FINITE ELEMENT MODEL FOR PREDICTING MODULUS OF ELASTICITY OF LUMBER MEASURED BY STRESS-GRADING MACHINES

Marcel Samson[1]
Research Scientist
Forintek Canada Corp., Western Laboratory, 6620 N.W. Marine Drive
Vancouver, B.C. V6T 1X2

(Received May 1984)

ABSTRACT

Development of a finite element model to predict modulus of elasticity of lumber as measured by bending-type stress-grading machines is discussed. The model is general in that lumber characteristics critical in machine grading, such as natural bow and local stiffness variations, can be taken into account. Simulations are carried out to compare the performance of two constant deflection machines employing different testing geometries. Results show that, depending on support conditions, grading machines may vary substantially in their ability to locate low stiffness zones along the length of lumber and identify the value of modulus of elasticity at these zones. Ability of the grading machines to test end portions of the lumber is also discussed, along with the influence of span on machine accuracy.

Keywords: Machine stress-rating, grading machines, lumber.

INTRODUCTION

Machine stress-rating of lumber is becoming established internationally. Today it is estimated that over two hundred grading machines have been installed worldwide. The number of grading machine manufacturers has also been steadily growing. As a result, a large number of grading machines employing different principles of operation and measuring different properties are now available. The concern of machine manufacturers is now to gain machine type approval in the major countries where machine stress-rating is practiced.

Modulus of elasticity (referred to in this article as E, rather than MOE) is universally used as a predicting parameter for machine stress-rating. Although E can be measured by a variety of mechanical and electronic means, most grading machines in operation today employ the principle of the laboratory bending machines to determine E. The various models of bending-type grading machines available differ essentially in their testing geometry (span, method of loading).

The increasing economic importance of machine stress-rated lumber, coupled with the need to select higher reliability lumber products, suggests that new grading machines will continue to come on the market. In order to discuss and compare existing machines and to provide a rational basis for designing optimum grading machines for the future, it is essential to have general models describing machine measurement of E in lumber. Suitable models for this do not exist at present. This paper presents a model that contains essential features for describing the measurement of E in any grading machines of the bending type.

BACKGROUND

In the last two decades, extensive research effort has been directed towards assessing the efficiency of machine stress-rating. Vast literature on the topic re-

[1] Currently Assistant Professor, Wood Science Department, Faculty of Forestry, Laval University, Ste-Foy, P. Quebec, Canada G1K 7P4.

Wood and Fiber Science, 17(4), 1985, pp. 490–503

viewed by Müller (1968), Endersby (1969) and more recently by Glos and Schultz (1980) reveals that most of this research effort has focused on demonstrating the correlation between strength and E. Very little attention has been devoted to studying the operating principles of the grading machines and assessing their ability to measure E.

Galligan et al. (1977) and Glos (1982) have reported on the operating principles of the grading machines used to date. All currently approved bending-type machines operate by deflecting each piece of lumber over a relatively short span and at short intervals throughout the length of the piece. Bending E is determined from measurement of either the deflection under a defined load (constant load machines) or the force necessary to maintain a certain deflection (constant deflection machines). Either the low point value (low point E) or a combination of the average (average E) and the low point value is used to decide the grade of the piece.

Most machines load the lumber on the wide face because large, more easily measured deflections can be obtained at low stress levels. The measured deflection or force is influenced by any bow or crook of the lumber. One method widely used to eliminate errors arising from deviation from straightness is to load the lumber on the two faces and average the results. This operation is carried out either in two passes, for machines using unidirectional bending, or in a single pass when bidirectional bending is used.

Factors such as natural bow, local defects, machine span and type of loading affect machine readings. Machine sensitivity to the presence of local defective zones has been studied by Thunell (1969) and Korneev (1980) for grading machines utilizing three different testing geometries. They derived closed-form solutions for the deflection of beams, containing a single zone of lower E, under constant bending moment, center-point loading over free supports and center-point loading over fixed supports. Influence of span and defect length on the machine sensitivity to low E zones has also been analyzed for the three testing geometries considered.

The load-deflection models provided in these papers are not general in that each of them represents only one testing geometry. Furthermore, conditions usual in practice such as natural bow and presence of more than one defective zone within the machine span were not accounted for.

MODEL DEVELOPMENT

For analysis purposes, the lumber is treated as a prismatic beam of rectangular cross section of thickness t and depth d. This beam is made of a heterogeneous but linearly elastic material whose bending modulus of elasticity E, constant within a cross section, varies along the length axis of the beam. Natural bow of the lumber is accounted for by assuming that the beam is initially deformed prior to loading. Dynamic effects arising from the motion of the lumber in the grading machine as well as effects of overhangs of the ends of the lumber outside the machine are disregarded.

A schematic representation of the beam in a grading machine utilizing possibly the most general testing scheme is shown in Fig. 1. Coordinate axes x (longitudinal) and z (vertical), as well as the initial deformation ($w_o(x)$) and final deformation ($w_f(x)$) of the centroidal axis of the beam, are shown. The machine span L is

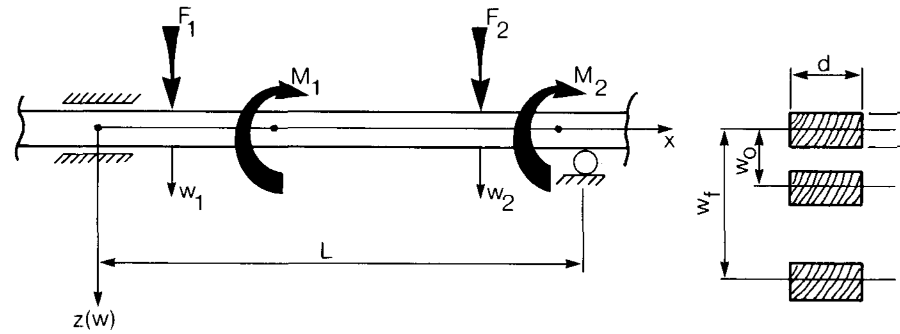

FIG. 1. Sketch of a beam in a grading machine showing loading and displacement sign conventions. Initial deviation from straightness not shown.

delimited at the right by a roller support resisting normal forces and at the left by a fixed support resisting all forces and moments.

The beam is divided into an assemblage of one-dimensional beam elements interconnected at a discrete number of nodal points situated on their boundaries. One of these elements is shown in Fig. 2 in its initially straight and displaced position. External forces (F_i, F_j) and moments (M_i, M_j) applied at the nodes i, j of the element are also illustrated. The relationship between the cartesian coordinate x and the natural coordinate ξ introduced in Fig. 2 is

$$x = \frac{x_i + x_j}{2} + \frac{H}{2}\xi \qquad [1]$$

where H is the element length.

The deflection w within the element is assumed in the form of a cubic polynomial, uniquely determined by the value of the deflection (w_i, w_j) and slope (θ_i, θ_j) at each of the nodes. In terms of the nodal displacements, the deflection is given by

$$w = \{N\}^T\{q\} \qquad [2]$$

where

$$\{N\}^T = \{N_1(\xi), \ldots, N_4(\xi)\}$$

and

$$\{q\}^T = \{w_i, \theta_i, w_j, \theta_j\}.$$

The functions $N_i(\xi)$ are the cubic hermitian polynomials expressed as a function of ξ (see e.g., Cook 1981). An expression similar to Eq. [2] can be written for the initial shape w_o of the element prior to loading. Assuming that this shape is also approximated by a cubic polynomial, we can write

$$w_o = \{N\}^T\{q_o\} \qquad [3]$$

where

$$\{q_o\}^T = \{w_{o_i}, \theta_{o_i}, w_{o_j}, \theta_{o_j}\}$$

and the subscript o denotes initial parameter values.

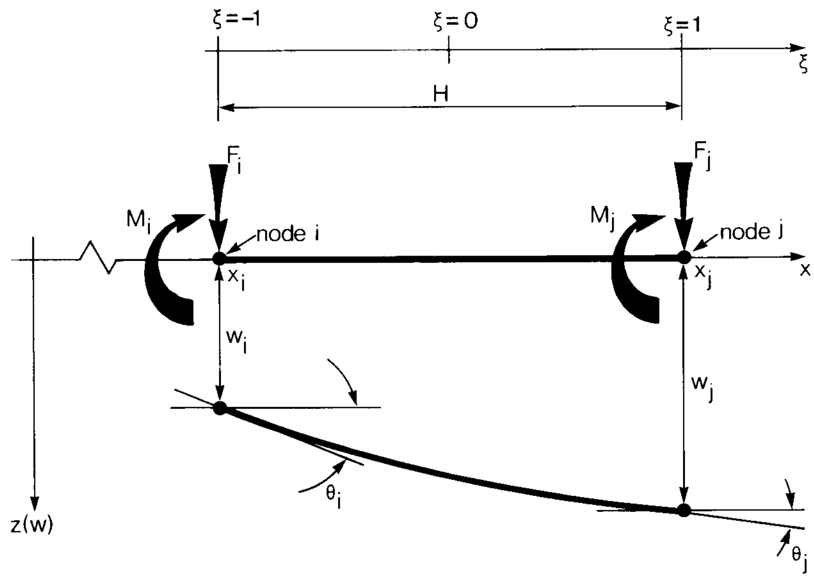

FIG. 2. Beam finite element in its originally straight and displaced positions.

In the present model, modulus of elasticity is assumed to vary linearly within the element to assure a smooth variation of this parameter along the length of the beam. Therefore, within the element, E can then be expressed as a function of the nodal values of modulus of elasticity E_i, E_j, as

$$E_e(\xi) = a_e + b_e\xi \qquad [4]$$

where

$$a_e = \frac{E_i + E_j}{2}$$

$$b_e = \frac{E_j - E_i}{2}$$

and e is a subscript denoting an element property.

By the principle of minimum potential energy, the variation δW in external work, due to the virtual deformation δq of the beam element must be equal to the variation in internal work δU due to the same virtual displacement. The external work on the element is

$$W = \{q\}^T\{Q_e\} - \{q_o\}^T\{Q_e\} \qquad [5]$$

where $\{Q_e\}$ is the element load vector given as

$$\{Q_e\}^T = \{F_i, M_i, F_j, M_j\}$$

Neglecting deformations due to shear, the internal work is simply the bending strain energy

$$U = \int_{x_i}^{x_j} \frac{E_e(\xi)I}{2}\left(\frac{d^2w}{dx^2} - \frac{d^2w_o}{dx^2}\right)^2 dx \qquad [6]$$

where I is the moment of inertia about the neutral axis. Upon introducing the natural coordinate ξ using Eq. [1] and substituting Eqs. [2], [3] and [4] into Eq. [6], the strain energy can be written as

$$U = \frac{4I}{H^3}\int_{-1}^{1}(a_e + b_e\xi)(\{q\}^T\{N''\}\{N''\}^T\{q\} - 2\{q\}^T\{N''\}\{N''\}^T\{q_o\} + \{q_o\}^T\{N''\}\{N''\}^T\{q_o\})\,d\xi \qquad [7]$$

where

$$\{N''\}^T = \left\{\frac{d^2N_1}{d\xi^2}, \ldots, \frac{d^2N_4}{d\xi^2}\right\}$$

Applying the principle of minimum potential energy, $\delta U - \delta W = 0$, we obtain

$$\{\delta q\}^T([K_e]\{q_o\} - [K_e]\{q_o\} - \{Q_e\}) = 0 \qquad [8]$$

where

$$[K_e] = \frac{8I}{H^3}\int_{-1}^{1}(a_e + b_e\xi)\{N''\}\{N''\}^T\,d\xi$$

is the element stiffness matrix (see Appendix I). Since the variation of the nodal displacements δq is arbitrary, the expression in parentheses must vanish. This gives the equilibrium equations for the element. The equilibrium equations for the individual elements can then be used to construct the equilibrium equations for the assemblage of elements, representing the complete beam, by matching the deflections and slopes at the nodes and adding the loads and stiffnesses at these locations. The equilibrium equations for the beam can be abbreviated in the following form

$$[K]\{r\} = [K]\{r_o\} + \{Q\} \qquad [9]$$

where [K] and {Q} are the beam stiffness matrix and load vector, respectively, and $\{r_o\}$ and {r} are the initial and final nodal displacements for the beam, again respectively.

These equations cannot be solved without the imposition of boundary conditions. The particular boundary conditions to be considered in modelling a specific grading machine are dictated by the type of supports used in the machine. In this study, the boundary conditions were taken into account by modifying the equilibrium equations [9] so that the vector {r} contained only unknown quantities (free displacements and reactions at the constrained displacements) and the right hand side contained only known quantities. Then, a Gaussian elimination procedure was used to solve the resulting simultaneous equations for the unknowns.

For the loading conditions under consideration, the finite element procedure developed allows for exact closed-form solutions for displacements and reactions at constraints only when E is constant within span. In the more general case where stiffness varies along the length axis of the beam, the finite element procedure yields only approximate solutions. However, the finite element solution converges rapidly to the exact solution as the beam elements are made shorter. Comparison of closed-form and finite element solutions for selected problems considering

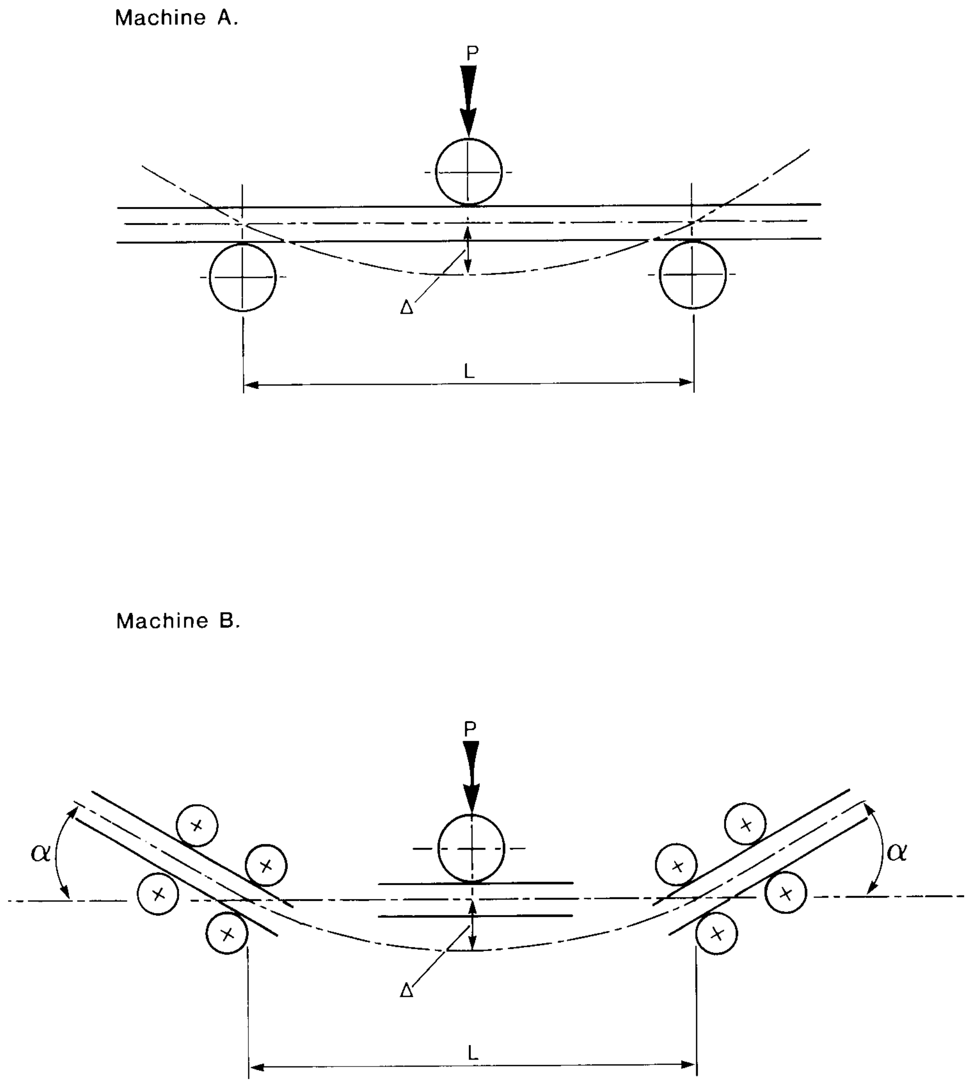

FIG. 3. Sketch of the two machines studied. Curvatures are exaggerated.

beams of varying stiffness showed that the finite element solutions are within 1% of the exact solutions when the size H of the elements is smaller than L/5.

APPLICATION

Simulations were carried out with the model to compare the effectiveness of two constant-deflection-type machines, illustrated in Fig. 3 and referred to as Machines A and B in the remainder of the study. The specific testing schemes shown in Fig. 3 are typical of those employed by two major machines now in commercial use.

The only difference between Machines A and B lies in the support conditions. In Machine A, the lumber is free to rotate at the supports. In Machine B, the slopes at the supports are fixed to an angle α. The value of α chosen will depend on machine span and deflection, as discussed in further detail in Appendix II. The parameter measured by both machines is the force P which can be calculated from Eq. [9], once the geometric boundary conditions at the supports and at midspan are introduced.

In the simulations, the lumber had clear zones of uniform modulus of elasticity, E_{clear}, and, for simplicity, a single defective zone of comparatively low modulus of elasticity, E_{low}. Therefore, the nodal modulus of elasticity was assigned the value E_{clear} for all nodes, except one which was given the value E_{low}. Initial shape of the lumber $w_o(x)$ was prescribed, within the span, by a parabola fitted through the three points

$$\begin{array}{ll} x = 0, & w_o = 0 \\ x = L/2, & w_o = b \\ x = L, & w_o = 0 \end{array}$$

where b is the natural bow measured over the machine span.

The actual grading operation was simulated by calculating a value of P each time the lumber moved a distance H through the machine. Machine assessment of modulus of elasticity, E_{mach}, was calculated from P, at every increment H, using the basic deflection formula for center-point loading

$$E_{mach} = \frac{L^3}{48I\Delta} P$$

where Δ is the deflection at midspan.

Since both machines use unidirectional bending, the grading operation was carried out in two passes and two series of E_{mach} values, associated with each face loaded, were obtained for each simulation run. The E values compensated for bow were calculated along the lumber by averaging the individual E_{mach} values of corresponding locations.

Maximum bending stress σ_m applied on the lumber during grading was calculated for each individual pass from the maximum bending moment M_m using the conventional flexure formula

$$\sigma_m = \frac{M_m c}{I}$$

where $c = t/2$. The value of M_m was determined from examination of the bending moment diagram, constructed for each problem from the reaction force and moment values provided in the finite element solution $\{r\}$.

Throughout the calculations, the following parameters were kept constant

$$\begin{array}{lll} d = 3.5 \text{ in.} & H = 3.0 \text{ in.} & E_{clear} = 1.5 \times 10^6 \text{ psi} \\ t = 1.5 \text{ in.} & \Delta = 0.3125 \text{ in.} & \\ L = 48.0 \text{ in.} & \alpha = 5/256 \text{ radian} & \end{array}$$

while b and E_{low} were assigned varying values.

DISCUSSION

Influence of bow and local defects

The effects of natural bow and presence of a local defect on machine readings are examined in Figs. 4 to 6 which show E profiles measured by Machines A and

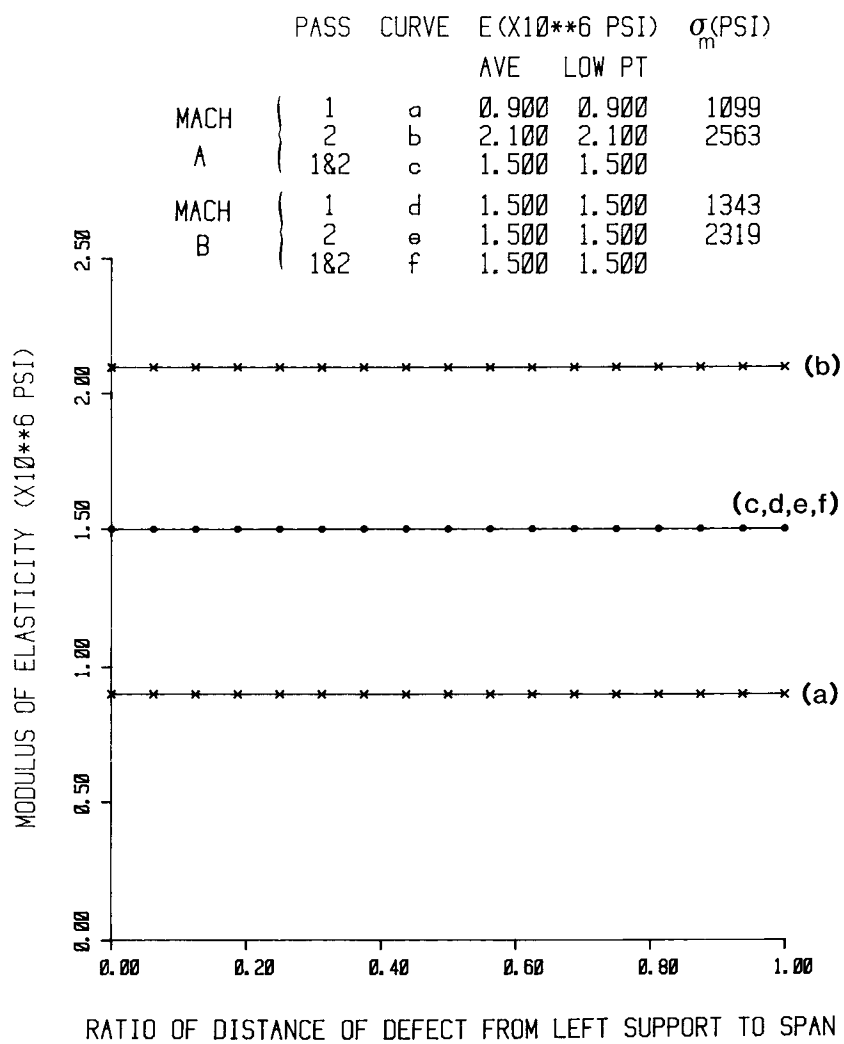

FIG. 4. E profiles measured by Machines A and B when $E_{low} = 1.5 \times 10^6$ psi and b = 0.125 in. in the simulations.

B along three pieces of lumber possessing the E_{low} and b characteristics listed in Table 1. Summary data on machine-measured E, including average and low point values, are provided on the figures for each individual pass and both passes combined. Maximum bending stress calculated during each pass when the defective zone was positioned at midspan is also given on the graphs.

As evident from Fig. 4, Machine B is capable of determining, in only one pass, the exact E of bowed lumber of uniform stiffness while this operation requires two passes in Machine A. This is due to our choice of both the function w_o describing the initial shape of the lumber and the value of the support angle α. In fact, it can be shown that, given a value of α, there exists a particular family of functions w_o which will make the force P, and consequently the machine-measured E, independent of bow. This condition is satisfied, for the particular

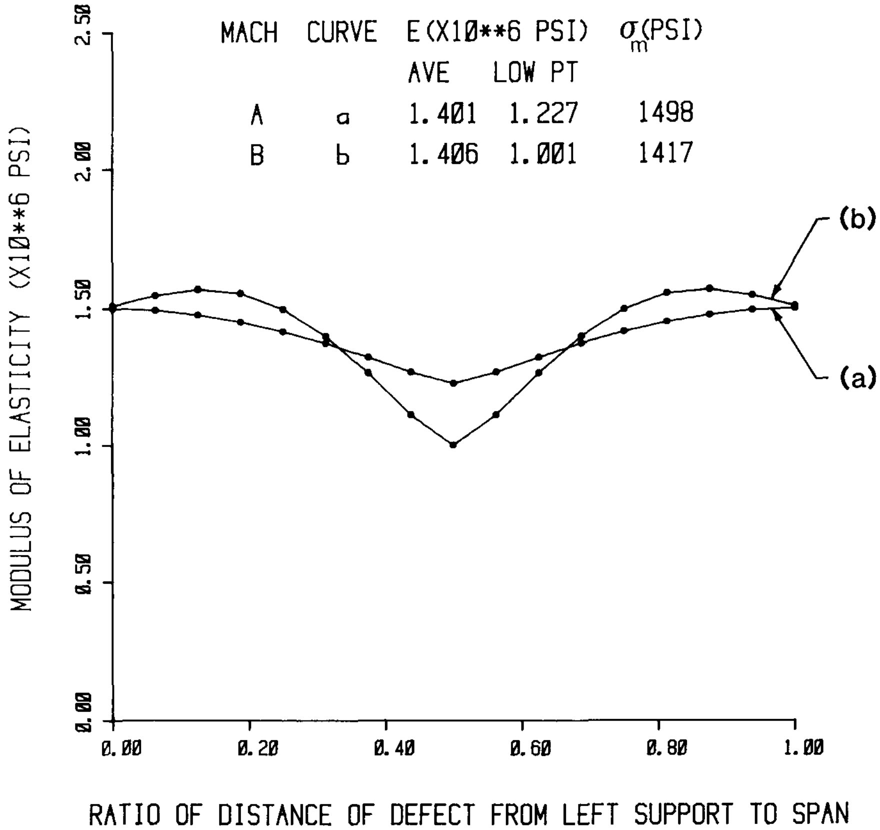

FIG. 5. E profiles measured by Machines A and B when $E_{low} = 0.5 \times 10^6$ psi and $b = 0$ in the simulations.

value of $\alpha = 3\Delta/L$ chosen here, when a parabola is used to represent bow. Adequacy of representing bow by a parabola cannot be verified at present since bow data pertaining to commercial lumber are nonexistent in the current literature. Although the parabola appears to be the most logical function to model natural bow, it is expected that, in practice, bow of commercial lumber will deviate to some extent from a true parabola. In such instances where w_o does not conform to a parabola, the results from the two passes in Machine B will no longer coincide. The averaged results from the two passes, however, will not be affected.

Sensitivity of the two machines to the presence of a local defect in initially straight lumber is examined in Fig. 5. Since $b = 0$, only one pass needs to be considered here. It is apparent from comparison of the two curves on the figure that the changing of the support angles from the free to the fixed condition results in a considerable change in the machine E profiles. While Machine A measures a gradual decrease in E as the defective zone proceeds towards midspan, Machine B first records an increase in E above the E_{clear} value, followed by a comparatively rapid decrease towards the low point. The E profile measured by Machine B may appear unrealistic in the sense that it indicates the presence of local zones stiffer than the lumber itself. This overestimation is due to the reaction moment at the support which is negative (i.e., tends to deflect the lumber upward) when the

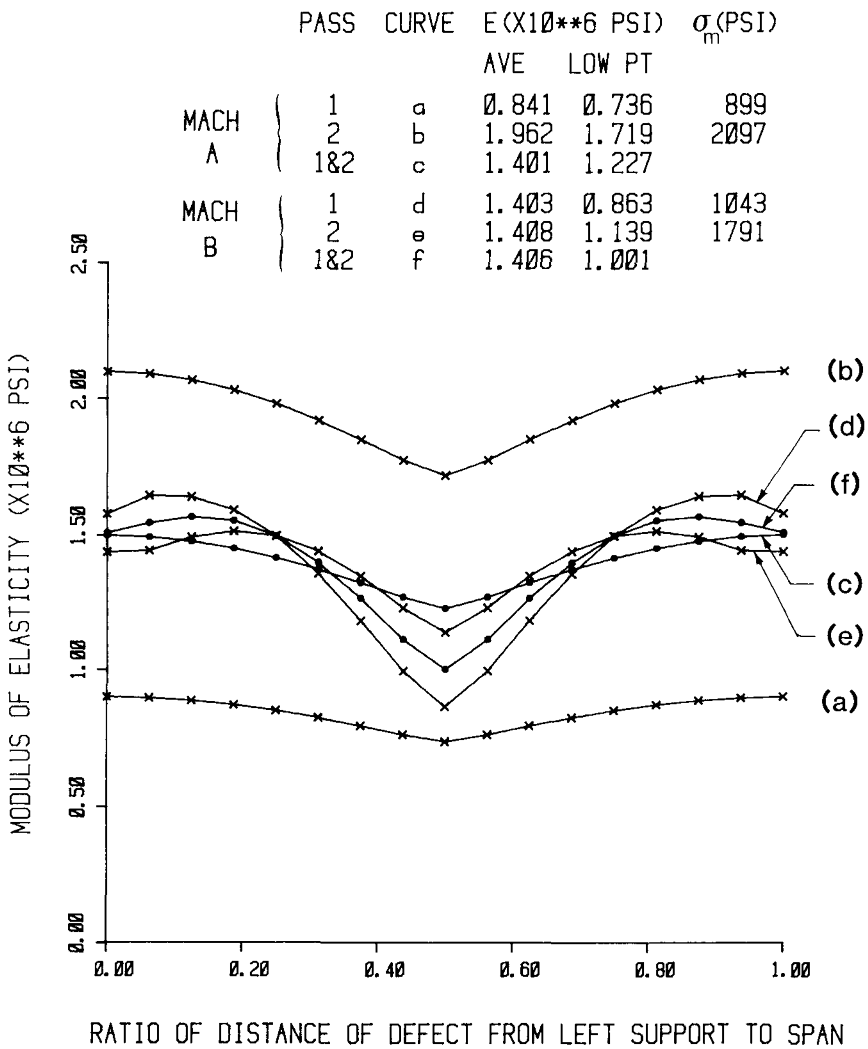

FIG. 6. E profiles measured by Machines A and B when $E_{low} = 0.5 \times 10^6$ psi and b = 0.125 in. in the simulations.

defective zone is in the vicinity of the support. A higher reaction force is then required at midspan to maintain the constant deflection, which explains why the machine indicates higher E values. On the other hand, as the defective zone approaches midspan, the reaction moment becomes positive, yielding lower E values. This reaction moment is advantageous since it permits Machine B to indicate a low point E value closer to the true E_{low} value than indicated by Machine A. Furthermore, because of the steeper descent to the low point, Machine B provides a clearer indication of the location of the low point E zone than Machine A does. Although Machines A and B differ in their abilities to identify low point E value and location, the data in Fig. 5 indicate that both machines provide the same assessment of average E value.

Ability of the two machines to measure local E variation in bowed lumber

TABLE 1. *Low point E and bow values used in the numerical examples.*

	E_{low} ($\times 10^6$ psi)	b (in.)
Piece 1 (Fig. 4)	1.5	0.125
Piece 2 (Fig. 5)	0.5	0
Piece 3 (Fig. 6)	0.5	0.125

containing one major defect is examined in Fig. 6. The E profiles for the two individual passes in Machine B no longer coincide as was the case for uniform lumber. This separation, however, is much less than what is observed for Machine A. The separation is so small that even the poorest assessment of low point E value that can be obtained in Machine B in a single pass (1.139×10^6 psi) is better than the low point E value indicated by Machine A in two passes (1.227×10^6 psi). As to average E, the assessment provided by Machine B in either pass is the same as that indicated by Machine A in two passes. Interestingly, the averaged profiles in Fig. 6 are identical to those measured when bow is absent (Fig. 5). This indicates that both machines are capable of totally overcoming the problem of natural bow in lumber of varying stiffness provided that two passes are used.

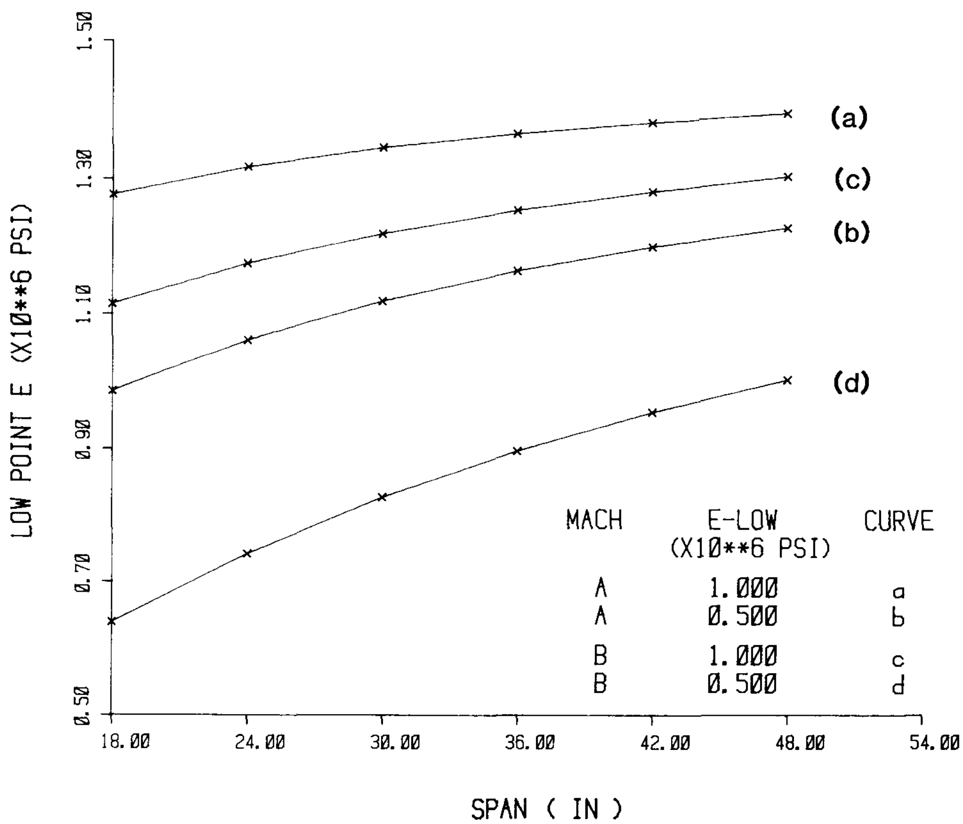

FIG. 7. Low point E versus span relationship for Machines A and B.

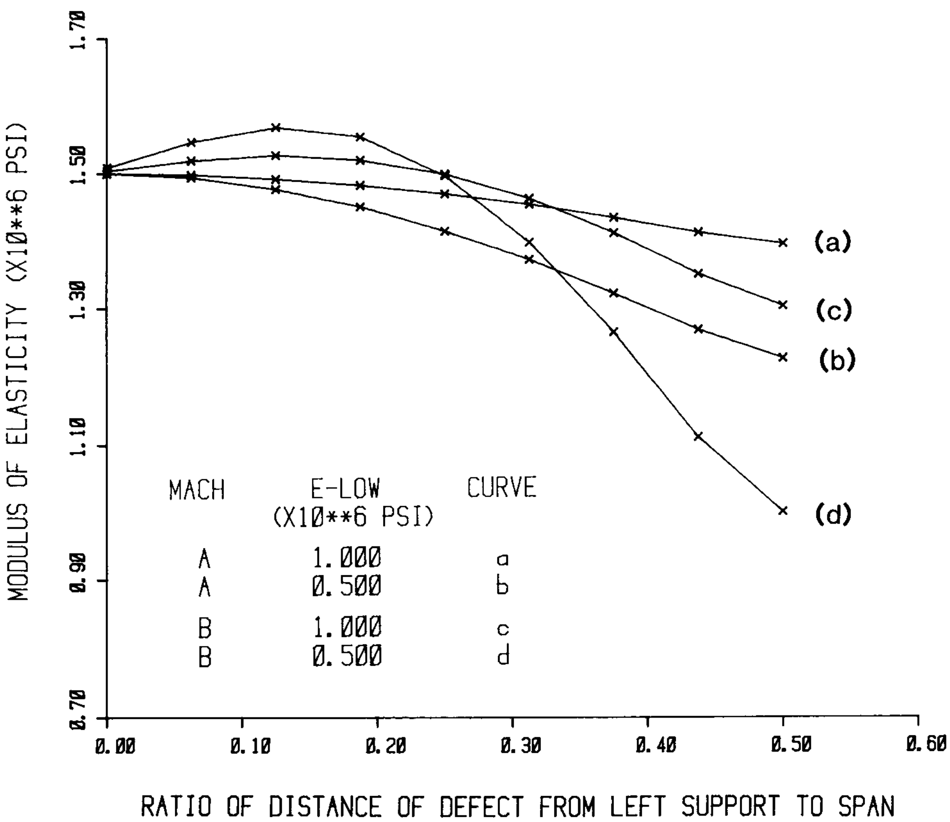

FIG. 8. E profiles measured by Machines A and B in the end portion of pieces of lumber with different E_{low} values.

Information on σ_m provided on Figs. 4 to 6 reveals that the maximum stresses induced in the lumber during grading are substantially lower in Machine B than in Machine A, regardless of the characteristics of the lumber. This is an advantageous feature for Machine B since the lower the stress, the lower the risk of causing permanent damage to the lumber during grading.

Influence of span

The ability of any bending-type machines to identify low point E improves as the machine span is shortened. As pointed out previously, when both machines use the same span, Machine A is inferior to Machine B in its ability to identify low point E. The question arising is then how short a span should Machine A use to achieve the performance of Machine B in identifying low point E. The problem is investigated in Fig. 7, which shows, for both machines, the relationship between machine-measured low point E value and span, for two E_{low} values. The low point E data plotted are the averaged values for the two passes combined. As evident from examination of Fig. 7, for the low point E measuring accuracy of Machines A and B to be identical, the Machine A span would have to be reduced from 48 inches to approximately 18 inches, that is 37.5% of Machine B span. Such a short span, however, is impractical since the deflections that could safely be produced would be too small, possibly the same order of magnitude as the surface irregularities of lumber. Therefore, under practical situations, a type B machine should always be better than a type A machine for indicating low point E.

Ability to test end portions

A concern often expressed about all bending-type grading machines is that, at the beginning and at the end of each piece, there are portions that cannot be adequately tested. This fact is illustrated in Fig. 8 which compares the sensitivity of the two machines to the presence of a defect located anywhere between the left support and midspan. The two curves pertaining to $E_{low} = 0.5$ million psi are sections of those shown on Fig. 5. As apparent from the figure, both machines are almost insensitive to the presence of defects located within L/3 from the end of the piece.

CONCLUSION

The finite element model developed can be used to evaluate modulus of elasticity, as statically measured by bending-type grading machines. The model proposed is general in that 1) lumber characteristics critical in machine grading, such as initial deviation from straightness and local stiffness variation, can be taken into account, and 2) any machines can be represented by specifying proper loading and support conditions.

The numerical examples carried out show that, depending on support restrictions, machines may differ substantially in their performance, mainly regarding their ability to identify low point E value and location. The examples also indicate that some machines are less likely to cause permanent damage to the lumber as they apply lower stresses during grading. It is also observed that, in the case of bowed lumber graded in machines using unidirectional bending, the difference in E values measured for the two passes largely depends on the testing geometry used.

The model developed could be a valuable aid in the design of new machines and in the investigation of grading machines for approval.

REFERENCES

COOK, J. D. 1981. Concepts and applications on finite element analysis. John Wiley and Sons, NY.

ENDERSBY, H. J. 1969. Survey of machine stress grading. Proceedings from IUFRO meeting, Working Group on Structural Utilization, vol. II, For. Prod. Res. Lab., Princes Risborough, England. Pp. 149–176.

GALLIGAN, W. L., D. V. SNODGRASS, AND C. W. CROW. 1977. Machine stress rating: Practical concerns for lumber producers. USDA For. Serv. Gen. Tech. Rep. FPL 7, For. Prod. Lab., Madison, WI.

GLOS, P. 1982. Die maschinelle Sortierung von Schnittholz, Stand der Technik—Vergleich der Verfahen. (The mechanical sorting of lumber, status of the technique—Comparison of the processes.) Holz-Zentralblatt 108:153–155.

———, AND H. SCHULTZ. 1980. Stand und Aussichten der maschinellen Schmittholzsortierung. (The state and prospects of mechanical sawn timber grading.) Holz Roh- Werkst. 38(11):409–417.

KORNEEV, V. I. 1980. Investigation of devices for grading lumber according to strength. Lesnoi Zhurnal 3:98–104. (In Russian.)

LOGAN, J. D. 1978. Machine stress rating. Proceedings of the 4th nondestructive testing of wood symposium. Vancouver, WA. Pp. 285–303.

MÜLLER, P. H. 1968. Mechanical stress-grading of structural timber in Europe, North America and Australia. Wood Sci. Technol. 2:43–72.

THUNELL, B. 1969. On the premises for mechanical grading of timber. Paper and Timber 51(4):373–377.

APPENDIX I

Performing the multiplications and integrations involved in Eq. [8], the element stiffness matrix can be more simply expressed as

$$[K_e] = \frac{I}{H^3}\begin{bmatrix} 12a_e & (6a_e - 2b_e)H & -12a_e & (6a_e + 2b_e)H \\ & (4a_e - 2b_e)H^2 & (-6a_e + 2b_e)H & 2a_eH^2 \\ & & 12a_e & (-6a_e - 2b_e)H \\ \text{Symmetric} & & & (4a_e + 2b_e)H^2 \end{bmatrix}$$

APPENDIX II

Fixing the support angles is a method used in certain machines to isolate the bending span from vibration of the piece at the in-feed and out-feed ends of the machine. This is achieved by clamping the lumber in a series of rollers which can be inclined to an angle α (Logan 1978). In the present simulations, α is set to the precise angle at which a simply supported beam, initially straight and of uniform stiffness, would lie in the machine. It can be shown that this angle is given by

$$\alpha = \frac{3}{L}\Delta$$

where L is the span and Δ is the deflection at midspan.