THEORETICAL WOOD DENSITOMETRY: II. OPTIMAL X-RAY ENERGY FOR WOOD DENSITY MEASUREMENT

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ABSTRACT

Using a comparable approach, we extended the theoretical X-ray wood densitometric models to a case encountered in practice. Optimal X-ray energy was treated as the photon energy of the X-radiation which produced the maximum radiation resolution, as measured by differential transmission probabilities detected in a densitometric experiment. Parametric representation of radiation resolutions revealed that the maximum resolution of a specific densitometric procedure is governed by the range of densities in a given wood. The maximum radiation resolution obtainable in a particular wood densitometric experiment can be calculated readily from two equations derived in this study. Examples show that under "good architecture" conditions (1) transmission probabilities for a given wood densitometric experiment increase their magnitudes as the X-ray becomes more energetic, yet the maximum radiation resolution remains constant for a given set of parametric values; (2) optimal X-ray energies, for nine types of coniferous wood, are in the range of 5.13–5.69 keV for 1.0-mm-thick samples; (3) wood with a broader density range results in higher maximum resolution when irradiated by the theoretically optimal X-ray energy; and (4) accurate wood density measurements could be achieved only if the radiation energy used was near the optimal level. Regarding the architecture of a radiation detection system, the advantage of using a monochromator to reduce the X-ray energy continuum so as to increase the accuracy of wood density measurements was examined.

Keywords: Optimal X-ray energy, wood densitometry, differential transmission probability, maximum radiation resolution, wood density.

INTRODUCTION

The search for the optimal X-ray energy to use in wood densitometric measurement has confronted forest scientists for many years. From a radiation detection point of view, the optimal X-ray energy for wood density measurement should be one that provides maximum resolution; namely, an X-ray having the highest power to distinguish high densities from low ones. This is especially true when a densitometric experiment is only a prelude to dendrochronological analyses. The search for such an optimal X-ray energy requires analytical tools that quantitatively describe interaction mechanisms occurring between X-ray photons and constituent atoms in wood. Liu and his coworkers (1987) used mathematical models to describe these interactions. This study is an extension of that work, devising a method for deciding the optimal X-ray energy.

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THEORY AND METHODS

First, a brief discussion of the concept of accuracy in X-ray densitometry is needed. Derivations of a parametric expression for measurement accuracy, or in this case, resolution in radiation detection, will then be performed. Since the present study utilizes the models developed by Liu et al. (1987), a review of their methodology will aid in understanding the current work.

Previous theoretical models

Within the framework of the previous study (Liu et al. 1987), an X-ray wood densitometric experiment was conducted under good architecture conditions in which a piece of wood with uniform thickness t and heterogeneous density distribution $\rho(r)$ was perpendicularly irradiated by narrow, monoenergetic X-rays of energy E and intensity $I_0(E)$. The irradiated surface was the transverse surface of the wood in which only radial density variations were of interest. In addition, the wood comprised four major elements (H, C, N, O) and a definite number of minor ones.

It was demonstrated that the mass attenuation coefficient ν_j of the j^{th} element in a heterogeneous material like wood can be calculated by

$$\nu_j(E) = N_a\sigma_j(E)/M_j \tag{1}$$

Here, N_a is Avogadro's number and M_j, the atomic weight of the j^{th} element. Values of the total photon cross section $\sigma_j(E)$ can be found in atomic data tables (Veigele 1973; Hubbell 1982). Assuming that w_j, the density weight fraction for the j^{th} element is constant and is equivalent to the percent dry weight of the j^{th} element in wood, and that the sum of w_j equals unity, Liu et al. introduced a theoretical model for the mass attenuation coefficient of an n-element wood

$$\nu(E) = \sum_{j=1}^{n} \nu_j(E)w_j \tag{2}$$

With this parametric representation of $\nu(E)$, they proposed

$$P(E, r, t) = I(E, r, t)/I_0(E) = \exp\{-\nu(E)\rho(r)t\} \tag{3}$$

and

$$\rho(r) = -\ln P(E, r, t)/\nu(E)t \tag{4}$$

for calculating transmission probability P and wood density $\rho(r)$ at a given wood thickness t. Note that both P and $\rho(r)$ are functions of spatial location r representing an irradiated point on a radial line (radius). Thus, density variations in the radial direction of wood can readily be obtained by substituting experimentally measured Ps and calculated $\nu(E)$'s into Eq. (4).

Theoretical derivations

The wood density equation given above indicates that the accuracy of density measurement depends on the accuracy of experimentally measured transmission probability, which in turn depends on the elemental concentration (fractions-by-weight) of wood, thickness of the irradiated sample, and X-ray energy [Eq. (3)]. This leads to a consideration of representing the concept of accuracy of transmission probability by a mathematical expression, which is presented in the paragraphs below.

General parametric model.—First, let $\rho_l = \min \rho(r)$ and $\rho_h = \max \rho(r)$. The range of density variation in a wood is thus given by

$$\Delta\rho = \rho_h - \rho_l$$

Second, designate the lowest and the highest transmission probabilities as

$$P_l(E, t) = \exp\{-\nu(E)\rho_h t\} \text{ and } P_h(E, t) = \exp\{-\nu(E)\rho_l t\} \tag{5}$$

respectively. Then write their difference as

$$\Delta P(E, t) = P_h(E, t) - P_l(E, t) \tag{6}$$

The quantity ΔP, the range of transmission probability, is defined here as the radiation resolution of a given wood X-ray experiment, in reference to the separability of transmission probabilities resulting from density variations in the radial direction of wood.

Let C (combined effect) designate the combination of X-ray energy E, wood thickness t, and other inherent characteristics of wood defined by

$$C(E, t) = \nu(E)t = t \sum_{j=1}^{n} \nu_j(E)w_j \tag{7}$$

From Eq. (3), Eq. (6), and Eq. (7), we know the ΔP is a function of C. Then, the radiation resolution under the condition of C, by definition, is

$$\Delta P(C) = P_h(C) - P_l(C) = \exp\{-C\rho_l\} - \exp\{-C\rho_h\} \tag{8}$$

By intuition, there exists a C_0 such that when $C = C_0$ the resolution of the radiation detection system assumes its maximum value. Symbolically,

$$\Delta P(C_0) = \max[\Delta P(C)] \tag{9}$$

By the method of extrema, we have

$$C(E, t) = C_0(E, t)$$

when

$$\frac{d\Delta P(C)}{dC(E, t)} = (-\rho_l)\exp\{-C\rho_l\} - (-\rho_h)\exp\{-C\rho_h\} = 0$$

Solving for C(E, t) leads to

$$C_0(E, t) = \frac{\ln(\rho_h/\rho_l)}{\rho_h - \rho_l} \tag{10}$$

This equation indicates that, when all parameters form a combination represented by C_0, the maximum radiation resolution is determined by density variations in wood.

Optimal energy model.—For ordinary X-ray densitometric experimentation, density variations in the z, or the vertical direction of the tree bole, are considered negligible. Conventionally, the thickness of wood samples is confined to one thickness, although different thicknesses have been used in different densitometric investigations (Echols 1970; Parker and Jozsa 1973; McNeely et al. 1973; Milson

1979; Jacoby and Perry 1981). It is conceivable, however, that if the effect of density variations in the z direction does exist, then it is minimized when thin samples are used. We will assume the use of thin (1.0 mm) samples in the models developed in this investigation.

Maximum ΔP, as defined in Eq. (9), is realized under the condition of optimal combination (C_0) which is a function of E, t and w_j's [Eq. (7)]. Because the primary interest of this study is to find the X-ray energy that, under a given combination of w_j's and t, possesses maximum radiation resolution, we can simplify the situation described in the general parametric model by (1) using one sample thickness and (2) assuming that the effect of changes in wood elemental concentration on transmission probabilities is insignificant. This is equivalent to the determination of the optimal X-ray energy for wood densitometry in which all parametric values are fixed except that of the radiation energy.

One immediately perceives that there exists an optimal radiation energy E_0 such that when E equals E_0 the radiation resolution is optimal. This can be written as

$$\Delta P(E_0) = \max[\Delta P(E)] \tag{11}$$

In the expression of ΔP [Eq. (6)], the only component relating to E [Eq. (5)] is the mass attenuation coefficient, $\nu(E)$. Thus, the ideal mass attenuation coefficient $\nu_0(E)$ can be solved by differentiating $\Delta P(E,t)$ with respect to $\nu(E)$ and by making the derivative equal to zero. After simplification, the result is

$$\rho_l \exp\{-\nu_0(E)\rho_l t\} = \rho_h \exp\{-\nu_0(E)\rho_h t\} \tag{12}$$

Solving for $\nu_0(E)$, we have

$$\nu_0(E) = \frac{\ln(\rho_h/\rho_l)}{(\rho_h - \rho_l)t} \tag{13}$$

Also from Eq. (2), the mass attenuation coefficient equation can be rewritten as

$$\nu_0(E) = \sum_{j=1}^{n} \nu_j(E) w_j \tag{14}$$

When Eq. (13) and Eq. (14) are simultaneously satisfied, $\nu_0(E)$ becomes $\nu(E_0)$ and E becomes the optimal radiation energy, E_0, which will provide maximum resolution for wood density measurement. However, because the relationship between $\sigma_j(E)$ and E is implicit, derivation of an explicit expression for E_0 like the one shown by Eq. (10) for C_0 is by no means trivial. On the other hand, the numerical solution for maximum radiation resolution is only a straightforward application of previously obtained results. Combining Eq. (5) and Eq. (6), we have

$$\Delta P(E_0) = P_h(E_0) - P_l(E_0) = \exp\{-\nu(E_0)\rho_l t\} - \exp\{-\nu(E_0)\rho_h t\} \tag{15}$$

in which $\nu(E_0) = \nu_0(E)$ when all w_j's are fixed quantities. Substitution of Eq. (13) into Eq. (15) results in

$$\Delta P(E_0) = \left(1 - \frac{\rho_l}{\rho_h}\right)\left(\frac{\rho_l}{\rho_h}\right)^{\frac{\rho_l}{\rho_h - \rho_l}} \tag{16}$$

This equation indicates that, in a densitometric experiment, the maximum

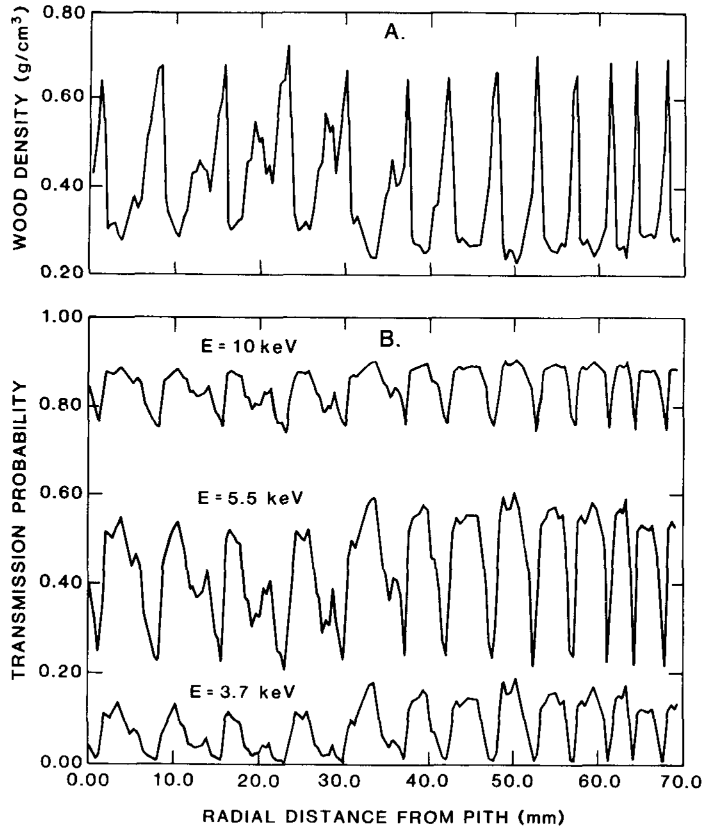

FIG. 1. Ponderosa pine (A) wood density (source: Echols 1972) and (B) transmission probabilities at three energy levels.

radiation resolution is associated only with density variations in wood, when $E = E_0$. This conclusion is analogous to that derived from Eq. (10). It becomes clear that, under good architecture conditions,

$$\Delta P(E_0) = \Delta P(C_0) = \max\Delta P \quad (17)$$

Thus, we generalize the mathematical problem considered in this section by stating that the optimal X-ray energy (E_0) is the radiation energy E when $\Delta P(E)$ assumes its maximum value.

APPLICATION EXAMPLES

As shown, the optimal X-ray energy for wood densitometric experiments can be determined by selecting an E corresponding to the $\max\Delta P(E)$ calculated by Eq. (16). We may now consider how the foregoing derived relationship between radiation resolution and wood densitometric parameters can be used in the application of X-rays to wood densitometry.

Data

To calculate $\Delta P(E)$s, four different data sets were used: (1) Percent contents of major elements and that of total ash for six different types of wood (Table 1a)

TABLE 1. *Elemental composition of selected woods.*

(a) Major elements and total ash[1]

Wood	Elemental concentration (% dry weight)				
	H	C	N	O	Ash
Pine sapwood	6.08	50.18	0.17	43.38	0.19
Pine heartwood	6.31	54.38	0.17[2]	38.99	0.15
Spruce sapwood	6.05	50.03	0.19	43.47	0.26
Spruce heartwood	6.18	49.55	0.18	43.89	0.20
Spruce (spp.)	6.20	50.31	0.04	43.08	0.37
Fir (spp.)	5.92	50.36	0.05	43.39	0.28

(b) Ash elements[3]

Wood	Elemental concentration (% of total ash)									
	Ca	K	Mg	Mn	Na	Cl	P	Al	Fe	Zn
Balsam fir	41.07	38.10	13.36	6.28					0.64	0.54
Red spruce	62.79	15.31	5.36	11.03			3.83		1.07	0.61
Pine (spp.)	69.96	3.57	10.07	8.88	2.56	4.40		0.55		

[1] Adapted from Hagglund (1951).
[2] Approximated value.
[3] Adapted from Fengel and Wegener (1984).

were taken from experimental data compiled by Hagglund (1951). Listed in Table 1b are percentages of known ash elements relative to that of total ash for wood of fir, spruce and pine and was adapted from Fengel and Wegener (1984). (2) Photon cross sections for four major and ten ash elements of all types of wood were first extracted from atomic data tables (Veigele 1973) and then interpolated to obtain sufficient data points for subsequent calculations (Liu et al. 1987). (3) A set of abridged radial density distribution data for ponderosa pine heartwood was taken from experimental results documented by Echols (1972) and is shown in Fig. 1a. (4) Density ranges for 15 softwoods (Table 3) were taken from the work of Cown and Parker (1978).

Optimal X-ray energy for pine heartwood

Data Set 3 contains a set of density measurements for a sample of ponderosa pine heartwood with a density range of 0.491 g/cm^3, varying from 0.229 to 0.720 g/cm^3. Values of $P_l(E)$, $P_h(E)$, and $\Delta P(E)$ were calculated for X-ray energies ranging from 1 to 100 keV [Eq. (16)]. With a given sample thickness of 1.0 mm, the optimal X-ray energy E_0 was found to be 5.377 keV.

Figure 1b illustrates behavior patterns of P for the ponderosa pine heartwood section when E equals 10.0 (top), 5.5 (middle), and 3.7 (bottom) keV, respectively. As shown in Fig. 1b, transmission probabilities increase when X-rays become more energetic; however, irrespective of large or small transmission probabilities associated with strong or weak penetrating power of X-rays, $\Delta P(E)$ assumes, in this case, its highest value when X-ray energy equals 5.5 keV.

This result can be further verified by examining Table 2. The $\Delta P(E)$ is 0.2966 for a 5.5 keV X-ray and 0.1329, 0.1109, and 0.0066, respectively, for 3.7, 10.0 and 60.0 keV X-rays. In addition, the theoretical optimal X-ray energy (E_0 = 5.377 keV) has ΔP equal to 0.2973. When $\Delta P(E)$ of E_0 is divided into those of various E's, we see that the comparative resolving powers of 60.0, 10.0, 3.7, and 5.5 keV X-rays to the optimal X-ray energy are approximately 0.022, 0.373, 0.447, and 0.998, respectively. As mentioned before, use of maximum radiation

TABLE 2. *Resolution and X-ray energy for 1.0-mm-thick pine heartwood.*

Transmission probability	Energy (keV)				
	3.7	5.5	10.0	60.0	$E_0 = 5.377$
$P_l(E)$	0.0118	0.2527	0.7930	0.9882	0.2293
$P_h(E)$	0.1447	0.5493	0.9037	0.9948	0.5266
$\Delta P(E)$	0.1329	0.2966	0.1109	0.0066	0.2973

resolution always provides more accurate wood density measurements. In terms of the comparative resolutions presented above, it is justifiable to state that, under given experimental conditions, accurate wood density measurements can be obtained only if the radiation energy used is near the optimal level.

The search for the optimal X-ray energy can also be done graphically. It can be seen in Fig. 2 that, when $P_h(E)$ and $P_l(E)$ and their difference, $\Delta P(E)$, for ponderosa pine heartwood and for a 1-mm-thick sample are plotted against the logarithm of E, ΔP has a maximum at approximately 5.5 keV and then decreases rapidly as E departs from this point. To generalize, we point out that the resolution resulting from the monoenergetic optimal X-radiation will be greater than that of an X-radiation whose energy is either higher or lower than the optimal energy.

Combined effect on the maximum resolution

As suggested by Eq. (17), the optimal X-ray energy is of most interest, either E_0 or C_0, characterized by the maximum resolution, maxΔP, that, in turn, is governed by the function of ρ_h and ρ_l [Eq. (10) or Eq. (16)]. To demonstrate this point, a set of density values (Cown and Parker 1978) was used in the calculation of C_0 and maxΔP for 15 softwoods. From Table 3, we detected a relationship between maximum resolutions and density ranges. For most woods, large maximum resolution is accompanied by a wide density range. Moreover, it is notice-

TABLE 3. *Combined effect on radiation resolution of 15 softwoods.*

Wood	Density[1] (g/cm^3)			C_0 (cm^3/g)	maxΔP
	Low	High	Range		
Balsam fir	0.24	0.76	0.52	2.22	0.40
Grand fir	0.25	0.68	0.43	2.33	0.35
Yellow cedar	0.33	0.60	0.27	2.21	0.22
Tamarack	0.34	0.86	0.52	1.78	0.33
Dawn redwood	0.27	0.56	0.29	2.52	0.26
Sitka spruce	0.22	0.65	0.43	2.52	0.38
Jack pine	0.24	0.58	0.34	2.60	0.31
Shore pine	0.31	0.72	0.41	2.06	0.30
Ponderosa pine	0.27	0.62	0.35	2.38	0.30
Red pine	0.27	0.65	0.38	2.31	0.31
Scots pine	0.23	0.52	0.29	2.81	0.29
Douglas-fir	0.24	0.81	0.57	2.13	0.42
Western red cedar	0.21	0.63	0.42	2.62	0.38
Western hemlock	0.34	0.77	0.43	1.90	0.29
Mountain hemlock	0.32	0.71	0.39	2.04	0.29

[1] Source: Cown and Parker (1978).

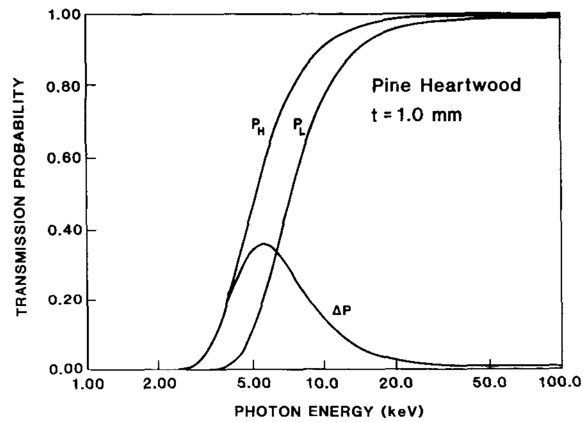

FIG. 2. Relationship between maximum transmission probability and X-ray energy. The maximum transmission probability corresponds to the maximum value of ΔP identified at the peak of the mound-shaped curve.

able that there is an underlying inverse relationship between maxΔP and the lowest density, although the precise mathematical relationship is not explicitly known at this time. Note also that the calculated C_0's represent the combined effects of all factors.

Optimal X-rays for softwoods

Optimal X-ray energies were calculated for nine different softwoods at a sample thickness of 1.0 mm; results are presented in Table 4. Optimal radiation energy tends to be confined within the range of 5.13–5.69 keV. With sample thickness held constant, differences in optimal X-ray energy can be attributed to differences in density variations in these softwoods (Table 3).

Effect of electron binding energy on optimal X-ray energy

Finally, we note there are situations in which two optimal X-ray energies can yield the same degree of accuracy. This is because, when two X-rays with photon energies very near the binding energy of the electron in the K shell of the atom, both the photon cross section and the mass attenuation coefficient of the irradiated

TABLE 4. *Optimal X-ray energies for nine softwoods (1.0-mm thick).*

Wood	keV
Balsam fir	5.621
Grand fir	5.531
Spruce (spp.)	5.407
Sitka spruce	5.379
Jack pine	5.266
Shore pine	5.690
Ponderosa pine	5.423
Red pine	5.472
Scots pine	5.127

element will suddenly increase—a phenomenon shown as a spike observable in the plotting of transmission probabilities against X-ray energy.

DISCUSSION AND CONCLUSIONS

Radiation detection is affected by characteristics of the radiation and the nature and structure of the absorber. In previous X-ray wood densitometric investigations, various methods using dissimilar X-rays (energy, intensity, and spectrum) and different woods (species, thickness, and other wood characteristics) have been explored. Based on results obtained from this study, it is apparent that different methods will produce density readings with varying degrees of accuracy and that accurate wood density measurement can be achieved only if the X-ray energy is near optimum.

Examples presented in previous sections indicate that, for coniferous wood density measurements, the appropriate X-ray photon energy is between 5.13 and 5.69 keV, with mean energy equaling 5.41 keV for 1-mm-thick wood samples.

It has been demonstrated that, for monoenergetic X-rays, the radiation resolution associated with the optimal X-ray energy is invariably greater than that associated with a radiation energy differing from the optimal energy. Symbolically, we write

$$\Delta P(E_0) > \Delta P(E), \text{ for } E \neq E_0 \tag{18}$$

For a polyenergetic X-ray with normalized spectral distribution f(E), the following condition holds:

$$\int f(E)df = 1 \tag{19}$$

Then, the average radiation resolution of a polyenergetic X-radiation with f(E) is its mathematical expectation, $\mathrm{Exp}(\Delta P_f)$, and is calculated by

$$\mathrm{Exp}\ (\Delta P_f) = \int \Delta P(E)f(E)dE \tag{20}$$

If the interest is to compare the resolution of an optimal monoenergetic X-radiation $\Delta P(E_0)$ with the mean radiation resolution of a polyenergetic one, $\mathrm{Exp}(\Delta P_f)$, the previous relationships [Eq. (18), Eq. (19), and Eq. (20)] can be used in the argument below.

First we substitute Eq. (18) into Eq. (20) to obtain the following inequality:

$$\mathrm{Exp}(\Delta P_f) < \int \Delta P(E_0)f(E)dE$$

Since $\Delta P(E_0)$ is a constant, we have

$$\mathrm{Exp}(\Delta P_f) < \Delta P(E_0)\int f(E)dE$$

With Eq. (19), we now obtain

$$\mathrm{Exp}(\Delta P_f) < \Delta P(E_0) \tag{21}$$

Thus, the radiation resolution of an optimal monoenergetic X-ray is, on the average, greater than the radiation resolution of polyenergetic X-radiation. Then, the closer the mean energy of a polyenergetic X-ray is to the optimal monoenergetic X-ray energy E_0 (Fig. 2), the higher the radiation resolution expected. Furthermore, only by collimation and monochromation can a polyenergetic X-ray be converted to a narrow beam of smaller spectral range. The radiation resolution reaches its maximum when the polyenergetic X-ray is converted to contain photons of the same energy E_0. It is, therefore, preferable to use monoenergetic optimal X-radiation for wood densitometric experiments.

Based on results obtained from this investigation, we suggest that in future instrumentation, the wood densitometer should use monoenergetic X-rays of near optimal photon energy corresponding to the thickness of the sample to be used. The relationship between maximum ΔP's and various combinations of X-ray energies and absorber thicknesses is currently under investigation.

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