AN ANALYSIS OF FLEXURAL STIFFNESS OF 5-PLY SOUTHERN PINE PLYWOOD AT SHORT SPANS PARALLEL TO FACE GRAIN

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ABSTRACT
A method for predicting deflection of 5-ply plywood strips in small span-to-depth ratios has been verified experimentally. This method first transforms the cross section of 5-ply plywood into a homogeneous double-I-beam and then calculates shear deflection by equating the internal work with the external work of the bend double-I-beam. This method predicts shear deflection with an average error of 18% and total deflection at span-to-depth ratio of 24:1 with an average error of less than 6%. After considering the nonhomogeneity of southern pine veneer and the deviation of the plywood from the assumed ideal structure, the agreement between predicted and observed total deflection should be regarded satisfactory, particularly in the absence of another method by which deflection can be predicted more accurately. From the above method a simpler equation has been derived and experimentally verified for predicting shear deflection. This simpler equation is based on the assumption that shear deflections in any two beams of the same length, height, and moment of inertia, similarly loaded, are proportional to the summations of the shear stresses on their respective vertical sections.

This paper presents partial results of a study that concerns flexural properties of southern pine plywood, including plywood overlaid with fiberglass reinforced plastics. More specifically, this paper presents an experimentally verified method by which the flexural stiffness of 5-ply southern pine plywood can be predicted accurately at short spans parallel to face grain.

It has been shown by both March (1936) and Biblis (1969) that the shear deflection at midspan of 5-ply plywood strips ½ inch thick and 24 inches span (48:1 span-to-depth ratio) is approximately 5 to 7% of the total deflection. For 12-inch spans (24:1 span-to-depth ratio) Biblis (1969) reported midspan shear deflections approximately 18% of the total. Since the use of structural plywood involves these span-to-depth ratios, shear deflections should be considered in their design.

March (1936) developed a theoretical method by which the effective stiffness of a plywood strip with any number of plies

including relatively small span-to-depth ratios can be predicted accurately. March's method requires, however, in addition to the knowledge of values of moduli of elasticity and rigidity of faces and core (E_f, E_c, G_LT, G_TR), the values of Poisson's ratios, which for most species are not known.

March's theoretical treatment for predicting shear deflection assumes plywood with plies of equal thickness. Since most of the commercial structural plywood consists of face and core plies with unequal thicknesses, rederivation of March's shear deflection equation will be required for application to plywood with plies of unequal thickness.

Biblis and Chiu (1969) developed and verified a simplified method that predicts accurately total deflection of 3-ply plywood strips with face grain parallel to span, even at span-to-depth ratios of 14. According to this method, the cross section of a 3-ply plywood rectangular strip is transformed into a hypothetical cross section of a homogeneous I-beam with grain direction parallel to span. The transformation is made by reducing the width of the core ply by the ratio of modulus of rigidity perpendicular to grain (rolling shear) to that
A SIMPLIFIED METHOD FOR PREDICTING DEFLECTIONS

The total deflection of a 5-ply plywood strip of rectangular cross section freely supported at the ends and centrally loaded consists of a portion due to pure bending \( \delta_1 \), which is caused by elongation and compression of fibers from axial stresses and a further deflection \( \delta_2 \) due to shear stresses.

The deflection caused by pure bending at midspan of the 5-ply plywood strip is equal to:

\[
\delta_1 = \frac{P L^3}{48 EI}
\]  

where: 
- \( P \) = load within the elastic region, lbs
- \( L \) = span of plywood strip, inches
- \( EI = E_f I_f + E_c I_c \)

\( E_f \) = pure modulus of elasticity of faces (grain parallel to span), psi.
\( I_f \) = moment of inertia of faces and core with respect to the neutral axis of the plywood, inches.\(^4\)
\( E_c \) = pure modulus of elasticity of crossbands (grain perpendicular to span), psi.
\( I_c \) = moment of inertia of crossbands with respect to the neutral axis of the plywood, inches.\(^4\)

The deflection caused by shear at the midspan is calculated as follows: the rectangular cross section of the 5-ply plywood strip is transformed into a hypothetical cross section of a homogeneous double-I-beam with grain direction parallel to span (Fig. 1). The transformation is made by reducing the width of the crossbands by the ratio of modulus of rigidity perpendicular to grain (rolling shear) to that parallel to grain, \( G_{TR}/G_{LR} \). Assuming a parabolic

\( G_{TR} \) is associated with shear deformation in the TR plane (rolling shear). \( G_{LR} \) is associated with shear deformations in the LR plane along the radial direction.

\(^4\) The subscripts \( L, T, \) and \( R \) refer to longitudinal, tangential and radial directions in wood, respectively. \( G_{TR} \) is associated with shear deformation in the TR plane (rolling shear). \( G_{LR} \) is associated with shear deformations in the LR plane along the radial direction.
distribution of shear stress along the depth as shown in Fig. 2, the deflection caused by shear deformation can be calculated by using the strain energy of shear. Thus by equating the internal work to the external work that causes shear distortion, we have:

Let $\tau =$ unit shear stress

$G_{LR} =$ modulus of rigidity parallel to grain

$b_1$ and $b_2 =$ widths of web and flange, respectively

$H_1$, $H_2$ and $H_3 =$ distances from the neutral axis as shown in Fig. 2

$V =$ total vertical shear $= P/2$

$y =$ distance from neutral axis

$I =$ moment of inertia of cross section.

The internal work per unit volume is:

$$\frac{\tau}{2} \frac{G_{LR}}{b dy} \frac{dA dx}{} = \frac{2}{2G_{LR}} \frac{\tau^2 dA dx}{},$$

where $dA = bdy$

for a beam of length $L$, loaded at the center with a load $P$, the external work is equal to $P\delta_2/2$. Since the external work equals the internal work then:

$$\frac{P \delta_2}{2} = 2 \left\{ \frac{\tau^2 b dy}{2G_{LR}} \right\} \int_0^H$$

where the unit shear stress $\tau_1$, in the outer flange is equal to:

$$\tau_1 = \frac{V}{I b_2} \left[ \int_0^{H_3} \frac{H_2}{y} dy + \int_{H_1}^{H_2} \frac{H_2}{y} dy \right]$$

or

$$\tau_2 = \frac{V}{I b_1} \left[ \int_{H_1}^{H_2} \frac{H_2}{y} dy + \int_{H_1}^{H_2} \frac{H_2}{y} dy \right]$$

or

$$\tau_3 = \frac{V}{I b_2} \left[ \int_{H_1}^{H_2} \frac{H_2}{y} dy + \int_{H_1}^{H_2} \frac{H_2}{y} dy \right]$$

for a beam of length $L$, loaded at the center with a load $P$, the external work is equal to $P\delta_2/2$. Since the external work equals the internal work then:
After substituting the appropriate values of unit shear stress \( \tau \) to each integral of equation (3), integrating within each definite limit and manipulating, we derive the following equation for the external work on the entire beam:

\[
\frac{P \delta}{2} = \frac{P L}{16 G L^2} \left[ b_2 \left( H_3^2 - H_2^2 \right) + b_1 \left( H_3^2 - H_1^2 \right) \right] + b_1 \left( H_3^2 - H_2^2 \right)
\]

and the deflection caused by shear is equal to:

\[
\delta = \frac{P L}{8 I^2 G L} \left( K \right)
\]

The above formula assumes a parabolic distribution of shear stress on the cross section of the transformed homogeneous double-I-beam, and the deflection caused by shear is determined by equating the external work to internal energy. Equation (7) involves numerous factors with high powers; however, with the use of an electronic computer, it can be solved easily for \( \delta \).

An equally accurate but simpler formula for determining shear deflection of the transformed homogeneous double-I-beam (and therefore of a 5-ply plywood strip) is also presented, particularly for those who do not have the services of an electronic computer. The principles on which this simpler method is based were used first by Newlin and Traycer (1924) for the development of a simple method for determining shear deflection of I-beams. In this simpler method, the fundamental assumption is that shear deflections in two beams of any cross section of the same length, height, and moment of inertia that are similarly loaded are proportional to the summations of shear stresses on their respective vertical sections.

Let us now assume a beam of rectangular cross section with the same length and depth of that of the homogeneous double-I-beam and of a width to make its moment of inertia equal to that of the double-I-beam. The shear stress distribution in each beam would be as shown in Fig. 2. Let us further assume that the shear deformations will be proportional to the areas under the stress curve. Knowing that shear deflection of a rectangular section beam freely
supported at the ends and loaded at the center is 0.3PL/AG, we can determine the shear deflection of a similarly loaded double-I-beam by multiplying this value by the ratio of the area under the shear stress curve of the double-I-beam to the area under the stress curve of the beam with rectangular cross section.

From equations (4), (5), and (6), the shear stress distribution across the depth of the double-I-beam was established as shown in Fig. 2. The area under the shear stress distribution curve is calculated as follows:

Area $ABCDGF = Area\ ABCZM - Area\ FDZG$

$$MT = \frac{V}{2I} \left( \frac{H_3}{3} \right)$$

and since $ABT$ is a parabola, the area

$$ABTM = \frac{2H_3}{3} \left( \frac{VH_3^2}{2I} \right) = \frac{VH_3^3}{3I}$$

Area $ABCDGF = Area\ ABCZM + Area\ BCRS$

$$BC = TZ = MZ - MT$$

$$MZ = \frac{V}{2I} \left( \frac{H_3}{3} \right) \left( \frac{b_2}{b_1} - 1 \right)$$

Area $ABCDGF = \frac{VH_3^3}{3I} + \frac{VH_2}{2I} \left( \frac{H_3}{3} \right) \left( \frac{b_2}{b_1} - 1 \right)$

Area $ABCDGF = \frac{VH_3^3}{3I} + \frac{VH_2}{2I} \left( \frac{H_3}{3} \right) \left( \frac{b_2}{b_1} - 1 \right)$

Area $FDZG = Area\ FDQN$

Area $FDQN = H_1(FD) = \frac{VH_1}{2I} \left( \frac{H_3}{3} \right) \left( \frac{b_2}{b_1} - 1 \right)$

The area under the shear stress distribution curve of the rectangular cross section beam with the same length, height and moment of inertia, which is similarly loaded, is:

$$\frac{2}{3}H_3 \left( \frac{VH_3^2}{2I} \right) = \frac{VH_3^3}{3I}$$

The ratio of the area under the shear distribution of the double-I-beam to the area under the stress curve of the rectangular beam is:

$$\frac{VH_3^3}{3I} + \frac{VH_2}{2I} \left( \frac{H_3}{3} \right) \left( \frac{b_2}{b_1} - 1 \right)$$

Therefore, the deflection caused by shear of the double-I-beam is:

$$\delta_2 = \frac{0.3PL}{AG} \left\{ 1 + \frac{3}{2H_3} \left[ \frac{H_3}{3} \frac{b_2}{b_1} - 1 \right] \right\}$$

where $A = area\ of\ the\ rectangular\ cross\ section\ beam.$
Since \( I \) of the double-I-beam = \( I \) of the rectangular beam = \( 2 \frac{bh^3}{3} \)

\[
I = \frac{3I}{2h^3} \quad \text{and} \quad A = \frac{3I}{2h^3} \left( \frac{2h}{3} \right) = \frac{3I}{H^2}.
\]

Therefore, shear deflection \( \delta_2 \) is:

\[
\delta_2 = \frac{P LH^2}{10 IG L R} \left\{ \begin{array}{c}
1 + \frac{3}{2H^3} \left[ \left( \frac{H^2 - H_1^2}{H_2 - H_1} \right) + H_1 \left( \frac{H_2 - H_1}{H_2 - H} \right) \right] \\
\left( \frac{b_2}{b_1} - 1 \right) + H_1 \left( \frac{b}{b_1} - 1 \right)
\end{array} \right\}
\]

EXPERIMENTAL PROCEDURE FOR VERIFICATION OF PROPOSED METHOD

A detailed experimental procedure describing fabrication and flexure test of the 5-ply plywood was presented in a previous paper (Biblis 1969). However, a summary of this procedure is presented here. Grade A rotary-cut veneer of southern yellow pine was used exclusively. Veneer was selected carefully in the mill to exclude all visible defects. Three panels of 5-ply plywood were glued with extended phenolic resin in a plywood mill. Plywood was constructed with \( \frac{1}{2} \)-inch-thick faces and core, and \( \frac{1}{8} \)-inch-thick crossbands. Total thickness of plywood varied from 0.528 to 0.561 inches. Three static bending specimens with face grain parallel to span and with dimensions as specified by ASTM Standards were machined from each panel. Each specimen was tested in static bending at the following span-to-depth ratios: 48, 24, 18, 14, 11, and 8. After each test, specimens were trimmed to the shorter length and relaxed for 24 hr in a conditioning room before being retested at the shorter span. The load applied to each span was only one-third of the estimated proportional limit load for each specimen. Specimens were supported freely at the ends and centrally loaded, using an Instron testing machine. Loading speeds used for each span were according to ASTM Standards (1968). Deflections were measured with an electric deflectometer attached to the core at midspan and recorded simultaneously with the corresponding loads on X-Y recorder.

ACTUAL DEFLECTIONS CALCULATED FROM OBSERVED TOTAL DEFLECTION

Observed test values of midspan total deflection were separated into actual pure bending and shear deflections by the first and second factors, correspondingly, of the following equation:

\[
\delta = \frac{PL^3}{48EI} + \frac{P3PL}{AG}
\]

where: \( \delta \) = total deflection at midspan; 
\( P \) = load corresponding to deflection \( \delta \); 
\( L \) = span of specimen; 
\( E \) = pure modulus of elasticity of plywood (free of shear deformations); 
\( I \) = moment of inertia plywood cross section = \( bd^3/12 \); 
\( A \) = cross section area of plywood = \( bd \); 
\( G \) = modulus of rigidity of the entire cross section of plywood in the direction of span.

Values of pure moduli of elasticity \( E \) and moduli of rigidity \( G \) for plywood of each panel were determined by the method used previously by Biblis (1965), from experimentally obtained values of effective moduli of elasticity at various span/depth ratios.
**Table 1.** Actual and predicted midspan deflections by a simplified method for 5-ply southern pine plywood strips

<table>
<thead>
<tr>
<th>Designation of specimen</th>
<th>Span-to-depth ratio</th>
<th>Pure bending deflection</th>
<th>Shear deflection</th>
<th>Total Deflection</th>
<th>Percentage of shear deflection to total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Actual in.</td>
<td>Predicted in.</td>
<td>Difference</td>
<td>Actual in.</td>
</tr>
<tr>
<td>4-P</td>
<td>48</td>
<td>0.07281</td>
<td>1.12432</td>
<td>+15.57</td>
<td>0.05603</td>
</tr>
<tr>
<td>4-P</td>
<td>24</td>
<td>0.24320</td>
<td>0.28108</td>
<td>+15.57</td>
<td>0.05603</td>
</tr>
<tr>
<td>4-P</td>
<td>14</td>
<td>0.08283</td>
<td>0.09573</td>
<td>+15.57</td>
<td>0.05603</td>
</tr>
<tr>
<td>5-P</td>
<td>48</td>
<td>0.80376</td>
<td>0.78645</td>
<td>-2.15</td>
<td>0.04119</td>
</tr>
<tr>
<td>5-P</td>
<td>24</td>
<td>0.20094</td>
<td>0.19661</td>
<td>-2.15</td>
<td>0.04119</td>
</tr>
<tr>
<td>5-P</td>
<td>14</td>
<td>0.06844</td>
<td>0.06696</td>
<td>-2.15</td>
<td>0.04119</td>
</tr>
<tr>
<td>6-P</td>
<td>48</td>
<td>0.81643</td>
<td>0.76113</td>
<td>-6.77</td>
<td>0.04396</td>
</tr>
<tr>
<td>6-P</td>
<td>24</td>
<td>0.20411</td>
<td>0.19028</td>
<td>-6.77</td>
<td>0.04396</td>
</tr>
<tr>
<td>6-P</td>
<td>14</td>
<td>0.06951</td>
<td>0.06487</td>
<td>-6.77</td>
<td>0.04396</td>
</tr>
<tr>
<td>Av.</td>
<td>48</td>
<td>0.86433</td>
<td>0.89063</td>
<td>+3.04</td>
<td>0.04706</td>
</tr>
<tr>
<td>Av.</td>
<td>24</td>
<td>0.21608</td>
<td>0.22286</td>
<td>+3.04</td>
<td>0.04706</td>
</tr>
<tr>
<td>Av.</td>
<td>14</td>
<td>0.07359</td>
<td>0.07583</td>
<td>+3.04</td>
<td>0.04706</td>
</tr>
</tbody>
</table>

1. Deflections correspond at proportional limit load.
2. Number designates plywood panel.
DEFLECTIONS PREDICTED BY THE SIMPLIFIED METHOD

Deflection caused by pure bending at midspan was calculated by equation (1).

Values of pure moduli of elasticity of faces $E_f$ (grain direction parallel to span) and of core $E_c$ (grain direction perpendicular to span) were determined experimentally from 5-ply unidirectionally laminated veneer strips of the same thickness with those of plywood. Laminated vencers were matched to those of plywood, and both plywood and laminated veneer panels were manufactured simultaneously with the same process. Values of pure $E_f$ and $E_c$ for equation (1) were determined by the same method used for determining values of $G$ and pure $E$ of plywood, used in equation (10).

Shear deflection at midspan was calculated by using equation (7) in a program fitted to an I.B.M.-360 computer. For transforming the 5-ply plywood strip into a homogeneous double-I-beam with grain direction parallel to span, a value for the ratio of moduli $G_{TR}/G_{LR}$ equal to 0.23 was used, as in previous work by the authors for 3-ply plywood. Values of $G_{LR}$ for equation (7) were determined experimentally by the same method described for determining $E_f$.

Shear deflection at midspan was also calculated by the simpler equation (9).

RESULTS AND DISCUSSION

Total deflection, pure bending deflection, and shear deflection at midspan of 5-ply plywood strips with face grain parallel to span were predicted at 48, 24, and 14 span-to-depth ratios by a method based on the equivalency of external and internal energy in a bend beam. The observed total deflections for the same span-to-depth ratios were separated into pure bending and shear deflections and compared with those predicted.

Comparisons between actual deflections and those predicted by equation (7) are shown in Table 1. Percentage difference is considered positive when the predicted value is larger than the actual. Pure bending deflection was predicted with a maximum error of +15.6%. Shear deflection was predicted with a maximum error of +29% and best prediction with an error of 14%. Total deflection was predicted with a maximum error of 15% while the average error at span-to-depth ratio of 14 was 9%. After considering the rather complicated distribution of bending and shear stresses in 5-ply plywood, the high degree of non-homogeneity of southern pine veneer and the variability in thickness of each veneer layer, then the agreement between the calculated and observed values of total deflection should be regarded as satisfactory.

The average error in predicting shear deflection by the method presented is 18.6%. The accuracy of this method compares favorably with that of the exact method developed by March (1936), where the shear deflection of 5-ply, ½-inch-thick Douglas-fir plywood was predicted with an average error of 24.5%. It should be pointed out, however, that the plywood used by March for verifying his theory was commercial and not matched to the solid material used for determining the required elastic constants.

Shear deflections predicted by equation (9) are shown in Table 2, together with those calculated by equation (7). It is shown that the percentage difference of predicted shear deflection by the two equations is less than 2%.

<table>
<thead>
<tr>
<th>Specimen designation</th>
<th>Shear deflection in inches by equation (7)</th>
<th>Shear deflection in inches by equation (9)</th>
<th>Percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-P</td>
<td>0.064056</td>
<td>0.065216</td>
<td>+1.81</td>
</tr>
<tr>
<td>5-P</td>
<td>0.053242</td>
<td>0.054118</td>
<td>+1.64</td>
</tr>
<tr>
<td>6-P</td>
<td>0.050104</td>
<td>0.050986</td>
<td>+1.76</td>
</tr>
<tr>
<td>Av.</td>
<td>0.055800</td>
<td>0.056770</td>
<td>+1.74</td>
</tr>
</tbody>
</table>
SUMMARY AND CONCLUSION

A method for predicting deflection of 5-ply plywood strip in small span-to-depth ratios has been verified experimentally. This method first transforms the cross section of 5-ply plywood into a homogeneous double-I-beam and then calculates shear deflection by equating the internal work with the external work of the bend double-I-beam. This method predicts shear deflection with an average error of 18% and total deflection at span-to-depth ratio of 24:1 with an average error of less than 6%. After considering the nonhomogeneity of southern pine veneer and the deviation of the plywood from the assumed ideal structure, the agreement between predicted and observed total deflection should be regarded as satisfactory, particularly in the absence of another method by which deflection can be predicted more accurately. From the above method a simpler equation has been derived and experimentally verified for predicting shear deflection. This simpler equation is based on the assumption that shear deflections in any two beams of the same length, height, and moment of inertia, similarly loaded, are proportional to the summations of the shear stresses on their respective vertical sections.

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