A GEOMETRICAL MODEL FOR THERMAL CONDUCTIVITY

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ABSTRACT

The application of a cellular model equation to describe the effect of wood density upon transverse thermal conductivity indicates that the presence of bound water does not appear to affect the thermal conductivity of cell-wall substance. Good fits with the published experimental data are obtained when values of 1.0×10^{-4}, 10.5×10^{-4} and 21.0×10^{-4} cal/(cm °C sec) are assumed for air and cell-wall substance perpendicular to and parallel to the fiber axis. An empirical regression equation is presented that yields results nearly equal to those of the theoretical model-based equation.

THE GEOMETRICAL MODEL APPLIED TO TRANSVERSE FLOW

A geometrical model has been used successfully by Siau et al. (1968) to predict the physical properties of wood-polymer composites from those of their three components: cell-wall substance, polymer, and air. This model has been simplified to represent the two-component system of cell-wall substance and air as illustrated in Fig. 1. This model has been assumed to apply to all longitudinal wood cells including tracheids, fibers, vessels, and longitudinal parenchyma. The tangential and radial cell walls are assumed equal in thickness, and the proportion of cell-wall thickness to cell diameter is assumed equal for all cells. The cell-wall substance at the ends of the cells, the pit openings, and the presence of transversely oriented cells are neglected.

The mathematical analysis of the model with the flux in the transverse direction may be clarified by reference to Fig. 2. The model is assumed to have unit outside dimensions, and the width of the lumen is represented by a. The cross section is then divided into two cross walls with a width equal to the full width of the model and a combined length in the flux direction of $(1–a)$. The side walls are those portions of the cell wall aligned parallel with the flux and with a length equal to that of the lumen (a), and a combined width of $(1–a)$. The lumen is a square having the dimension a on each side.

It is clear, then, that the parameter, a, is equal to the square root of the porosity of the wood.

$$a = \sqrt{V_a} = \sqrt{1 - V_w}, \qquad (1)$$

or

$$V_a = a^2, \qquad (2)$$

where V_a = porosity of wood;
V_w = volume fraction of cell-wall substance in wood.

The porosity of wood may be calculated from the following relationship.

$$V_a = a^2 = 1–G(0.685 + 0.01M/G_s), \quad (3)$$

where G = specific gravity of wood on a dry weight and moist volume basis;
$0.685 = 1/1.46$ where 1.46 = specific gravity of dry cell-wall substance;
M = moisture content, per cent of oven-dry weight;
G_s = specific gravity of bound water according to Mac Lean (1952).

When the model is considered as shown in Fig. 2, the same flux flows in Section (1) as in Sections (2) and (3) in parallel. If the misalignment of cells and the flow of flux across the boundary between Sections (2) and (3) are neglected, the following conductivity equation may be derived.

$$K_{gT} = \frac{(1-a)\,K_{wT}^2 + aK_aK_{wT}}{(1-a)^2K_{wT} + a(1-a)\,K_a + aK_{wT}}, \quad (4)$$

where K_{gT} = transverse thermal conductivity of gross wood;
K_{wT} = transverse thermal conductivity of cell-wall substance;

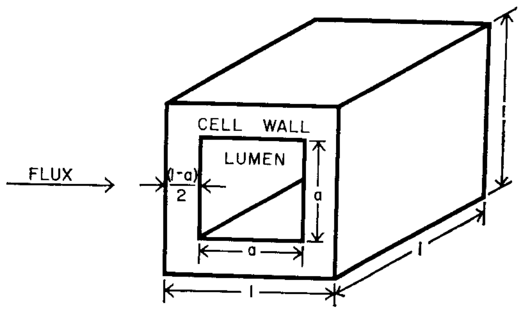

FIG. 1. Geometrical model for single wood cell.

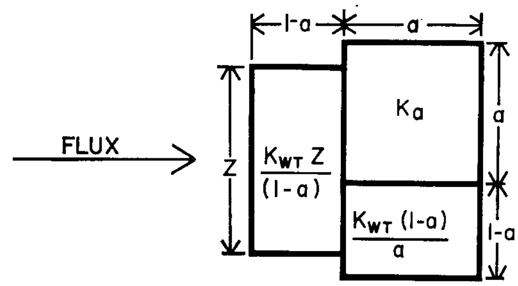

FIG. 3. Conductivity model after modification for reduced conductive efficiency of the crosswalls.

K_a = thermal conductivity of air in voids.

Stamm (1964), Maku (1954), and Choong (1965) have discussed similar transverse conductivity models but have derived their equations for the flux at 90 degrees to the direction indicated in Fig. 2.

Since the cross wall is not completely effective for conduction because of the low concentration of flux adjacent to the lumen, a factor, Z, is introduced to account for this deficiency. When this factor is introduced into Equation (4),

$$K_{gT} = \frac{(1-a)ZK_{wT}^2 + aZK_aK_{wT}}{(1-a)^2K_{wT} + a(1-a)K_a + aZK_{wT}}, \quad (5)$$

where Z = fraction of cross wall that may be considered effective for conduction.

Equation (5) applies to a modified model as depicted in Fig. 3.

Hart (1964) has investigated the fringing current effect in the cross walls of the model, assuming no conductivity for the air space. He derived an empirical equation for calculating the fraction of the cross-wall width that may be considered effective for conduction in which

$$w_1 = \frac{0.48\,(1-a)}{a}[1-\exp(-.208a/(1-a))], \quad (6)$$

where w_1 = fraction of cross wall, located adjacent to the lumen, which is effective for conduction.

There is a significant heat flux in the lumens of wood, and it is assumed that this flux flows unimpeded through the portion of cross wall of width a. The concentration of flux in the lumen is K_a/K_{wT} times the concentration in the cell-wall substance of the side walls. Because of this relatively low flux concentration, the additional cross-wall width effective for conduction because of the lumen flux may be calculated as $(a\,K_a/K_{wT})$. Therefore the total fraction of cross wall that is effective for conduction will be

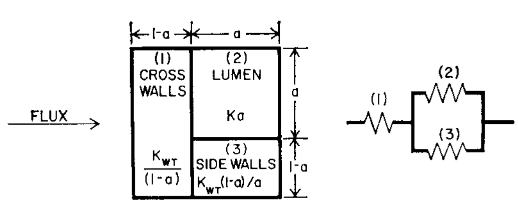

FIG. 2. Equivalent configuration of conductivity model showing conductivities of sections and equivalent electric circuit.

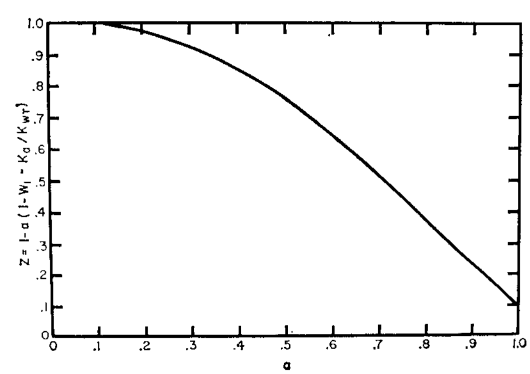

FIG. 4. Z as a function of a.

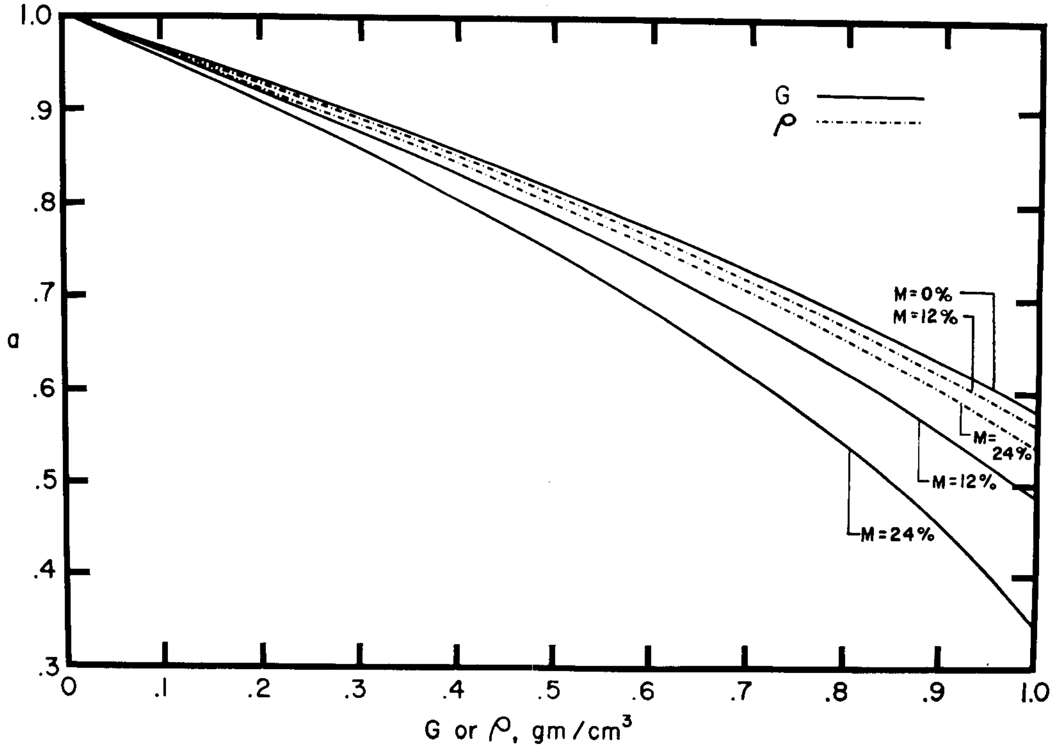

FIG. 5. Parameter a as a function of specific gravity and density for $M = 0$, 12, and 24%.

$$Z = 1 - a\left(1 - w_1 - \frac{K_a}{K_{wT}}\right). \qquad (7)$$

The value of Z as a function of a may be determined from Fig. 4.

Since Equation (5) expresses K_{gT} as a function of a, it is useful to relate this latter variable to the specific gravity and to the density of wood. The value of a is obtained from the specific gravity using Equation (3). The relationship between density and specific gravity may be expressed as follows,

$$\rho = G\ (1 + .01\ M) \times 1\ \text{g/cm}^3 \qquad (8)$$

where ρ = density of wood, g/cm³

The relationship between a and specific gravity and density for moisture contents of 0, 12, and 24% is presented in Fig. 5.

RESULTS OF PREVIOUS INVESTIGATIONS

Much of the previous work on the thermal conductivity of wood has been done using the British engineering system and the m.k.s. system. Therefore, data of other investigators have been converted to the c.g.s. system using conversion factors listed in Table 1.

TABLE 1. *Conversion Factors*

1 BTU in/(ft² hr °F)
= 3.44×10^{-4} cal/(cm °C sec)
1 kcal/(m hr °C)
= 2.78×10^{-3} cal/(cm °C sec)

Mac Lean (1941) presents an empirical equation that may be written in the form

$$K_{gT} = [G(5.18 + 0.091M) + 0.57V_a] \times 10^{-4}, \qquad (9)$$

where K_{gT} = transverse thermal conductivity of gross wood, cal/(cm °C sec);
M = moisture content, % ($M <$ 40%);
V_a = porosity of wood. Mac Lean

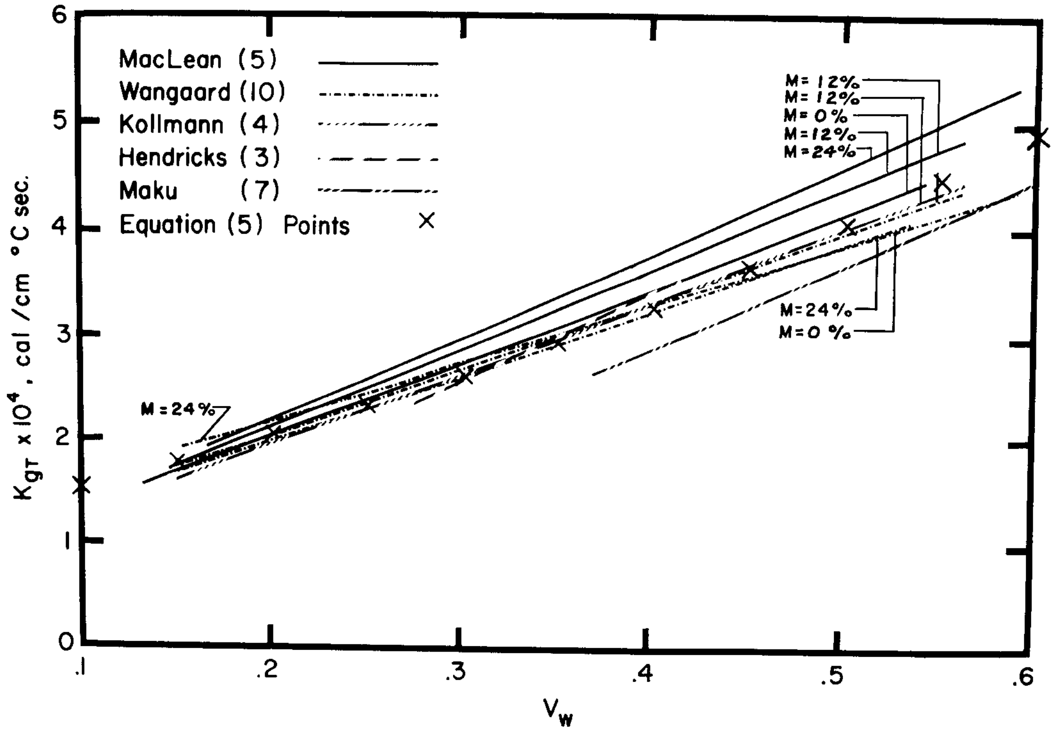

FIG. 6. Transverse thermal conductivities from literature as a function of volume fraction of cell-wall substance.

(1941) calculated this quantity as $[1.0\text{-}G(0.685 + 0.01M)]$.

Equation (9) may be considered to represent a model consisting of parallel conductive paths of cell-wall substance, bound water, and air. Equation (9) predicts values of 7.6×10^{-4}, 9.1×10^{-4}, and 0.57×10^{-4} cal/(cm °C sec) for cell-wall substance, bound water, and air when the specific gravity of cell-wall substance is assumed to be 1.46.

Kollmann (1951) gives an empirical equation for the transverse thermal conductivity of wood at $M = 12\%$.

$$K_{gT} = (4.67\rho + 0.61) \times 10^{-4}, \qquad (10)$$

where ρ = density of moist wood, g/cm^3.

This predicts thermal conductivities of 7.3×10^{-4} and 0.61×10^{-4} cal/(cm °C sec) for moist cell-wall substance ($M = 12\%$) and air, respectively. The value for moist cell-wall substance is based upon 1.46 g/cm^3 as the density of dry cell-wall substance and 1.18 g/cm^3 as the density of sorbed water, according to Mac Lean (1952). This gives a density of moist cell-wall substance ($M = 12\%$) of 1.43 g/cm^3.

Maku (1954) made measurements of the transverse thermal conductivity of compressed wood samples and calculated a value of 10.0×10^{-4} cal/(cm °C sec) for cell-wall substance, using a cellular model equation based upon a flux direction 90 degrees to the direction indicated in Fig. 2. Thus it is apparent that the assumption of a cellular model results in a higher conductivity for cell-wall substance than those obtained from Equations (9) and (10). Maku also found that the agreement between his model equation and experimental results at low wood specific gravities was improved when a conductivity of 1.1×10^{-4} cal/(cm °C sec) was assumed for air. This value is

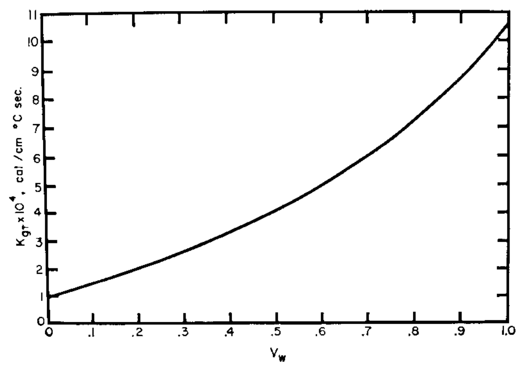

FIG. 7. Transverse thermal conductivity of wood vs. volume fraction of cell-wall substance according to model Equation (5).

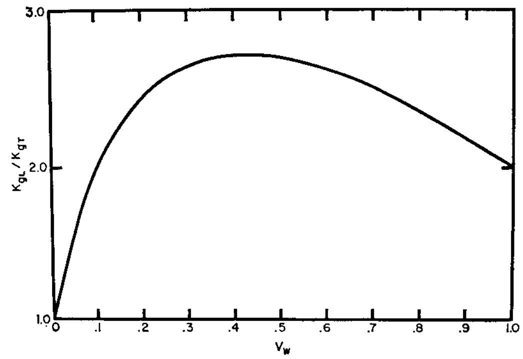

FIG. 9. Ratio of longitudinal to transverse thermal conductivities as a function of volume fraction of cell-wall substance.

higher than that for still air, and the apparent discrepancy was attributed to radiation and convection.

Kollmann and Malmquist (1956) employed a layered model from which they calculated a thermal conductivity of 9.7×10^{-4} cal/(cm °C sec) for dry cell-wall substance.

Transverse thermal conductivity data obtained from the literature are presented in Fig. 6 on a fractional cell-wall basis. The values were converted to this form using Equation (3).

The data of Wangaard (1943) were determined from his alignment chart for values of M of 0, 12, and 24%. These lines are very closely coincident. Equation (10) of

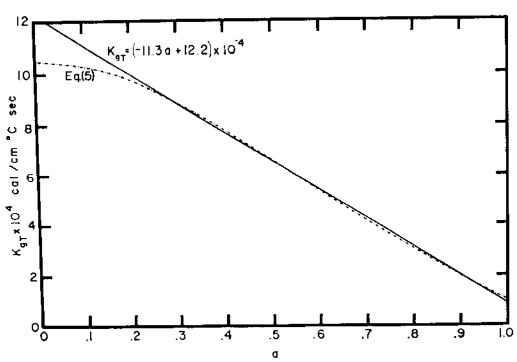

FIG. 8. Transverse thermal conductivity of wood vs. a according to model Equation (5) and linear regression equation (11).

Kollmann (1951) and Hendricks' data (1962) are in close agreement with Wangaard. Equation (9) of MacLean was also plotted for the same values of M, with the higher moisture contents yielding somewhat higher conductivities. Since these data show essentially that the change in thermal conductivity with moisture content is small when calculated on a volume-fraction-of-cell-wall basis, there is a good indication that the value of K_{wT} is independent of the moisture content and that it may be considered so for practical purposes for wood. Theoretical points based upon Equation (5) are plotted for comparison with the literature values. Fairly good agreement is obtained when values of 1.0×10^{-4} and 10.5×10^{-4} cal/(cm °C sec) are assumed for K_a and K_{wT} respectively, with the latter value being assumed independent of moisture content. It is clear, however, that the value of K_{gT}, as calculated from Equation (5) is not independent of moisture content because an increase in moisture content decreases the porosity of the wood (and a) resulting in an increase in K_{gT}.

Equation (5) is plotted in Fig. 7, using V_w as the independent variable. This shows a nearly linear function up to V_w of 0.5, with increasing slope as K_{gT} approaches 10.5×10^{-4} cal/(cm °C sec) for solid cell-wall substance.

Equation (5) is plotted with a as the independent variable in Fig. 8. It is clear that this function is very nearly a straight line between values of a from 0.95 to 0.225 (G from 0.15 to 1.4). Since this covers almost all wood specific gravities, a useful regression equation may be obtained that has a maximum deviation of 2.8% from Equation (5) within the above limits of G.

$$K_{gT} = (-11.3a + 12.2) \times 10^{-4} \qquad (11)$$

THE GEOMETRICAL MODEL APPLIED TO LONGITUDINAL FLOW

When the model of Fig. 1 is applied in the direction of the fiber axis, the following theoretical equation results.

$$K_{gL} = K_{wL} V_w + K_a (1 - V_w), \qquad (12)$$

where K_{gL} = longitudinal thermal conductivity of gross wood;
K_{wL} = longitudinal thermal conductivity of cell-wall substance.

Maku (1954) presented a longitudinal model equation identical with Equation (12). Values of K_{gL} originally reported by Griffiths and Kaye (1923) and Kollmann (1936) were substituted into the model equation with a value of K_a of 0.61×10^{-4} cal/(cm °C sec). This resulted in a calculated value of 15.6×10^{-4} cal/(cm °C sec) for cell-wall substance in the longitudinal direction.

When a value of 21.0×10^{-4} cal/cm °C sec is assumed for K_{wL}, the resulting ratios of K_{gL}/K_{gT} are shown in Fig. 9. The values are within the range of 2.25 to 2.75 for usual gross wood specific gravities, in agreement with the findings of Mac Lean (1952).

CONCLUSION

In conclusion it may be stated that thermal conductivity equations based upon the cellular geometrical model result in good agreement with experimental data when values of 1.0×10^{-4}, 10.5×10^{-4}, and 21.0×10^{-4} cal/(cm °C sec) are assumed for K_a, K_{wT}, and K_{wL}, and when the thermal conductivity of cell-wall substance is assumed to be independent of moisture content. In addition the ratios of longitudinal to transverse conductivities of wood are in agreement with the findings of Mac Lean (1952). It may also be stated that the model presented here is an approximation that does not consider the effects of such factors as extractive content, microfibrillar orientation, and nonuniformity of the cell-wall structure that may occur between different species of wood.

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