STOCHASTIC DAMAGE ACCUMULATION AND RELIABILITY OF WOOD MEMBERS

David Rosowsky

Assistant Professor
School of Civil Engineering
Purdue University
West Lafayette, IN 47907

and

Kenneth Fridley

Assistant Professor
Wood Research Laboratory
Department of Forestry and Natural Resources
Purdue University
West Lafayette, IN 47907

(Received October 1991)

ABSTRACT

The mechanism leading to the reduction in strength of a wood member under sustained loads is one of creep rupture. Damage models have been developed to assess static damage accumulation in wood. Forms of these models are being used in reliability analyses of wood members and systems and in the development of probability-based design codes. Questions as to the validity of these models and the appropriateness of one or more of the models in the context of reliability studies have been raised. This paper presents the current state-of-the-art with respect to damage models for wood and offers observations concerning the limitations on the use of some of these models in reliability studies. Particular attention is paid to single members in flexure and the stochastic progressive accumulation of damage as predicted by the different models.

Keywords: Creep rupture, damage models, load duration, reliability, wood.

INTRODUCTION

Damage models for wood are being used in lifetime reliability analyses of wood members and systems and in the development of probability-based design codes (Ellingwood and Rosowsky 1991; Foschi et al. 1989; Rosowsky and Ellingwood 1990). While these models have continued to evolve over the years, there are significant shortcomings associated with their forms and use. Specifically, differences exist between the various forms of the damage models and thus their predictive capabilities for damage accumulation. Furthermore, these models have been developed from high-stress, single load-pulse, time-to-failure tests. Limited verifications of these models have been made with ramp, step-constant, and cyclic load histories (Barrett and Foschi 1978b; Gerhards 1979). Questions as to the validity of these models in the context of reliability studies have been raised.

This paper presents the current state-of-the-art with respect to damage models for wood, provides some indication of how these models have been used in reliability analyses, discusses relative sensitivity of reliability results to choice of damage model, and offers observations concerning limitations of these models. Particular attention is paid to the stochastic progressive accumulation of damage as predicted by the different models.

DAMAGE MODELS FOR WOOD

The mechanism leading to the reduction in strength of a wood member under sustained loads is one of creep rupture (Barrett and Foschi 1978a; Gerhards 1979). Creep rupture arises from the propagation of voids in the microstructure of the wood at a stress level that is lower than the short-term static strength (Barrett and Foschi 1978a). A number of creep rupture models involving a state-variable similar to that used in the analysis of metal fatigue (Miner 1945) have been proposed to assess static damage accumulation in wood structural members. These models are empirical because of the complexities associated with the creep rupture failure mechanism. The more accepted models have been calibrated with data from extensive laboratory tests (Barrett and Foschi 1978a, b; Foschi and Barrett 1982; Foschi and Yao 1986; Gerhards 1979, 1988; Wood 1951).

Early damage accumulation models took into account the fact that wood strength is a function of the time the load is held at a constant intensity. The Madison Curve (Wood 1951) is an example of this type of model and is based on tests of small clear specimens. Subsequent research (Madsen 1973, 1975) showed that this effect in dimension lumber differed somewhat from what it was with small clear specimens. State-variable models evolved once it was understood that wood structural members may fail as a result of progressive accumulation of damage and not just due to overload. This state-variable, $\alpha(t)$, takes on values from 0 to 1, where $\alpha(t) = 1$ represents failure. These observations led to experimental testing programs to establish the load-duration effect for structural size lumber. The specimens were subjected to long-term loadings, and times-to-failure were recorded. Relationships between stress ratio (defined as the ratio of the applied stress to the stress assumed to cause failure in a conventional short-term strength test) and time-to-failure were established, and damage rate models were developed. Some experimental work has resulted in models proposing the existence of a threshold level below which no damage accumulates (Barrett and Foschi 1978a; Foschi and Barrett 1982; Madsen 1973). Other studies, however, failed to indicate any such threshold (Gerhards 1979, 1988; Gerhards and Link 1986). Finally, in more recent models, the damage rate has been cast as a function of the existing damage as well as the current stress (Barrett and Foschi 1978a; Foschi and Yao 1986).

Examples of the more widely used damage accumulation models are the following: "*Madison Curve*[1]" (Hendrickson et al. 1987):

$$\frac{d\alpha}{dt} = A(\sigma - \sigma_0)^B \qquad (1)$$

Barrett and Foschi Model I (B/F-I) (Barrett and Foschi 1978a):

$$\frac{d\alpha}{dt} = A(\sigma - \sigma_0)^{B,C} \qquad (2)$$

Barrett and Foschi Model II (B/F-II) (Barrett and Foschi 1978a):

$$\frac{d\alpha}{dt} = A(\sigma - \sigma_0)^B + C\alpha \qquad (3)$$

Exponential Damage Rate Model (EDRM, Gerhards) (Gerhards 1979):

$$\frac{d\alpha}{dt} = \exp(-A + B\sigma) \qquad (4)$$

in which σ is the stress ratio (dimensionless) defined as the applied stress at time t divided by the stress known to cause failure in a conventional short term strength test. A, B and C are experimental parameters based on failure data from various load duration tests. The parameter σ_0 is the damage threshold stress ratio. For values of $\sigma < \sigma_0$, no damage occurs. In some models, the rate of damage accumulation, $d\alpha/dt$, is a function of the applied stress only (Eqs. 1 and 4), whereas in others, $d\alpha/dt$ is a function of the existing damage state, α, as well (Eqs. 2 and 3).

Another model proposed recently for damage accumulation has the form: *Foschi and Yao Model (F/Y)* (Foschi and Yao 1986):

$$\frac{d\alpha}{dt} = A[\tau(t) - \sigma_0\tau_s]^B + C[\tau(t) - \sigma_0\tau_s]^n\alpha \qquad (5)$$

---

[1] Damage rate implied by original Madison Curve.

where $\tau(t)$ is the applied stress, τ_s is the short-term strength of the member, α is the existing damage, and A, B, C, and n are model parameters. The model parameters B, C, n, and σ_0 are treated as random variables, and A is functionally related to B and the ramp loading rate, k, as

$$A = \frac{k(B + 1)}{[\tau_s(1 - \sigma_0)]^{B+1}} \tag{6}$$

$$k = \frac{\tau_s}{T_s} \tag{7}$$

where T_s = time-to-failure in the short-term test. This model differs from the earlier models in that it is not expressed in terms of stress ratios.

A plot of constant stress ratio vs. time-to-failure for several damage models is shown in Fig. 1. These models are all based on static load tests. A comparison of the Madison Curve model and the EDRM (Gerhards) model for Douglas-fir lumber in bending reveals that these models are similar in the short-duration, high stress range and diverge as stress decreases. The Barrett and Foschi Model II, with threshold ratio $\sigma_0 = 0.5$, is based on tests of western hemlock. As illustrated in Fig. 1, between about 7 hours and 2 years, there is a significant difference between the EDRM and B/F-II models, with the EDRM tending to overestimate the stress ratio relative to the B/F-II model. Beyond about 2 years, the models begin to converge until about 8 years, the time at which the B/F-II model reaches its threshold stress ratio of 0.5, after which they diverge again. The Foschi and Yao model can be integrated to solve for time-to-failure for a given applied stress (failure defined as $\alpha = 1$), with model parameters taken from Karacabeyli (1987). The curves in Fig. 1 for the F/Y model suggest: (1) significant differences exist between damage models for lumber in bending; (2) while trends (shapes) are similar, some differences exist between curves for Eqs. 5–7 for different species; and (3) differences exist for different limit states (flexure, compression, tension).

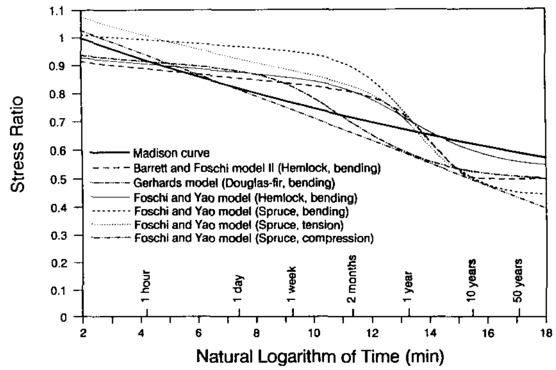

FIG. 1. Comparison of damage accumulation models.

SPECIES EFFECTS

Each of the damage models was developed based on experimental test results involving specific lumber species. The Madison Curve was developed originally from small clear Douglas-fir test data (Wood 1951) and has since been used for other species, as well as full size structural lumber. The EDRM (Eq. 4) was developed and calibrated to data from structural Douglas-fir lumber (Gerhards 1988). The Barrett and Foschi Models I and II (Eqs. 2 and 3) were originally calibrated to the same small clear Douglas-fir data used for the development of the Madison Curve (Barrett and Foschi 1978a), and were subsequently calibrated to data from extensive tests of western hemlock lumber (Foschi and Barrett 1982). The Foschi and Yao model (Eq. 5) was fit to data from tests of structural grade western hemlock and spruce lumber (Karacabeyli 1987).

The data presented in the literature (e.g., Foschi and Barrett 1982; Gerhards 1988; Karacabeyli 1987; Madsen 1973) suggest the existence of a species effect in the load-duration behavior of lumber. This effect is evidenced by the forms of the resulting damage models (Fig. 1). For example, Douglas-fir lumber exhibits a general log-linear trend in damage accumulation, while western hemlock and spruce exhibit more of a nonlinear response. Furthermore, a stress threshold is observed and included in the models for western hemlock and spruce data, but no such observation or model parameter is included for the Douglas-fir data.

OTHER FACTORS AFFECTING DAMAGE ACCUMULATION

Other factors can contribute to damage accumulation in structural wood members. For example, the type (i.e., bending, tension, compression, etc.) and direction (i.e., parallel to the grain, perpendicular to the grain) of loading may affect the damage accumulation process. This is partially illustrated in Fig. 1 for the Foschi and Yao model calibrated for spruce lumber in bending, tension, and compression.

Temperature and moisture content are known to affect the short-term strength and stiffness characteristics of lumber (Fridley et al. 1992a). These conditions can also affect damage accumulation. An extensive research effort was conducted to evaluate the effects of environmental conditions on damage accumulation (Fridley et al. 1992b). It was concluded for constant environmental conditions that damage accumulation was not sensitive to temperature and moisture content *if* the short-term strength was accurately adjusted for the current hygrothermal condition of the lumber. For cyclic moisture conditions, however, significant damage resulted that may be related to so-called mechano-sorptive creep strain often observed in wood. Mechano-sorptive creep strain is the nonlinear interaction between changing moisture content and applied stress resulting in excess creep (Bažant 1985).

OTHER MODELING APPROACHES

Nielsen and Kousholt presented a load-duration strength model for wood based on viscoelastic fracture mechanics (Nielsen and Kousholt 1980). This model was reviewed and presented again by Johns and Madsen with particular reference to full size lumber (Johns and Madsen 1982). Nielsen later justified his model and presented results from his studies as well as results from studies which utilized his model (Nielsen 1986a, b).

The Nielsen fracture mechanics model was developed using the Dugdale crack model and predicts the time to catastrophic failure as

$$t_f = (8/\pi^2)(\phi Q + 1)(\sigma_1/\sigma_{cr})^2(\sigma_{cr}/\sigma_a)^2 \times \\ \times \int_1^{(\sigma_{cr}/\sigma_a)^2} \left(\frac{F[\theta]}{\theta}\right) d\theta + F[(\sigma_{cr}/\sigma_a)^2] \quad (8)$$

where t_f is the time to failure, $\theta = (\sigma_{cr}/\sigma_a)^2 c/c_{cr}$, ϕ is a weighted creep function constant, Q is a crack growth model constant, σ_1 is the ultimate Dugdale crack tip stress, σ_{cr} is the critical value of the externally applied stress which is approximately equal to the Griffeth stress capacity, σ_a is the applied stress, $F(\theta)$ is the inverse of a crack tip creep function, c is the half crack length, and c_{cr} is the critical half crack length. Knowing an appropriate crack tip creep function allows the inverse creep function, $F(\theta)$, to be formulated.

Johns and Madsen applied Nielsen's model (Eq. 8) to full size lumber (Johns and Madsen 1982). A creep model of the form

$$\epsilon(t) = \sigma(1 + at^b)/E \quad (9)$$

was used to write the inverse creep function

$$F(\theta) = (1/a)^{1/b}[(\theta - 1)^{1/b}/\theta] \quad (10)$$

where a and b are empirical constants. This fracture model, as formulated above, predicts different load-duration responses for different applied stress ratios (Johns and Madsen 1982; Nielsen 1986b), similar to the Foschi and Yao damage model (Eq. 5).

Numerical and statistical approaches to the load-duration problem in wood have been used by several researchers. Wood's original model (Eq. 1) was actually a best fit to a hyperbolic equation that matched observed data; it was later recast as a damage accumulation model (Hendrickson et al. 1987). Martin discussed the stress-life relationship for wood from a mathematical and statistical point of view (Martin 1980). Saunders and Martin presented three numerical load-duration models and illustrated that their models, developed from statistical considerations, echoed the trends predicted by the EDRM model (Eq. 4) and the Barrett and Foschi Models I and II (Eqs. 2 and 3) (Saunders and Martin 1988).

A model based on chemical reactions during plastic deformation was proposed (van der Put 1989b) that relates fracture times to bond breaking processes in the wood. The model is based on physical parameters and is quite flexible; however, the governing equations are very complex and their application to structural lumber with its inherent defects and high material variability may be difficult. This model is applicable only to discrete zones of deformation (van der Put 1989a).

A strain energy model to predict load-duration effects in lumber has recently been developed (Fridley et al. 1991). A critical strain energy density function was established from experimental observations. Failure was defined as the exceedence of a critical strain energy density, u_{cr}, which was assumed to correspond to the initiation of nonlinear material behavior. For example, u_{cr} in a standard ramp load strength test would correspond to the proportional limit; in a creep test, u_{cr} would correspond to the initiation of tertiary creep. The strain energy definition of failure was found to be invariant with respect to load (history, duration, magnitude), environment (temperature, moisture content, constant, cyclic), and material (grade, stiffness, strength). Species effects on the strain energy model have not been addressed.

DAMAGE MODELING AND RELIABILITY ANALYSIS

Although many approaches are available to describe load-duration effects in lumber, most of the recent work has been directed toward the development and calibration of damage accumulation models. Therefore, greater confidence is placed in the more popular damage models (i.e., Eqs. 1–5) owing to the simple fact that more experimental data went into the development of these models. Furthermore, some of the non-damage (i.e., statistical, thermodynamic, energy) models have been shown to echo trends predicted by damage models (Saunders and Martin 1988). From a computational standpoint, the damage models, with the exception of the Foschi and Yao model (Eq. 5), offer the convenience of being nondimensional since they are expressed in terms of a stress ratio and not unique values of applied stress and strength. The discussion that follows will focus on the use of damage models in predicting the stochastic accumulation of damage and in evaluating lifetime reliability.

In structural reliability analysis, failure is defined as the exceedence of a particular limit state; failure is not limited to loss of load-carrying capacity, but could relate to unserviceability (excessive deflection, vibration, etc.) as well.

The limit state is formulated as,

$$g(\mathrm{X}) = g(X_1, X_2, \ldots, X_n) = 0 \qquad (11)$$

in which X is the vector of engineering variables, including loads, material strengths and dimensions. The function $g(\mathrm{X})$ is formulated, by convention, so that failure occurs for combinations of variables producing $g < 0$; the structure is safe when $g > 0$ (Melchers 1987). Therefore, the limit state probability is the probability that $g < 0$. This probability is found by integrating the joint probability density function of all of the load and resistance variables involved in the reliability function over the domain of $(X_1, X_2, \ldots, X_n)$ where $g < 0$. If the loads or strength are time-dependent, g is also a function of time, t. In many cases, because of the large number of random variables involved, the exact form of the joint density function is not known. Monte-Carlo methods can be used as an alternative to numerically integrating the joint PDF in order to evaluate failure probabilities. Alternatives to Monte-Carlo simulation are first-order second-moment (FOSM) methods (Melchers 1987).

Damage in wood accumulates stochastically (in accordance with the assumed damage accumulation model) owing to the variation of structural loads in time. Simple pulse models have been used to model the temporal variation of static gravity loads due to the permanent weight of the structure and the permanent attachments, occupancy live loads, and snow loads (Hendrickson et al. 1987). Typical load process statistics and a detailed background of their development may be found in (Ellingwood et al. 1980; Rosowsky and Ellingwood 1990).

Using the state-variable representation of damage accumulation, as represented in Eqs. 1–5, the state of cumulative damage in a member at time t can be expressed as the sum of the incremental damage due to the applied loads up to time t:

$$\alpha(t) = \sum_i \Delta\alpha_i \tag{12}$$

The increment of damage that accumulates under load pulse i of duration τ is expressed,

$$\Delta\alpha_i = \int_0^{\tau_i} \left(\frac{d\alpha}{dt}\right) dt \tag{13}$$

The damage rate is defined by the appropriate damage model. Mathematically, $\alpha(t)$ is a monotonically increasing stochastic process. Failure occurs if $\alpha \geq 1$ at the end of the service life of the structure, or the otherwise specified reference period (T). The cumulative damage limit state can now be written,

$$g(\mathrm{X}) = 1 - \alpha(T) = 1 - \sum_i \Delta\alpha_i \tag{14}$$

Equation 11 is in the form of a first passage problem with failure being defined as the first passage of the state-variable α through 1 in the interval (0, T). Solutions to first passage problems have not been obtained in closed form, except for a few simple cases (Larabee and Cornell 1981). Monte-Carlo simulation can be used to estimate the probability of failure (P_{fdol}) due to damage accumulation. The associated reliability index, β_{dol}, is estimated as (Melchers 1987),

$$\beta_{dol} = \Phi^{-1}(1 - P_{fdol}) \tag{15}$$

in which Φ^{-1} = inverse standard normal probability density function.

Another reliability measure, β_{ovld}, considers overload failure due to the 50-year maximum load event, and does not include time-dependent damage accumulation. β_{ovld} can be computed using first-order second-moment (FOSM) reliability analyses of the type used previously for steel and reinforced concrete (Ellingwood et al. 1980; Melchers 1987). These values can be compared with those obtained from the stochastic damage accumulation analysis, β_{dol}, as described above.

ANALYSIS OF EXISTING CRITERIA FOR WORKING STRESS DESIGN

A series of reliability assessments of current Working Stress Design (WSD) criteria were performed in order to gain a fundamental understanding of how damage accumulates stochastically and to provide reliability benchmarks for subsequent code development (Ellingwood and Rosowsky 1991; Rosowsky and Ellingwood 1990). The members considered were designed according to the following safety check for Working Stress Design (National Forest Products Association 1986):

$$\zeta F_b S_x \geq D_n + X_n \tag{16}$$

in which S_x = section modulus in bending, F_b = allowable stress in bending, based on an assumed 10-year duration, ζ is a factor which reflects the assumed duration-of-load (DOL) effect for the design load in the current NDS (National Forest Products Association 1986), D_n = nominal dead load, and X_n = varying load (occupancy live, snow, roof, as appropriate). The stochastic stress ratio at time t is given by,

$$\sigma(t) = \frac{D + X(t)}{S_x F_r} \tag{17}$$

in which D = dead load, $X(t)$ = time varying load, F_r = short-term modulus of rupture, obtained by loading the beam to failure over a period of approximately 5–10 minutes. Substituting from Eq. 16, the stress ratio for use in the damage accumulation and reliability analyses can be written,

$$\sigma(t) = \frac{D + X(t)}{D_n + X_n} \frac{1}{(F_r/\zeta F_b)} \tag{18}$$

where D, X, and F_r are random variables.

A Bernoulli pulse process is used to model the roof snow load. Temporal and spatial mod-

TABLE 1. *Comparison of damage models for dead + snow loads.*

Damage model	Species	β_{dol}	β_{ovld}
Madison Curve	DF (small clear)	2.14	2.68
EDRM	DF	2.14	2.68
Barrett and Foschi, Model I	DF (small clear)	2.31	2.68
Barrett and Foschi, Model II ($\sigma_0 = 0.5$)	WH	2.33	2.68
Foschi and Yao	WH	2.36	2.68

el parameters are developed from load surveys and discrete process theory. A detailed description of these models and their use in reliability studies is presented in Rosowsky and Ellingwood (1990). The sensitivity of reliability of single members subjected to dead + snow loading to the choice of damage model is evaluated by comparing reliabilities obtained using the Madison Curve, the EDRM (Gerhards), the Barrett and Foschi Models I and II, and the Foschi and Yao model. The forms of these models were discussed previously and are shown in Fig. 1. The recent Foschi and Yao damage model (Eq. 5) is more difficult to implement in a reliability analysis than the other models because it cannot be nondimensionalized, owing to the presence of the terms τ_s^B and τ_s^n. An approximate expression is used for cumulative damage during a single pulse, *i*, of assumed constant intensity (Foschi et al. 1989):

$$\alpha_i = \alpha_{i-1}K_i + L_i \tag{19}$$

where,

$$K_i = \exp[C(\tau_i - \sigma_0\tau_s)^n \Delta t] \tag{20}$$

$$L_i = \frac{A}{C}(\tau_i - \sigma_0\tau_s)^{B-n}(K_i - 1). \tag{21}$$

The damage can now be calculated as the sum of the damage increments accumulated under each pulse, as was done using the other models. In order to directly compare this model with the Barrett and Foschi Model II, with $\sigma_0 = 0.5$, the model parameters for western hemlock are taken from Foschi et al. (1989). The median values are selected and used as deterministic values for the parameters: $B = 37.16$, $C = 2.465 \times 10^{-7}$, $n = 1.290$, $\sigma_0 = 0.533$. Mean values for the short-term strength, τ_s, for western hemlock were taken from Karacabeyli (1987). The parameter A is computed to be $4.82 \times 10^{-16}\ \text{h}^{-1}$ for $k = 388.5$ ksi/h. Values for parameters of the other models are taken from the literature, but only the B/F-II and the F/Y models were actually calibrated to the same data sets (i.e., species). Table 1 indicates reasonable agreement in the reliability levels associated with a combined dead + snow loading using the different damage models. The close agreement of the Madison Curve with the EDRM (Gerhards) results, and of the B/F-II model with the F/Y model results is apparently a consequence of the close agreement of these pairs of models in the range of durations typical for snow pulses (1–4 weeks) as seen in Fig. 1.

A parametric study using the snow pulse model was performed in order to investigate the effect of the total number of pulses retained (largest to smallest) in the load model on the reliabilities. Table 2 presents DOL reliabilities for the different damage models, considering damage accumulation during both the full snow load process and the maximum pulse only. Figure 2 shows the relative frequency of the number of pulses required to fail a beam (using the EDRM), given that the beam fails. This table and figure support the hypothesis that

TABLE 2. *Comparison of damage models for dead + snow loads, full process and maximum pulse.*

Damage Model	β_{dol} (full)	β_{dol} (max.1)
Madison Curve	2.14	2.14
EDRM	2.14	2.15
Barrett and Foschi, Model I	2.31	2.31
Barrett and Foschi, Model II ($\sigma_0 = 0.5$)	2.33	2.34

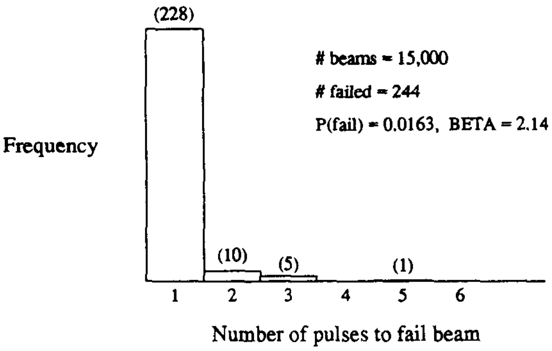

FIG. 2. Relative frequency of number of pulses to fail a beam under dead + snow loads.

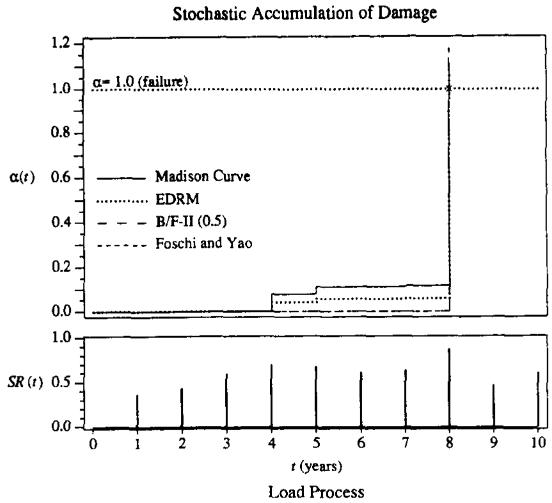

FIG. 3. Damage accumulation model comparison for load process "A."

most of the failures occur under a single load pulse, and therefore the reliability based on damage accumulating during the maximum snow load should be nearly the same as that for the full snow load process. The reduction of the full snow load process to the maximum one or two load pulses appears valid because of the high coefficient of variation in the load intensity and highly nonlinear nature of the damage accumulation.

A Bernoulli process is also used (as a simplification to the assumed underlying Poisson process) for each of the light-occupancy live load components (a sustained component representing occupancy, and a transient extraordinary component for overcrowding situations). Details of this model can be found in Rosowsky and Ellingwood (1990). Table 3 presents reliabilities for single members under combined dead + live load considering the EDRM, B/F-II, and F/Y models for comparison. The B/F-II and F/Y models produce effectively the same reliabilities for pulse processes where the pulses significant in causing damage to accumulate have relatively short duration. This is consistent with what was observed for the dead + snow combination (Table 2). From the comparison of the models shown in Fig. 1, some differences may be expected with long-duration pulses (i.e., heavy occupancy live load). Because the Foschi and Yao model involves significantly more computation than the Barrett and Foschi model, as a consequence of the increased number of parameters and the dimensional form, the B/F-II (σ_0 = 0.5) was the preferred damage model in subsequent comparisons in (Rosowsky and Ellingwood 1990).

TABLE 3. *Comparison of damage models for dead + live loads.*

Model	β_{dol}	β_{ovld}
EDRM	2.66	3.18
Barrett and Foschi, Model II (σ_0 = 0.5)	2.84	3.18
Foschi and Yao	2.89	3.18

An analysis of characteristics of damage accumulation revealed that only the combined sustained and extraordinary pulses (L_s + L_e rather than just L_s) needed to be considered in combination with the dead load (Rosowsky and Ellingwood 1990). For both single member and system studies, it was found that the pulse processes representing the structural load histories could be greatly simplified for the purpose of cumulative damage failure probability analysis. Owing to the sparse nature of the structural loads (Hendrickson et al. 1987; Rosowsky and Ellingwood 1990) and the highly nonlinear nature of the damage accumulation models, only the largest few pulses drive the damage accumulation process. Therefore, an order-statistics approach to generating the *n* largest pulses could be employed rather than generating the complete load history.

STOCHASTIC ACCUMULATION OF DAMAGE

In order to visualize the stochastic accumulation of damage as predicted by the different damage models, a series of load processes are considered. These processes, along with the associated progressive accumulation of damage, are illustrated in Figs. 3–8. The load processes are 10 years long and consist of a single pulse arrival per year. All pulses have a duration of 1 week unless otherwise specified. This form is based on the combined sustained and extraordinary occupancy live load model discussed previously, and the observation that for damage accumulation reliability studies, only the combined pulses ($L_s + L_e$) needed to be considered. The load magnitudes do not necessarily coincide with a typical 10-year live load history, but rather are selected simply for illustrative purposes (i.e., for visualization of the damage accumulation process). Sustained components are not considered. These pulses, such as would represent the dead and sustained live load components, have been shown not to contribute to the damage accumulation process (Hendrickson et al. 1987; Rosowsky and Ellingwood 1990), as the magnitudes are significantly lower than would be required to accumulate damage based on the models. Four damage models are considered: the Madison Curve, the exponential damage rate model (EDRM), the Barrett and Foschi Model II (σ_0 = 0.5, western hemlock), and the Foschi and Yao Model (western hemlock, bending).

Figure 3 illustrates a load process for which all the damage models considered predict the *same* time-to-failure. However, it can be seen that the "damage paths" are quite different. For example, the Madison Curve and the EDRM both have distinct paths that appear to be the result of progressive damage accumulation, whereas the B/F-II and F/Y models seem to indicate failure as a result of a single large pulse. Recall that these models purport the rate of damage to be a function of existing damage in addition to the stress ratio. A closer

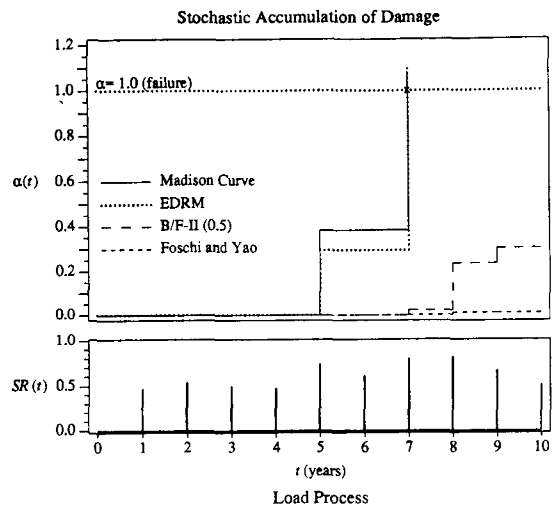

FIG. 4. Damage accumulation model comparison for load process "B."

inspection of the damage path for these two models reveals that some damage is induced prior to the pulse at year 8, but that this amount is very small. The question might now be asked, is this failure an "overload" type[2] or a cumulative (progressive) damage type? Visual inspection would suggest that it is the largest single pulse *only* that fails the member according to the B/F-II and F/Y models.

Figure 4 shows a case in which two of the models predict member failure at year 7, but the other two models predict the member survives the load process. Figure 5 shows a case of two different times-to-failure. Here, only the pulses at years 8 and 9 are changed slightly with respect to those in Fig. 4, resulting in all models predicting failure. The damage path for the F/Y model still appears to be associated with a single pulse.

Figures 6–8 consider the same series of load magnitudes, but vary the pulse duration (τ) from 1 day to 3 months. Figure 6 (τ = 1 day) shows failure predicted by two of the four

[2] This should not be confused with the definition of "overload" used in reliability studies (Ellingwood and Rosowsky 1991; Foschi et al. 1989; Hendrickson et al. 1987; Rosowsky and Ellingwood 1990) which refers to the condition of stress ratio >1.0.

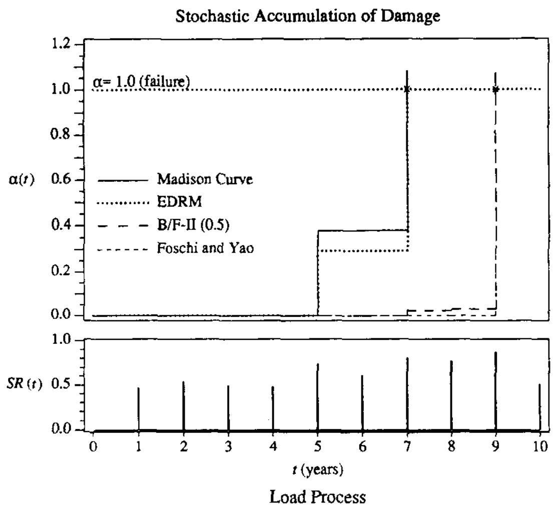

FIG. 5. Damage accumulation model comparison for load process "C."

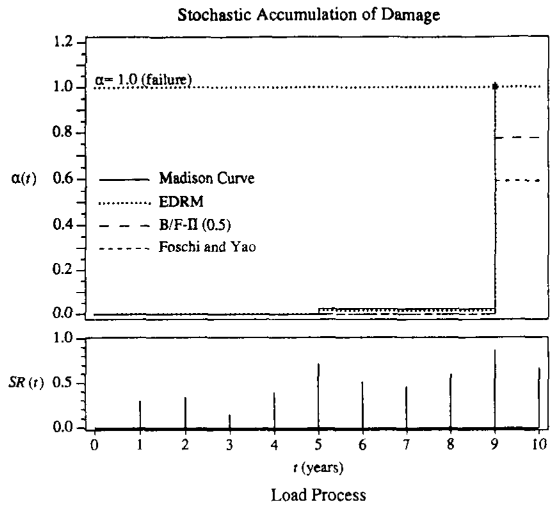

FIG. 6. Damage accumulation model comparison for load process "D" and τ = 1 day.

models, with similar damage paths. Figure 7 extends the pulse durations to 1 week, and while all models indicate failure, slightly different damage paths are observed. The B/F-II and F/Y models suggest failure is due to a single pulse at year 9. When the durations are extended to 1 month (not shown), all models again predict failure, and the differences between damage paths for the Madison Curve and EDRM are exaggerated. Finally, Fig. 8 depicts the damage due to the load process with pulse duration $\tau = 3$ months. In this case, all models predict failure, though two different times-to-failure are observed, and (nearly) all failures appear to be due to a single pulse.

Recall that the Barrett and Foschi model assumes the existence of a stress threshold, σ_0. This threshold has not been observed in other testing programs. However, as Fig. 1 illustrates, and as evident from these figures, this threshold is insignificant for damage analyses based on critical pulses of duration on the order of days to months. Figure 1 shows that the threshold influence (the steep decline in the damage model) does not occur until about 1 year.

While it has long been accepted that the trends shown in Figs. 3–8 are indicative of how damage accumulates based on the available models, no such collection of figures has been published previously.[3] These figures graphically illustrate the great differences that exist between these models in predicting the state of damage in a wood member.

LIMITATIONS OF EXISTING MODELS

As a consequence of the testing programs to develop the currently available damage accumulation models, a state-variable representation of damage is used rather than a member property degradation model. Furthermore, these models include no provision for changing environmental conditions. Only recently have provisions been made in the damage models for variation in temperature or moisture-content, for example (Fridley 1992b).

The mechanics of creep rupture suggest that a degradation of member stiffness occurs with load-duration, as voids are propagated through the length of the wood. Existing damage mod-

[3] A few figures showing $\log_{10}(\alpha)$ vs. time were presented by Corotis and Sheehan (1986) for a specific damage model and load process. Plotting the logarithm of damage accentuated the progressive accumulation process. This study was one of the initial justifications for the use of such damage models with stochastic load models in reliability analyses of wood members.

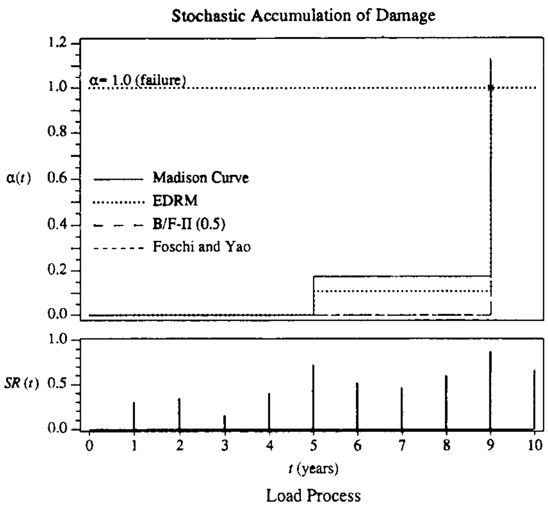

FIG. 7. Damage accumulation model comparison for load process "D" and τ = 1 week.

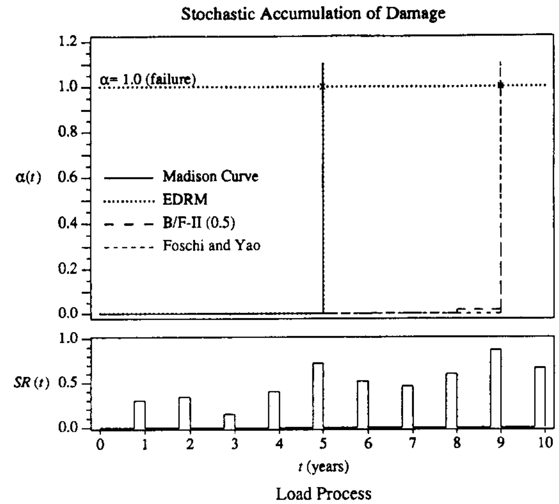

FIG. 8. Damage accumulation model comparison for load process "D" and τ = 3 months.

els tacitly assume that the member properties of the wood are the same *just prior* to failure as they are in the undamaged state. While time-dependent stiffness may be a significant factor in system strength studies, as the inherent load-sharing in multi-member systems is a function of the relative member stiffnesses, this dependency may be particularly important for deflection serviceability studies of both single members and systems.

SUMMARY AND CONCLUSIONS

The current state-of-the-art with respect to damage accumulation models developed to describe load-duration effects in wood has been presented. These models were developed for specific materials under specific load histories, and thus predict different responses. The use and implications of these models in reliability analyses have been presented, and the relative sensitivity of the reliability levels to the use of the various damage models was discussed. Assumed in the reliability analyses is the validity of summing incremental damage. The stochastic progressive accumulation of damage as predicted by the different models was discussed and illustrated. It was shown that the damage accumulation process was highly dependent on the magnitude of the loads. Furthermore, in many situations, the damage process did not appear to be *cumulative* in nature, but may be the result of a single pulse of some critical magnitude and duration. Therefore, the problem may be reduced to a process consisting of a *single* pulse. This differs from an *overload* problem in that the duration of the pulse is significant. It may, however, indicate that entire load histories need not be considered in a reliability analysis including load-duration effects, but rather only the duration and magnitude of a single *critical* pulse. Further experimental and analytical research is required to confirm this hypothesis and quantify the important parameters with respect to defining the critical pulse.

REFERENCES

BARRETT, J. D., AND R. O. FOSCHI. 1978a. Duration of load and failure probability in wood. Part I. Modelling creep rupture. Canadian J. Civil Eng. 5(4):505–514.

———, AND ———. 1978b. Duration of load and failure probability in wood. Part II. Constant, ramp, and cyclic loadings. Canadian J. Civil Eng. 5(4):515–532.

BAŽANT, Z. P. 1985. Constitutive equation of wood at variable humidity and temperature. Wood Sci. Technol. 19:159–177.

COROTIS, R. B., AND D. P. SHEEHAN. 1986. Wood damage accumulation by stochastic load models. ASCE J. Struct. Eng. 112(11):2402–2415.

ELLINGWOOD, B., AND D. ROSOWSKY. 1991. Duration of load effects in LRFD for wood construction. ASCE J. Struct. Eng. 117(2):584–599.

———, T. V. GALAMBOS, J. G. MACGREGOR, AND C. A. CORNELL. 1980. Development of a probability based load criterion for American National Standard A58. National Bureau of Standards Special Publication SP No. 577. Washington, DC.

FOSCHI, R. O., AND J. D. BARRETT. 1982. Load duration effects in western hemlock lumber. ASCE J. Struct. Div. 108(7):1494–1510.

———, AND Z. C. YAO. 1986. Another look at three duration of load models. Report by Working Commission W18—Timber Structures, International Council for Building Research Studies and Documentation. Florence, Italy.

———, B. R. FOLZ, AND F. Z. YAO. 1989. Reliability-based design of wood structures. Structural Research Series Report No. 34. Department of Civil Engineering, University of British Columbia, Vancouver, Canada.

FRIDLEY, K. J., R. C. TANG, AND L. A. SOLTIS. 1991. Load-duration effects in structural lumber: A strain energy approach. Submitted to ASCE J. Struct. Eng.

———, ———, AND ———. 1992a. Hygrothermal effects on the mechanical properties of lumber. ASCE J. Struct. Eng. 118(2) (in press).

———, ———, AND ———. 1992b. Hygrothermal effects on load-duration behavior of structural lumber. ASCE J. Struct. Eng. 118(4) (in press).

GERHARDS, C. C. 1979. Time-related effects of loading on wood strength. A linear cumulative damage theory. Wood Sci. 11(3):139–144.

———. 1988. Effect of grade on load duration of Douglas-fir in bending. Wood Fiber Sci. 20(1):146–161.

———, AND C. L. LINK. 1986. Effect of loading rate on bending strength of Douglas-fir 2 by 4's. Forest Prod. J. 36(2):63–66.

HENDRICKSON, E. M., B. ELLINGWOOD, AND J. MURPHY. 1987. Limit state probabilities for wood structural members. ASCE J. Struct. Eng. 113(1):88–106.

JOHNS, K., AND B. MADSEN. 1982. Duration of load effects in lumber. Part I. A fracture mechanics approach. Canadian J. Civil Eng. 9(4):502–514.

KARACABEYLI, E. 1987. Duration of load research for lumber in North America. Report prepared for the Canadian Wood Council Lumber Properties Steering Committee. Forintek Canada Corp., Canada.

LARRABEE, R. D., AND C. A. CORNELL. 1981. Combination of various load processes. ASCE J. Struct. Div. 107(1):223–239.

MADSEN, B. 1973. Duration of load tests for dry lumber in bending. Forest Prod. J. 23(2):21–28.

———. 1975. Strength values for wood and limit states design. Canadian J. Civil Eng. 2(3):270–279.

MARTIN, J. W. 1980. The analysis of life data for wood in the bending mode. Wood Sci. Technol. 14:187–206.

MELCHERS, R. E. 1987. Structural reliability: Analysis and prediction. Ellis Horwood Ltd., distributed by John Wiley and Sons, New York, NY.

MINER, M. A. 1945. Cumulative damage in fatigue. ASME J. Appl. Mech. 67:A159–A164.

NATIONAL FOREST PRODUCTS ASSOCIATION. 1986. National design specification for wood construction. NFPA, Washington, DC.

NIELSEN, L. F. 1986a. Wood as a cracked viscoelastic material. Part I. Theory and application. Proceedings of the International Workshop on Duration of Load in Lumber and Wood Products. SP-27. Forintek Canada Corp., Canada.

———. 1986b. Wood as a cracked viscoelastic material. Part II. Sensitivity of justification. Proceedings of the International Workshop on Duration of Load in Lumber and Wood Products. SP-27. Forintek Canada Corp., Canada.

———, AND K. KOUSHOLT. 1980. Stress-strength-lifetime relationship for wood. Wood Sci. 12(3):162–174.

ROSOWSKY, D. V., AND B. R. ELLINGWOOD. 1990. Stochastic damage accumulation and probabilistic codified design for wood. Cvil Eng. Rept. No. 1990-02-02. The Johns Hopkins University, Baltimore, MD.

SAUNDERS, S. C., AND J. W. MARTIN. 1988. The problem of strength and duration of load for structural timbers. Proceedings of the 1988 International Timber Engineering Conference. Forest Products Research Society, vol. 1, Madison, WI.

VAN DER PUT, T. A. C. M. 1989a. Theoretical explanation of the mechanosorptive effect in wood. Wood Fiber Sci. 21(3):219–230.

———. 1989b. Deformation and damage processes in wood. Delft University Press, Delft, The Netherlands.

WOOD, L. W. 1951. Relation of strength of wood to duration of loads. Rept. No. 1916. U.S. Forest Products Laboratory, Madison, WI.