THEORETICAL WOOD DENSITOMETRY: I
MASS ATTENUATION EQUATIONS AND
WOOD DENSITY MODELS

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ABSTRACT
Theoretical linear and mass attenuation equations that model X-ray attenuation processes of both
monoatomic and polyatomic absorbers with homogeneous or heterogeneous structures have been
derived. The reaction mechanisms of photons to atoms, as well as relevant atomic parameters, have
been used in the development of parametric expressions that relate attenuation coefficients to important
radiation parameters—including X-ray energy, absorber thickness, incident angle, and absorber den-
sity. To better understand the physical process to be modeled, systematic analyses and critical com-
parisons of attenuation equations of different X-ray sources and various absorbers are reported in
simple physical and mathematical contexts. To address the wood densitometric problems that we
encounter, theoretical formulas for transmission probabilities and wood density calculation have also
been developed. Examples are provided to demonstrate uses of these theoretical models in calculating
(1) atomic mass attenuation coefficients for major elements as well as minor ash elements in wood,
(2) mass attenuation coefficients of coniferous wood, (3) transmission probabilities under various
combinations of radiation parameters, and (4) wood density of pine heartwood. Throughout this
mathematical treatment, emphases have been placed on the microscopic point of view for modeling
and the parametric formulation for describing X-ray transmission experiments.

Keywords: Wood densitometry, attenuation coefficients, transmission probability, wood density,
X-radiation, atomic parameters, mathematical modeling.

INTRODUCTION

Among radiometric techniques that have been broadly used for measuring wood
density, X-ray radiometry has been regarded as the best (Parker and Kennedy
1973); however, theoretical treatment of X-ray wood densitometry has received
little attention. Previous studies, making limited technological advancement, have
mainly concentrated on laboratory experimentation. The present investigation
attempts to establish, by mathematical modeling, theoretical foundations for wood
X-ray densitometry, which will stimulate new approaches to research.

In an X-ray transmission experiment, photons in the incident beam are being
scattered from, absorbed by, or transmitted through the absorber. If the scattered
and secondary photons are excluded from reaching the detector, then the trans-
mitted beam must include only quanta that have not been changed (in wavelength
and direction) when passing through the absorber. The removal of quanta from
the incident beam by whatever process reduces its intensity and is called X-ray
attenuation.
Wood X-ray densitometry is a radiation detection method in which X-ray attenuation is converted to wood density measurement. To construct simple, yet sufficiently representative, X-ray attenuation models of wood and to make mathematical manipulations amenable, efforts should be concentrated on modeling an X-ray detection system with "good architecture." In such a system, an absorber is exposed to X-ray radiation when placed in the path between a source and a detector. Before striking and after exiting the absorber, incident and emergent X-rays are collimated to narrow beams perpendicular to the absorber's surface.

Another component used in the good architecture system is a monochromator, which modifies the energy continuum of the X-radiation. Therefore, beams are assumed monoenergetic in the present study unless otherwise specifically stated. An absorber is of uniform thickness whether it be monoatomic or polyatomic. The term absorber in this study refers to matter of definite (element or substance) or indefinite (mixture or alloy) composition that absorbs X-ray photons, even though a portion of quanta is known to be removed by a scattering process. To simplify terminology throughout the study, attenuation coefficient and cross section refer to, respectively, total decrease in X-ray intensity and the sum of proportions of quanta removed from the beam, regardless of whether absorption or scattering or both have been the interaction mechanisms in the transmission experiment.

MODELING ATTENUATION EQUATIONS

Laboratory measurement of attenuation is possible by detecting and recording intensities of both the incident and the emergent X-ray beams. Modeling attenuation experiments can be done by using parametric expressions to describe changes in X-ray intensity.

The difference between the intensity of the incident X-ray \( I_0 \) and that of the transmitted beam \( I \) is known as energy attenuation and is defined as \( \Delta I = I_0 - I \). When unit path length \( \Delta \ell \) is small compared to the total distance \( \ell \) that the X-ray traverses, then X-ray attenuation may be considered almost constant for each \( \Delta \ell \). Thus, there exists a linear relationship between \( \Delta I \) and \( \ell \) such that \( \Delta I / \ell = \mu \), the linear attenuation coefficient, represents the sum of probabilities of occurrence per unit path length that the X-ray photon is removed from the beam (Knoll 1979). Integration over the distance \( \ell \) leads to

\[
I = I_0 \exp \left( -\int_0^\ell \mu d\ell' \right)
\]

Now, we begin by formulating the commonly quoted (Olson and Arganbright 1981) macroscopic linear attenuation equation that motivated the problem in the first place.

Macroscopic linear attenuation equation

Because the X-ray is incident perpendicular to the absorber, the distance the X-ray penetrates is equal to the physical thickness \( t \) of the absorber. Substituting \( \ell \) for \( t \) results in

\[
I = I_0 \exp \left( -\mu t \right)
\]
This linear attenuation equation is macroscopic because all parameters in the model are macro-quantities, i.e., characteristic of the absorber.

Strictly speaking, this equation is applicable to radiation experiments under the good architecture condition. Deviations from this ideal condition require special considerations. Several special conditions have been treated in this investigation and are reported later in this paper.

Microscopic attenuation equations

X-ray attenuation processes depend on the (1) energy, intensity, and spectral distribution of the incident radiation; (2) density, atomic number, atomic weight, and microscopic cross section of the constituent elements in the absorbing medium; and (3) incident angle of the beams, with all these factors requiring careful consideration in the theoretical treatment of transmission experiments. Furthermore, interactions between X-ray photons and matter are single, identifiable processes, each associated with an individual atom. Accordingly, models of attenuation equations incorporating atomic parameters or microscopic quantities will provide an explicit and more precise description of X-ray transmission experiments. Thus, attenuation models for monoatomic absorbers are developed below.

Monoatomic absorbers.—Consider a monoenergetic X-ray beam of energy $E$ and intensity $I_0(E)$ that is reduced by an amount $\Delta I(E)$ on passing through an absorber of thickness $t$ and density $\rho$, consisting of one kind of atom with atomic number $Z_i$ and atomic weight $M_i$. Following from the definition of energy attenuation, we get $\Delta I_i(E) = I_0(E) - I_i(E)$. Similarly, the elemental linear attenuation coefficient, $\mu_i$, is related to $\Delta I_i(E)$ and $I_i(E)$ by $\Delta I_i(E)/I_i(E) = -\mu_i \Delta t$.

Since the number of photons eliminated from the beam in traversing a distance $\Delta t$ of the absorber is proportional to $\Delta t$ and to the number of incident photons, the transmission versus distance curve is an exponential function (Price 1964; Knoll 1979). After integration, the linear attenuation equation of the $i^{th}$ element in the periodic series is given by the following exponential function:

$$I_i(E) = I_0(E) \exp \left[ -\int_0^t \mu_i \, dt \right]$$

(3)

Use of the linear attenuation coefficient is limited by the fact that it varies with the density of the absorber, even though the absorber material is the same (Knoll 1979). The macroscopic mass attenuation coefficient is much more widely used and is defined as the ratio of the linear attenuation coefficient to the density of the absorber. To develop mass attenuation coefficients at the microscopic level, we must introduce into the function quantities pertaining to a single atom.

According to atomic theories (Glasstone and Edlund 1966), the number of atoms per unit volume $\eta_i$ equals $N_a \rho_i / M_i$, and the linear attenuation coefficient for the $i^{th}$ element $\mu_i(E)$ is equal to $\eta_i \sigma_i(E)$. Here, $N_a$ is Avogadro's number, $\rho_i$ the density of the $i^{th}$ element, and $\sigma_i$ the microscopic atomic cross section of the $i^{th}$ element which represents the relative probability that an interaction process takes place. Note that dependence on the photon energy and absorber type is included in the cross section.

With these understandings, $\mu_i$ can be expressed as

$$\mu_i(E) = N_a \rho_i \sigma_i(E) / M_i$$

(4)
Now, let $\nu_i$ denote the mass attenuation coefficient of the $i^{th}$ element; then by definition

$$\mu_i(E) = \nu_i(E) \rho_i$$  \hfill (5)

We now substitute Eq. (4) into Eq. (5) and obtain

$$\nu_i(E) = N_i \sigma_i(E)/M_i$$  \hfill (6)

The microscopic mass attenuation coefficient defined above is a function of the energy of the photons, the atomic number, and the atomic weight of the absorber element (Price 1964). It measures the cross section of the atom for X-ray absorption and is unaffected by the density of the interacting medium. The exponential function of the mass attenuation equation for homogeneous absorbers with uniform density becomes

$$I_s(E) = I_{0s}(E) \exp\{-\nu_s(E)\rho_s t\}$$  \hfill (7)

Note that both $\rho_s$ and $\nu_s$ are independent of $t$. The product $\rho_s t$, known as the mass thickness of the absorber, has the same number of atoms in absorbers of equal mass thickness and is now the significant parameter that determines the degree of attenuation.

**Polyatomic absorbers.**—In modeling mass attenuation equations for polyatomic absorbers, materials of homogeneous and of heterogeneous structure (i.e., density) will be considered separately. For homogeneously structured absorbers consisting of $n$ different elements, the density of the whole absorber is a constant quantity equal to the sum of constant densities of each element in the absorber, i.e., $\rho = \sum \rho_j$, for $j = 1, 2, \ldots, n$. The number of photons removed by the homogeneous absorber equals the number of photons eliminated by all constituent elements; thus, $\Delta I_s(E) = \sum \Delta I_s(E)$. The additive property holds true for the linear attenuation coefficient: $\mu(E) = \sum \mu_j(E)$.

From the definition of mass attenuation coefficient, we have

$$\mu(E) = \sum_{j=1}^{n} \nu_j(E) \rho_j$$  \hfill (8)

Now, let $w_j$ represent the weight fraction of element $j$ in the compound or mixture, then $w_j = \rho_j/\rho$. Since $\rho_j$ and $\rho$ are constant quantities in homogeneous mixture, $w_j$ is also a constant. It is thus apparent that $\sum w_j = 1$. The mass attenuation coefficient of a polyatomic absorber can now be calculated from

$$\nu(E) = \sum_{j=1}^{n} \nu_j(E) w_j$$  \hfill (9)

Substituting Eq. (9) into Eq. (8), we obtain

$$\mu(E) = \nu(E) \rho$$  \hfill (10)

Here, $\nu(E)$ designates the mass attenuation coefficient of a homogeneously structured absorber. Now, the mass attenuation equation of a homogeneous mixture takes the following form:

$$I_s(E) = I_{0s}(E) \exp\{-\nu(E)\rho t\}$$  \hfill (11)

For precise mathematical formulation of a mass attenuation equation of a heterogeneous absorber, we assume that there exists a density distribution, $\rho(x, y, z)$, in three dimensions. The distribution function $\rho$ then gives a complete
description of the density at any location in the absorber. To describe the chemical
content of the absorber, we designate $\rho_j(x, y, z)$ as the density distribution and
$w_j(x, y, z)$ as the density weight fraction of the $j^{th}$ element at $(x, y, z)$. Thus, $\rho_j$
is related to $w_j$ by $w_j(x, y, z) = \rho_j(x, y, z) / \rho(x, y, z)$. To get the total density at a
given spatial location, we must calculate the sum of the densities of all elements
at that location. This is written as $\rho(x, y, z) = \sum \rho_j(x, y, z)$. It is clear that both $\rho$
and $\mu$ are functions of spatial location. Also, the $z$-axis is in the forward direction
of X-ray because of the normal irradiation condition; hence, $dt = dz$. The
microscopic mass attenuation equation for the heterogeneous absorber now takes
the following form:

$$I(E, x, y, t) = I_0(E) \exp \left\{ - \sum_{j=1}^{n} \nu_j(E) \int_0^t w_j(x, y, z) \rho(x, y, z) \, dz \right\}$$

(12)

When density in the forward direction is uniform for all elements in the absorber, then Eq. (12) assumes the following form:

$$I(E, x, y, t) = I_0(E) \exp \left\{ - \sum_{j=1}^{n} \nu_j(E) w_j(x, y) \rho(x, y) \, dt \right\}$$

(13)

Special conditions

So far we have considered attenuation models under the good architecture
assumption. We will now present two equations for situations differing from this
condition.

In the case of oblique irradiation, the incident beam impinges on the surface
with an angle $\theta > 0$. The effective thickness $t_e$, therefore, relates to the physical
thickness of the absorber by $t_e = t \sec \theta$. When an X-ray of energy $E$ with intensity
$I_0(E)$ entering a homogeneous n-element absorber of thickness $t$ with incident
angle $\theta$, the mass attenuation equation at a point $(x, y, t_e)$ is

$$I(E) = I_0(E) \exp \{ - \nu(E) \rho t \}$$

(14)

When the incident X-ray is polyenergetic with normalized spectral distribution
$f(E)$, ranging from minimum energy $E_m$ to maximum energy $E_0$, the following
heuristic expression for the quantity $I_0$, representing the sum of intensities of
X-rays with different energies, is suggested:

$$I_0 = \int_{E_m}^{E_0} I_0(E) f(E) \, dE$$

(15)

Using this expression, the mass attenuation equation of a polyenergetic source
and either a homogeneous or a heterogeneous absorber can be deduced.

WOOD DENSITY CALCULATION FORMULA

When an X-ray densitometer of good architecture is used for wood density
measurement, the monoenergetic beam will radiate perpendicularly on a piece of
thin wood with uniform thickness and heterogeneous density structure, consisting
of four major elements (H, C, N, O) and various ash (minor) elements.

In conventional wood densitometry, a sample is a core extracted in a random
direction from the tree bole. If density variation in the radial direction is of interest,
we may conveniently use cylindrical coordinates \((z, r)\) to represent the spatial location of points in wood samples. When cylindrical coordinates (Korn and Korn 1961) are used, the \(z\)-axis represents the vertical axis of the tree bole in which direction the thickness of the wood sample is measured. The vector \(r\) is a radial line (radius) in the \(x-y\) plane and is perpendicular to the \(z\)-axis. When wood samples are very thin, density variations may be considered to be negligible in the vertical direction but easily detectable in the radial direction. Under these conditions, interest focuses on the measuring of density distribution in the radial direction, \(\rho(r)\).

Using neutron activation analyses to access effects of elemental concentrations (of individual tree-rings) on wood density measurements, Kouris et al. (1981) found that the influence from changing elemental concentrations on wood density measurements is negligible. Under this premise, we may equate, for each element in wood, a density weight fraction with relative elemental content (percent dry weight). Symbolically, we designate \(w_j(x, y)\) in Eq. (13) as \(w_j\). Rewriting Eq. (13) in cylindrical coordinates and substituting \(w_j\) for \(w_j(x, y)\), we get the mass attenuation equation of wood

\[
I(E, r, t) = I_0(E)\exp\left\{-\sum_{j=1}^{n} \mu_j(E)w_j \rho(r)t\right\}
\] (16)

Substituting Eq. (9) into Eq. (16) and then solving for \(\rho\), we obtain the equation for calculating wood density at location \(r\),

\[
\rho(r) = -\ln \frac{P(E, r, t)}{\rho(E)t}
\] (17)

In Eq. (17), \(P(E, r, t)\) is the laboratory-measured wood transmission probability that equals \(I(E, r, t)/I_0(E)\). In addition, the theoretical representation for the transmission probability can be derived by rewriting Eq. (16) as

\[
P(E, r, t) = \exp\{-\mu(E)\rho(r)t\}
\] (18)

which is the reciprocal function of Eq. (17).

**VERIFICATION AND APPLICATION**

Several applications are presented to verify or demonstrate the utility of models and formulas developed in the present study. Examples given below illustrate calculations of (1) elemental mass attenuation coefficient, (2) wood mass attenuation coefficient, (3) transmission probability, and (4) wood density.

Before performing various calculations, we used Lagrange’s procedure (Korn and Korn 1961) to interpolate, for energies ranging from 0.1 to 1,000 keV and for common elements in wood, experimentally measured total photon cross sections (Veigele 1973). After interpolation, approximately 231 data points for each curve were available for subsequent calculations.

Two criteria were used for verification: (1) the exponential transmission curve and (2) the critical absorption wavelength. The exponential law of X-radiation transmission states that increasing radiation energy is accompanied by decreased cross sections (Liebafsky et al. 1972; Knoll 1979). Photoelectric absorption occurs most readily if the binding energy is comparable with the photon energy, but the photoelectric absorption cross section falls sharply as the photon energy decreases through this critical value, i.e., critical absorption wavelength or energy. Thus, a chemical element can be identified by locating the edge that marks the
abrupt transition to a region of lower absorbance in the direction of increasing wavelength (Liebhafsky et al. 1972).

**Mass attenuation coefficients of major elements**

Elemental mass attenuation coefficients have been calculated [Eq. (5)] and are presented in the upper portion of Fig. 1 within which absorption edges are clearly depicted. Locations of these absorption edges correspond to the binding energy of K-shell electrons in the elements; therefore, the validity of the elemental mass attenuation coefficient model is established through successful identification of each of the elements used in the example.

Knowledge concerning critical absorption wavelengths of chemical elements is necessary for the selection of the appropriate X-ray energy for wood densitometry. Specifically, choosing an X-ray energy not in the proximity of absorption jumps of elements in wood will reduce the variation of the photoelectric cross section and subsequent wood density calculation.

**Mass attenuation coefficient of wood**

This model can be verified by comparing the shape of the calculated attenuation curve with that specified by the exponential transmission law. Calculation of wood mass attenuation coefficients [Eq. (11)] requires the prior information of elemental concentrations of wood. In this example, we used concentration data compiled by Hagglund (1951) to calculate mass attenuation coefficients of spruce heartwood.
Results have been plotted in the lower portion of Fig. 1. On this exponential curve, the locations of two absorption jumps correspond to those for C and O shown above. Thus, obviously X-ray attenuation in wood densitometric experiments is strongly affected by the large quantity of oxygen and carbon present. Furthermore, because of relatively low contents (small weight fractions), effects of other elements in wood are either relatively insignificant or masked (absence of absorption edges).

**Mass attenuation coefficients of ash**

The ash content of wood is normally 0.1% to 0.5% of the oven-dry weight, but it may be greater (Panshin and deZeeuw 1980). To demonstrate the effect of ash elements on wood mass attenuation coefficients, the ratio of mass attenuation coefficients of ash to that of spruce heartwood were determined at varying energy levels and are shown by the curve in Fig. 2. Elemental concentrations of H, C, N, O, Si, Na, Mg, P, S, K, Ca, Mn, and Fe were adapted from those contained in Hagglund (1951) and Fletcher and Hughes (1970).

As shown in Fig. 2, discontinuities on the curve represent absorption jumps that identify the 13 elements used. The maximum contribution of ash to the mass attenuation coefficient of spruce heartwood, diagrammed in Fig. 2, is approximately 3.4% when observed at X-ray energy of $11 \pm 1$ keV. When a softer X-ray of energy less than 6 keV is used, the contribution of ash is much less than 1%. Information contained in the single-peaked curve can be used effectively and should be strategically adapted for future wood densitometric investigations. For example, to minimize the confounding effect on wood density of environmentally induced high iron content, presumably unevenly distributed along the radial direction of the wood, one may choose a low energy X-ray (e.g., 5.41 keV) that will not react to iron atoms, thus maintaining a small ash to total wood transmission probability ratio.
Note that the transmission probability formula [Eq. (18)] is an e-based power function in which mass attenuation coefficient (\(\mu\)) is a negative component of the exponent. Because of this feature and the relatively low ash contribution to total mass attenuation coefficients, minute fluctuations of individual or total ash content should not appreciably influence either the magnitude of transmission probability or the wood density measurement [Eq. (17)].

Transmission probability

Properties of transmission probability can be studied by varying parametric values in the transmission probability model [Eq. (18)]. The analytic power of this equation is illustrated by the relationships between transmission probability and various parameters that affect X-ray transmission. Figure 3 shows that transmission probability (a) increases with increasing X-ray energy, (b) decreases with increasing sample thickness, (c) decreases with increasing density, and (d) decreases with increasing incident angle. These patterns—consequences of characteristics of the X-ray energy, and the nature and property of the absorber—offer important clues to the nature of wood transmission experiments.

Calculation of wood density

The proposed density model can be used to calculate wood density as long as parameters in the equations are measured or otherwise known. This example calculates density of pine heartwood, whose elemental concentration (fractions-by-weight) was given by Hagglund (1951). To apply Eq. (17), we have transformed wood density data (Echols 1972) into transmission probabilities (Fig. 4a) for 5.41 keV energy and for a 1-mm-thick sample. Corresponding wood density values derived from the new theoretical formula are plotted in Fig. 4b. The rise and fall of the wood density curve in Fig. 4b resemble those published previously (Polge 1965; Parker et al. 1974; Conkey 1979; Cown and Clement 1983; Kanowski 1985). This example demonstrates the possibility of using the newly developed density equation [Eq. (17)] for direct wood density calculation without undergoing the process of density calibration which is indispensable to previous experiments (Polge 1965; Ferrand 1982; Cown and Clement 1983).

SUMMARY AND DISCUSSION

An X-ray transmission experiment is governed by atomic radiation theory in which the rate of interaction is a characteristic of the radiation and the nature and structure of the absorber. Henke and his colleagues (1982) pointed out that it is possible to mathematically represent a monoatomic absorber's linear and mass attenuation coefficients by using the atom's cross section. In this investigation, the atomic cross section has been used together with atomic number and atomic weight for the derivation of microscopic models, including (1) linear attenuation equation for a monoatomic absorber [Eq. (3)], (2) mass attenuation equation for a monoatomic absorber [Eq. (7)], (3) mass attenuation equation for a polyatomic and homogeneous absorber under perpendicular irradiation [Eq. (11)], and (4) mass attenuation equation for a polyatomic and heterogeneous absorber with either nonuniform [Eq. (12)] or uniform [Eq. (13)] density in the forward direction. In practice, good results can be approximated by using Eq. (11), in which case homogenety is assumed for thin samples of wood, in lieu of Eq. (12) or Eq. (13).
FIG. 3. Transmission characteristics of ponderosa pine. Relationships between transmission probability $p$ and (A) X-ray energy $E$, (B) wood thickness $t$, and (C) wood density $\rho$. The transmission probability ratio of oblique to normal irradiation (D) is also given.
A mass attenuation equation for a polyatomic and homogeneous absorber subjected to oblique irradiation [Eq. (14)] has been proposed. Furthermore, a procedure for the construction of a mass attenuation equation of polyenergetic X-rays [Eq. (15)] has been suggested. With regard to wood densitometric applications, we have successfully formulated an equation for wood density [Eq. (17)] and its reciprocal function, the transmission probability [Eq. (18)]. Examples given in this report provide validation and suggest applications.

The conventional linear attenuation equation is perceived at the macroscopic level. By contrast, mathematical modeling of microscopic attenuation equations provides a more precise parametric description of the X-ray attenuation process occurring in absorbers in general and in wood in particular. Differences in the parametric representation of the same physical process often lead to different interpretations of the same phenomenon. In addition to an increased comprehension of X-ray attenuation, the theoretical models reported in this paper are significant because of their enhanced analytical and predictive power. These models should prove very useful in future studies involving density measurement.

As formulated in this report, the transmission probability equation contains several parameters among which radiation energy, absorber density, elemental concentration and absorber thickness are important factors in wood densitometry. Carbon and oxygen content have a dominant role in wood densitometric calculations, due to their large elemental weight fractions. Prior knowledge indicates that the weight fractions of individual ash elements do not increase or decrease significantly. Fluctuations in percent dry weight of a particular ash element should
not influence wood density calculations appreciably if the X-ray radiation energy is less than 6 keV. The transmission probability-photon energy curve (Fig. 3a) given in this report is characteristic of the relationship portrayed. This curve demonstrates that transmission probability of wood increases with increasing rate in the energy region between 5 and 10 keV, but increases with decreasing rate when photon energy is larger than 10 keV. The parametric equation [Eq. (17)] that models energy transmission of wood can readily and directly be used to calculate radial density distribution of wood. It is then feasible to develop a direct wood X-ray densitometer for direct measurement of wood density—a technique that is highly preferable to the radiographical method used in previous wood densitometric studies.

New parametric models constructed in this study offer a framework on which X-ray transmission experiments under various conditions can be studied more effectively. We hope that work will form a foundation and motivation for future wood X-ray densitometric research.

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