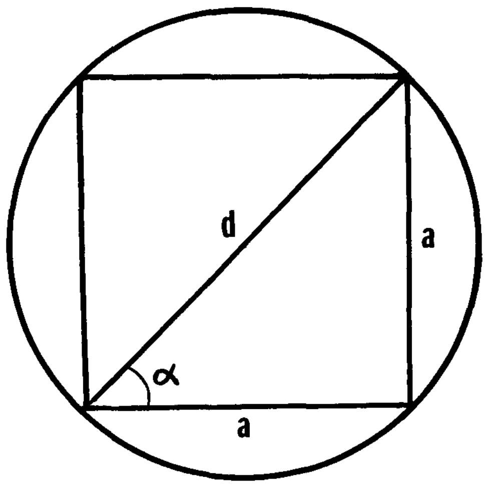

FIG. 1. Circular log with an inscribed square cant where: d = diameter of the circle, a = length of a side of the square, and α = 45° angle.

to be truncated cones and uses an iterative approach to optimizing sawline placement. Because of BOF's complexity and long run time, investigators recently explored methods for simplifying the computation of BOF position (Steele et al. 1987). They learned that centered sawing solutions are excellent predictors of BOF position. It was also learned that the predominant geometric factor determining optimum sawline placement is the two-dimensional geometry of the log.

With this background, a two-dimensional geometric theory for maximizing lumber yield from logs was developed. A centered-cant sawing solution for both circular and elliptical shaped logs was derived. This theory uses direct calculations for sawline placements and should substantially reduce computation time when applied to computerized log breakdown decisions.

PROCEDURE

Circular logs

Elementary geometry teaches that a square has the greatest area of any quadrilateral inscribed within a circle. It follows that a square cant inscribed within a circular log will also have greater area than any other four-sided cant. Therefore, the basis for this two-dimensional geometric theory is the inscription of a square cant within a circular log to obtain maximum yield.

The largest square that can be wholly inscribed within a circular log may be calculated as follows: (see Fig. 1).

$$a = d \cos \alpha = d \cos 45°$$
$$a = 0.707d$$

where:

d = diameter of the log
a = length of each side of the square cant
α = the 45° angle of a triangle formed by a diagonal and one side of the inscribed square cant

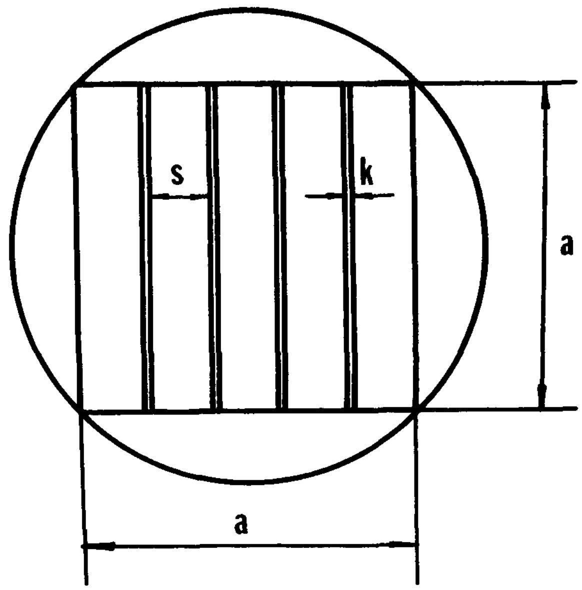

FIG. 2. The number of boards that can be sawn from a square cant is dependent upon the size of the cant (a), the thickness of the lumber to be sawn (s), and the thickness of the resaw kerf (k).

Therefore, the largest square cant that can be sawn from a circular log is equal to 0.707 times the diameter of the log. In practice, the diameter of the log may be measured at the small-end of the log or at some set-point along the length of the log. Wane limitations and log taper may dictate the point of measurement.

The number of boards that can be sawn from a square cant is dependent upon the size of the cant, the thickness of the lumber to be sawn, and the thickness of the resaw kerf. The number of boards may be calculated by the following equation: (see Fig. 2).

$$N = (a + k)/(s + k)$$

where:

N = number of boards
a = length of each side of the cant
s = thickness of each board
k = thickness of the resaw kerf

Of course, the number of boards must be a whole number. Because s and k are fixed, the size of the cant should be adjusted until N equals a whole number. Preferably, the size of the cant should be increased until N equals a whole number and wane restrictions are not exceeded. Obviously, increasing the size of the cant will increase the volume of lumber recovered from the cant.

Placement of the first sawline in the log during sawing is critical to the maximization of lumber yield (Hallock and Lewis 1971, 1976; Steele et al. 1987). Therefore, it is important to calculate the thickness of the slabs produced during the squaring of the cant. The thickness of the slabs is dependent upon the diameter

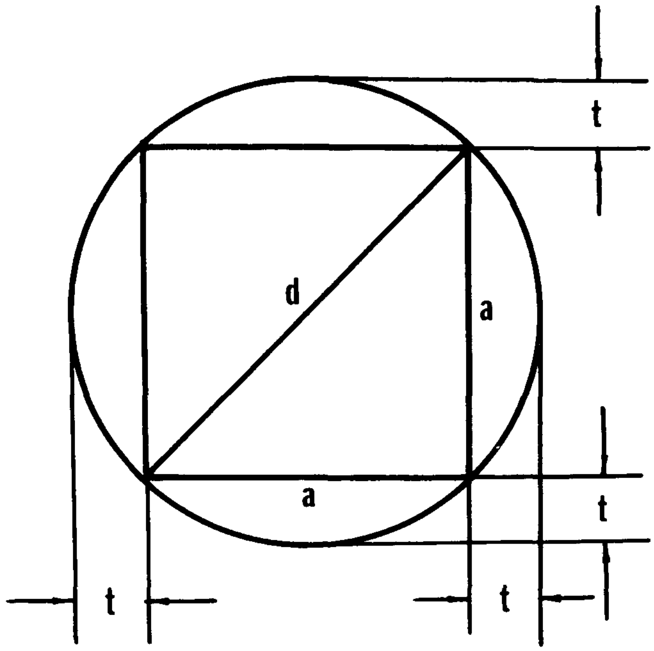

FIG. 3. The thickness of the slabs (t) sawn to produce the maximum size cant from circular logs is dependent upon the diameter of the log (d) and the size of the square cant (a).

of the log and upon the size of the maximum square cant to be produced. Since an equation for the length of each side of the square cant has already been shown, the thickness of the slab may be calculated as follows: (see Fig. 3).

$$d = 2t + a$$
$$t = (d - a)/2 = 0.147d$$

where:

t = thickness of the slab
d = diameter of the log
a = length of each side of the cant

Therefore, to produce a cant of optimum size, the sawline for each face of the cant should be (t) distance from the surface of the log at the set-point. The width of each cant face should be equal to (a).

For large logs, slabs will be relatively thick and it may be possible to recover lumber from each thick slab. To determine the thickness and width of the largest board that may be sawn from a thick slab, the following equations may be used: (see Fig. 4).

$$b/2 = \sqrt{r^2 - (c + a/2)^2} = \sqrt{r^2 - (c + (\sqrt{2}/2)r)^2}$$
$$b = 2\sqrt{r^2 - (c + a/2)^2} = 2\sqrt{r^2 - (c + (\sqrt{2}/2)r)^2}$$

where:

a = length of each side of the cant = 1.414r

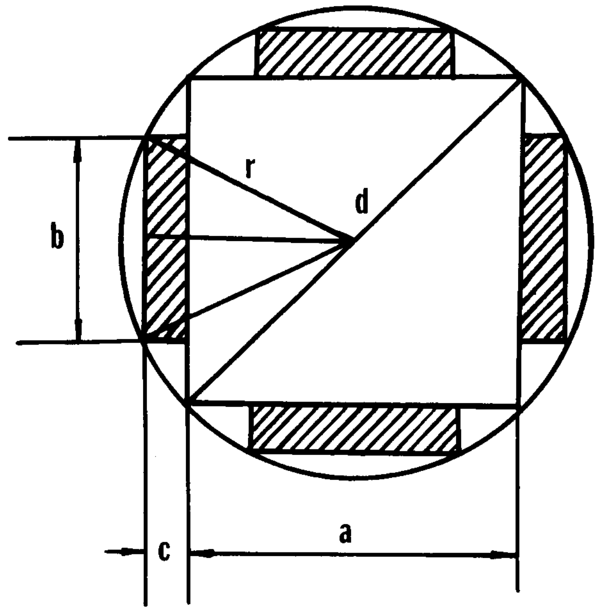

FIG. 4. The width (b) and thickness (c) of the largest board that may be sawn from a slab of a circular log is dependent upon the diameter (d) of the log and the size (a) of the inscribed cant.

b = width of the largest board that may be sawn from a thick slab
c = thickness of the largest board that may be sawn from a thick slab
r = radius of the log

Solving for the largest cross sectional area of the board (F) and replacing (b) with the above equation, we obtain the following equations:

$$F = bc$$
$$F = 2c\sqrt{r^2 - (c + (\sqrt{2}/2)r)^2}$$

Maximizing the area of (F) by taking the derivative of (F), setting it equal to zero and simplifying, we obtain the following equations:

$$dF/dc = (d/dc)2c(r^2 - (c + r\sqrt{2}/2)^2)^{1/2} = 0$$
$$(2(r^2/2 - c^2 - rc\sqrt{2}) - 2c^2 - rc\sqrt{2})/(r^2/2 - c^2 - rc\sqrt{2})^{1/2} = 0$$
$$2(r^2/2 - c^2 - rc\sqrt{2}) - 2c^2 - rc\sqrt{2} = 0$$
$$-4c^2 - 3rc\sqrt{2} + r^2 = 0$$

Solving for c:

$$c = (3r\sqrt{2} \pm \sqrt{18r^2 + 16r^2})/(-8)$$
$$c = r(3\sqrt{2} \pm \sqrt{34})/(-8) = r(4.243 - 5.831)/(-8) = 0.199r = 0.099d$$

Replacing (c) with the above expression and solving for (b):

$$b = 2\sqrt{r^2 - (c + a/2)^2} = 2d\sqrt{0.045}$$
$$b = 0.426d$$

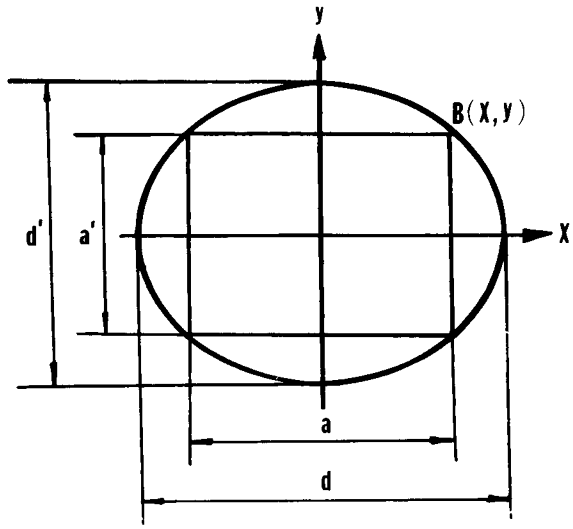

FIG. 5. The lengths of the sides (a and a') of the largest wholly inscribed rectangular cant within an elliptical log are dependent upon the lengths of the axis of the ellipse (d and d').

Both (b) and (c) may be increased to produce lumber of standard width and thickness so long as wane limitations are not exceeded.

Elliptical logs

Practical experience teaches that not all logs are circular in cross section. In fact, a recent study in China has shown that over seventy-percent of the logs examined were actually elliptical or oval in shape (Zheng 1979). Despite this knowledge, most current process-control decisions are based upon the assumption that logs are circular in cross section. This includes the widely used BOF computer program. For those logs that are not circular in cross section, improved sawing decisions and maximization of yield may result if a more realistic log shape, such as an ellipse, is assumed.

If the horizontal axis of an ellipse is (d) and the vertical axis is (d'), the cross-sectional area of the largest wholly inscribed rectangle may be calculated by the following equation: (see Fig. 5).

$$A = aa'$$

where:

A = area of the rectangular cant
a = length of the horizontal axis of the rectangle
a' = length of the vertical axis of the rectangle

If the coordinates for the intersection of the rectangle and the ellipse are (x, y), then:

$$a = 2x$$
$$a' = 2y$$
$$A = (2x)(2y) = 4xy$$

where:

x = intercept of the of the ellipse and rectangle in the x direction
y = intercept of the ellipse and rectangle in the y direction

The equation for an ellipse is:

$$x^2/(d/2)^2 + y^2/(d'/2)^2 = 1$$

Therefore:

$$y = d'/2\sqrt{1 - x^2/(d/2)^2}$$
$$A = 4x(d'/2)\sqrt{1 - x^2/(d/2)^2}$$

Maximizing the area of (A) by taking the derivative of (A), setting it equal to zero, and simplifying, we obtain the following equations:

$$dA/dx = 4(d'/2)(\sqrt{1 - x^2/(d/2)^2} - (x^2/(d/2)^2)(1/\sqrt{1 - x^2/(d/2)^2}))$$
$$\sqrt{1 - x^2/(d/2)^2} - (x^2/(d/2)^2)(1/\sqrt{1 - x^2/(d/2)^2}) = 0$$
$$1 - x^2/(d/2)^2 - x^2/(d/2)^2 = 0$$

Simplifying further and solving for (x):

$$x = (d/2)/\sqrt{2} = 0.707(d/2)$$
$$x = 0.354d$$

Because (y) has already been solved in terms of (x) above, the simplified expression for (y) is:

$$y = (d'/2)/\sqrt{2} = 0.707(d'/2)$$
$$y = 0.354d'$$

The length of each side of the largest wholly inscribed rectangle may now be calculated.

$$a = 2x = 0.707d$$
$$a' = 2y = 0.707d'$$

Therefore, the size of the largest rectangular cant that can be wholly inscribed within an elliptical log may be calculated in the same way as the size of the largest square cant inscribed within a circular log except that log diameter must be measured in two directions (d and d') for elliptical logs. The lengths of the sides of a wholly inscribed rectangular cant should be 0.707 times the parallel axis of the ellipse. As was the case with square cants, the number of boards that can be sawn from a rectangular cant is dependent upon the size of the cant, the thickness of the lumber to be sawn, and the thickness of the resaw kerf. The same equation may be used for the calculation. Again, the size of the cant may be increased so that (N) will equal a whole number so long as wane limitations are not exceeded.

The thickness of the slabs that must be removed to produce a rectangular cant with maximum area from elliptical logs may also be calculated in much the same

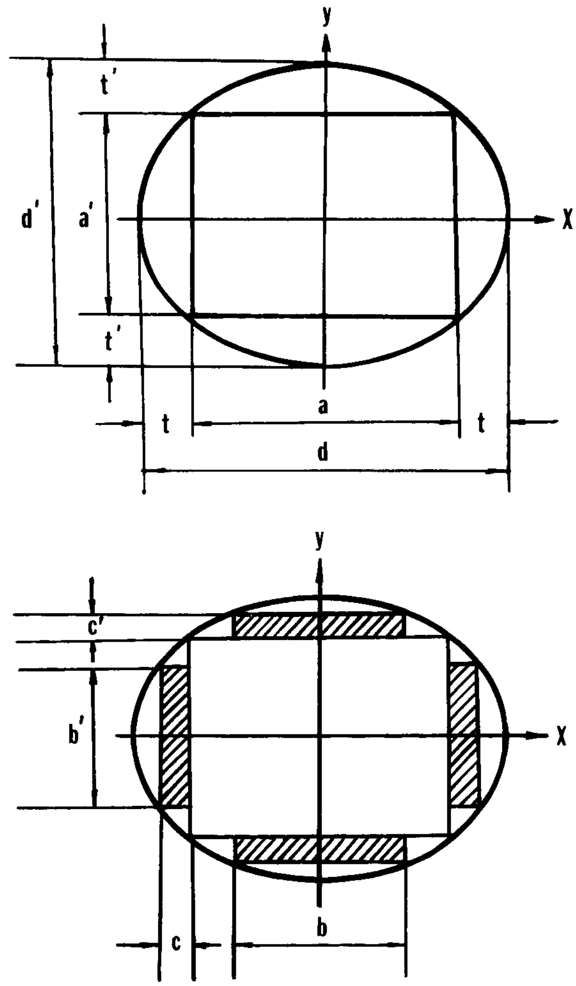

FIG. 6. The thicknesses of the slabs (t and t') sawn to produce the maximum size cant from elliptical logs are dependent upon the lengths of the axis of the ellipse (d and d') and upon the size of the rectangular cant (a and a'). The widths (b and b') and thicknesses (c and c') of boards with maximum volume that may be sawn from the slabs of large elliptical logs are dependent upon the lengths of the axis of the ellipse (d and d') and upon the size of the rectangular cant (a and a').

way that slab thickness was calculated for circular logs. Slab thickness is dependent upon the length of the perpendicular axis of the ellipse and upon the size of the maximum rectangular cant produced (See Fig. 6).

$$t = (d - a)/2 = 0.147d$$
$$t' = (d' - a')/2 = 0.147d'$$

where:

t = thickness of vertical slabs
t' = thickness of horizontal slabs
d = length of the horizontal axis of the ellipse
d' = length of the vertical axis of the ellipse

a = length of the horizontal side of the rectangle
a′ = length of the vertical side of the rectangle

Therefore, to produce a cant of maximum size, the sawline for the horizontal faces of the cant should be (t′) distance from the surface of the log and the sawline for the vertical faces of the cant should be (t) distance from the surface of the log at the set-point.

As with circular logs, it may be possible to recover side lumber from the slabs of large elliptical logs. For cants that are the largest rectangle that can be wholly inscribed within an elliptical log, board width and thickness from slabs may be derived as was done previously for logs of circular cross-section (See Fig. 6).

$$b = 0.426d$$
$$b' = 0.426d'$$
$$c = 0.099d$$
$$c' = 0.099d'$$

where:

b = width of the largest board that may be sawn from a thick horizontal slab
b′ = width of the largest board that may be sawn from a thick vertical slab
c = thickness of the largest board that may be sawn from a thick vertical slab
c′ = thickness of the largest board that may be sawn from a thick horizontal slab

The width and thickness of this side lumber may be increased to produce lumber of standard width and thickness so long as wane limitations are not exceeded.

SUMMARY

A two-dimensional geometric theory for maximizing lumber yield from logs was developed. A centered cant sawing solution for both circular- and elliptical-shaped logs was derived.

For circular logs, a square cant will produce maximum yield. The length of each side of a wholly inscribed square cant would be 0.707 times the diameter of the log at the small end of the log or at some set-point along the length of the log. The thickness of the slabs produced during the squaring of the cant is equal to 0.147 times the diameter of the log. The width of the largest board that may be sawn from each slab is equal to 0.426 times the diameter of the log and the thickness is 0.099 times the diameter.

For elliptical logs, a rectangular cant will produce maximum yield. The lengths of the sides of a wholly inscribed rectangular cant should be 0.707 times the parallel axis of the ellipse. The thickness of the slabs produced during the forming of the cant is equal to 0.147 times the length of the perpendicular axis of the elliptical shaped log. The width of the largest board that may be sawn from each slab is equal to 0.426 times the length of the parallel axis of the log and the thickness is 0.099 times the length of the perpendicular axis.

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