MODELING THE RELIABILITY OF WOOD TENSION MEMBERS EXPOSED TO ELEVATED TEMPERATURES

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ABSTRACT

The merit of approaching fire safety design from the standpoint of reliability is the impetus of this paper. Reliability, a direct function of time to failure, is a measure of performance that falls naturally under a performance-based code. The objectives of this study focus on advancing our understanding of the structural behavior of light-frame wood members subject to tension and elevated temperatures, and on the time to failure under a given stress and temperature history. A model based on linear damage accumulation theory was developed to predict the time to failure. This model is based on a kinetic theory for strength as a function of temperature and stress, coupled with a kinetic term, to express the pyrolytic process as a form of damage. The model, which requires the short-term strength as an input, fits well to experimental data on nominal 2×4 structural lumber tested at three different rates of tension loading, and at 150, 200, and 250°C, and room temperature. The model also predicts, with reasonable accuracy, the behavior of lumber under constant-load at 250°C. It predicts that lower-grade material generally has a lower reliability index; however, those differences are insignificant as far as current design practices are concerned. The reliability is sensitive to variability in temperature but not to variability in stress.

Keywords: Modeling, reliability, wood, lumber, tension member, elevated temperature, fire.

INTRODUCTION

Modern engineered wood structural systems are usually assembled from structural lumber having thickness 38 mm and width dimensions from 89 to 235 mm. While our knowledge of their structural performance has been steadily advanced, the design for fire safety has remained relatively unchanged and has not considered redundancy lost due to improved structural efficiency. Consequently, the ability of modern engineered wood-frame systems to maintain adequate fire endurance has been questioned and debated (Brannigan 1992; Corbett 1988; Mittendorf and Frannigan 1991; Schaffer 1988). While such concerns are legitimate, the issue must be addressed from the viewpoint of reliability. Knowledge of the behavior of wood elements exposed to fire must therefore be advanced to permit calculation of reliability. This paper discusses a study on the tensile behavior and reliability of typical light-framing members exposed to elevated temper-

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atures, and the development of a damage accumulation model to predict the reliability of such members at elevated temperatures.

BACKGROUND

Probabilistic design and reliability analysis

The reliability of a structural system is the probability that it will successfully perform a specific function under a given set of operating conditions for a specified duration. With respect to fire safety, these functions relate mainly to time-based criteria such as time to flash over, time to collapse, and time available for the orderly egress of occupants. In this regard, reliability analysis may take the form of $G = t_f - t_r$, where t_f is the failure time of a specific function and t_r is a required endurance time to perform that function. In reliability analysis, we seek the probability that G is greater than zero. This value is the reliability (probability of survival) of the system. The survival probability can be determined from actual observations of the variable time-to-failure (t_f).

Time to failure.—When wood is exposed to elevated temperatures, its load-carrying capacity is reduced partly by charring, which is considered to have no strength, and partly by strength degradation of the unburnt region (as a result of pyrolysis). For a member stress in tension, we may express the failure event as an equality between strength in reserve and increase in stress due to charring in the following form:

$$\sigma_u(T(t_0 \rightarrow t_f), \tau(t_0 \rightarrow t_f), \sigma_0) - k\sigma_0 = \frac{q(q_0, t_f)}{A(e(t_f))} - \frac{q_0}{A_0} \quad (1)$$

where

$\sigma_u(\cdot)$ = apparent strength at failure,

σ_0 = initial strength,

$\tau(t_0 \rightarrow t_f)$ = stress history from the beginning of the fire (t_0) to the time of failure (t_f),

$T(t_0 \rightarrow t_f)$ = temperature history from the beginning of the fire (t_0) to the time of failure (t_f),

k = initial stress ratio of load to capacity,

$q(\cdot)$ = load during the fire expressed as a function of the initial load q_0 and t_f,

$A(\cdot)$ = unburnt portion of the member expressed as a function of the char thickness e.

This equation can provide a solution for the variable t_f. The distribution form of f_f depends on the distribution forms of the other variables and their interdependence. Norén's work (1988) on lumber exposed to fire indicated that partially pyrolyzed wood can lose up to 75% of its strength. Interacting with this effect is the effect of stress and its history. In wood, the stress effect is referred to as the load-duration factor, which is expected to increase in severity with increasing temperature.

In order that Eq. (1) can be solved, $\sigma_u(\cdot)$ is needed to be known for all likely combinations of temperature and stress histories. This is awkward. A model based on damage accumulation concept was developed, which used the initial strength of the member, and the temperature and stress histories (load duration) as input to predict t_f.

Time-dependent lumber strength models

Damage accumulation approach.—For a material subject to repeated loads, Miner's theory (1945) postulates that a finite amount of "damage" is incurred each time the load is applied. By assuming that the damage incurred per cycle as the reciprocal of the number of cycles required to fail the specimen at the same load, Miner was able to show experimentally that failures tended to occur as the sum of such fractions more or less equaled one. The criterion for failure is

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \cdots + \frac{n_n}{N_n} = 1 \quad (2)$$

where n_1, n_2, and n_3 are the number of cycles at stress levels S_1, S_2, and S_3, and N_1, N_2, and N_3 are the number of cycles required to fail the specimen when only the corresponding

TABLE 1. *Damage accumulation models for duration of load effect in wood.*

Wood (1951)	$\frac{\partial \alpha}{\partial t} = A\,(\tau - \tau_0)^B$
Barrett and Foschi (1978)	$\frac{\partial \alpha}{\partial t} = A\,(\tau - \tau_0)^B\,\alpha^C$
Barrett and Foschi (1978)	$\frac{\partial \alpha}{\partial t} = A\,(\tau - \tau_0)^B + C\,\alpha$
Foschi et al. (1989)	$\frac{\partial \alpha}{\partial t} = A\,(\sigma - \tau_0\sigma_u)^B + C\,(\sigma - \tau_0\sigma_u)^D\,\alpha$
Gerhards (1979)	$\frac{\partial \alpha}{\partial t} = e^{-A + B\tau}$

stress level is applied. Since the ratio $1/N_i$ can be viewed as the rate of damage per cycle at the stress level S_i, we can express the failure criterion as

$$\int_0^{t_f} \frac{\partial\alpha}{\partial t}\, dt = 1 \tag{3}$$

where α represents the damage state. The models developed for wood are listed in Table 1. All are empirical in the sense that they are not based on any particular failure mechanism. With the exception of Gerhards (1979), the models include a stress threshold τ_0 below which stress has no effect.

Other theoretical approaches.—Other approaches to model the time-dependent behavior include a fracture model (Nielsen 1978), which considers crack-opening and crack-lengthening as the main cause of damage, a chemical kinetics model (Caulfield 1985; van der Put 1989), which recognizes the effect of stress on the potential energy barriers impeding molecular motion, Liu and Schaffer's application of statistical theory for the absolute reaction rate of bond formation and breakage (1991), and Fridley's proposal of a "strain energy density" reaching a certain critical level as a criterion for impending failure (Fridley et al. 1992). The approach proposed by Liu and Schaffer (1991) was an extension of earlier work by Hsiao and others (Hsiao 1966; Hsiao et al. 1968; Hsiao and Ting 1966; and Schaffer 1973). Hsiao's work on kinetic strength theory forms the basis for the model developed in this paper and will be discussed later.

Empirical approaches.—Several empirical models have been proposed for prediction of the structural resistance of lumber members in fire exposures. One, developed by White (1996), expresses the mean effect on tensile strength as an exponential function of the mean temperature of the wood. However, this model is restricted to predicting behavior under the same fire-exposure used to develop the model. It can not predict heating-history effects or variability. In their thermal-degrade model for lumber strength, Shrestha and Cramer (1995) accounted for the effect of heating history by incorporating area under time-temperature curve as one parameter in addition to time of exposure. However, the appropriateness of this approach to model cumulative effect of heating history is yet to be verified. As is the case with White (1996), this model predicts only mean behavior. Bender et al. (1985) proposed a model for prediction of t_f of glue-laminated beams. This model takes into account the strength variations of lumber and end-joints between laminates. The ultimate moment carrying capacity of the beams was calculated on the basis of randomly generated strength properties of individual laminates, and laminate length data, which affect the number and location of end-joints. This information was used with a fire endurance model to predict the t_f of the beam during ASTM E 119 fire exposure. Using Monte Carlo simulation, the distribution of t_f was estimated. The charring rate and the strength degradation factor for unburnt wood were assumed to be con-

TABLE 2. *Experimental design and sample size of each different combination of variables of the experiment.*

Rate of loading (kN/s)	Temperature of exposure (°C)			
	20	150	200	250
1.85	180	60	60	60
0.2	180	60	60	60
0.067	180	60	60	60

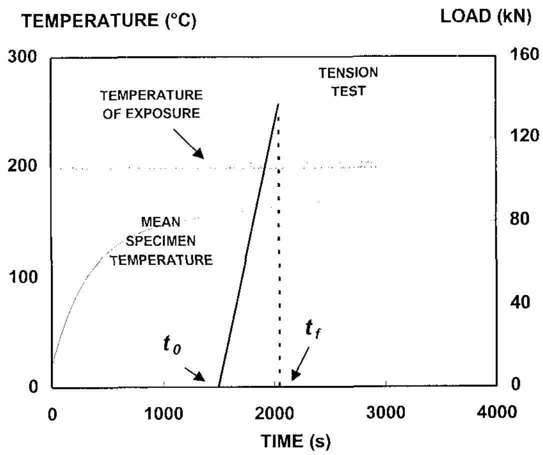

FIG. 1. A typical heating and loading sequence of the experiment.

stant. Gammon (1987) outlined the general requirements for probabilistic design of wall assemblies for fire safety but did not actually perform any reliability calculations due to lack of data. However, it was noted that, as was the case with Bender et al. (1985), an approach based on conditional reliability, in which some of the variables are treated as constant, may be used.

EXPERIMENTAL DESIGN

To yield data to calibrate a t_f model, "matched" samples of structural lumber were exposed to elevated temperatures. Part-way through the exposure, the specimens were subjected to a tension test at a constant rate of loading until failure. The ramp-load history mimics stress histories experienced by wood members exposed to fire because char formed reduces cross-sectional area. Table 2 shows the experimental variables and sample sizes for individual test groups. The loading rate of 1.85 kN/s produced, on average, a failure time of one minute at room temperature. The other load rates (0.2 and 0.067 kN/s) led to mean failure times of approximately 10 and 30 minutes, respectively, at room temperature. Figure 1 shows schematically a typical heating and loading sequence in the experiment. In this case, the exposure temperature was 200°C. The test materials were heated by conductive heat transfer with a pair of electrically heated aluminum platens, 2,440 mm long by 100 mm wide by 50 mm thick (Fig. 2). The length of specimen subjected to exposure was approximately 2,440 mm (exposed region). The origin of time ($t = 0$) was set when contact was made between the platens and the specimen. The loading in tension was initiated at 1,500 seconds (t_0 = 1,500 s). The temperatures of the specimen at two points, 610 mm from the center, was measured with ungrounded, type-K thermocouple probes. These probes had an outside diameter of 1.6 mm. Holes of the same diameter were drilled to 17.5 mm deep from the narrow face of the specimens to accommodate the probes. The heating and loading histories were recorded. Values computed were the maximum tension load and the time to failure.

Assumptions.—The analytical procedures assume that the sample groups have the same distribution of room-temperature tension strength, and that the location of the weakest point in the member does not change with temperature. Mean temperature is assumed to be a sufficient parameter for predicting temperature effects. The first assumption was confirmed with Kolmogorov-Smirnov (K-S) statistics (Press et al. 1992).

Material sampling.—Approximately 1,600 pieces of Machine Stress Rated (MSR), 2 × 4 Spruce-Pine-Fir (SPF) lumber were sampled. This species group constitutes a major source of softwood lumber in North America. The materials were taken from three MSR grades in equal quantities. The final moisture content of the specimens was approximately 9–11%.

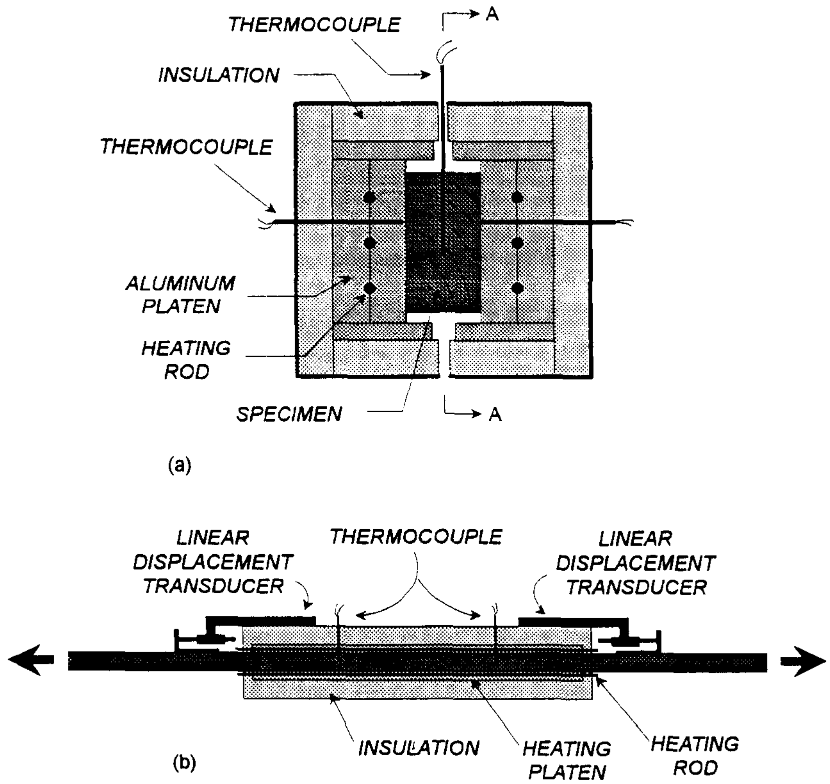

FIG. 2. Cross- (a) and longitudinal-sectional (b) views of the heating apparatus and the test specimen.

The specimens were planed to a thickness of 35 mm so that the surfaces were uniform for heating. Finally, each specimen was evaluated for the modulus of elasticity (MOE) value using an MSR grading machine. Based on this ranking of MOE values, groups of 60 specimens were formed in a systematic fashion.

MODELING APPROACH

Model development

In generalizing the approach based on damage accumulation, we may write

$$\frac{\partial \alpha}{\partial t} = F(\tau(t), T(t), \alpha) \tag{4}$$

where the rate of damage, $\partial\alpha/\partial t$, in a unit volume of material at any time t is expressed as a function of stress $\tau(t)$ and temperature $T(t)$ at time t, and the damage α accumulated to the time. In this approach, t_f is the time at which $\alpha = 1$.

If f denotes the fraction of unbroken elements or bonds in the direction of applied stress, then, according to Hsiao et al. (1968), the rate of change of f is given by

$$\frac{\partial f}{\partial t} = K_r(1 - f) - K_b f \tag{5}$$

where

$$K_r = \omega_r \exp\left(-\frac{E_r}{RT} - \gamma\varphi(t)\right) \tag{6}$$

is the rate of reformation of broken elements, and

$$K_b = \omega_b \exp\left(-\frac{E_b}{RT} + \beta\varphi(t)\right) \tag{7}$$

is the rate of rupturing of unbroken elements;

ω_r and ω_b are, respectively, the frequencies of the jump motion of the elements with respect to forming and breaking processes; E_r and E_b are the activation energies for bond reforming and bond rupture; R the universal gas constant; T the absolute temperature; γ and β are positive quantities that modify the energy barrier as a consequence of the applied stress in the direction of each element; and φ is the longitudinal stress per bond. Following the arguments made by Liu and Schaffer (1991), the value of K_r is ignored since loading rates are generally fast enough that bond reformation is unlikely and since strain is seen to be proportional to stress in lumber subject to tension at room temperature. Under elevated temperatures, bond reformation is even less likely since strain rate increases with increasing stress (an observation made in our experiment).

Assuming that bonds are oriented in the direction of the applied stress, bond stress $\varphi(t)$ and external stress $\tau(t)$ are related by

$$\varphi(t) = \frac{\tau(t)}{f(t)} \tag{8}$$

provided f is independent of bond orientation. Eq. (5) then becomes

$$\left(\frac{df}{dt}\right)_\tau = -K_b f \tag{9}$$

The subscript τ indicates the bond rupture process is an effect due to stress.

Intuitively, bonds can also be broken as a result of pyrolysis, and the process is independent of the effect of stress. Assuming that this effect is similar to rate of loss of density, we can express the effect by an Arrhenius expression (Roberts 1970):

$$\left(\frac{df}{dt}\right)_T = -\psi \exp\left(-\frac{E_T}{RT}\right) \tag{10}$$

where ψ is a coefficient depending on the anatomical property of the material (we may refer to this as a species effect). The subscript T signifies that this is a pyrolytic process. The apparent rate equation for f—assuming the two effects can be added—then becomes

$$\frac{df}{dt} = \left(\frac{df}{dt}\right)_\tau + \left(\frac{df}{dt}\right)_T = -\psi \exp\left(-\frac{E_T}{RT}\right) - f\omega_b \exp\left(-\frac{E_b}{RT} + \beta\varphi(t)\right) \tag{11}$$

Zhurkov (as cited in Bartenev and Zuyev 1968) found that activation energies in thermal degradation are of the same order of magnitude as those derived from strength tests. On the basis of several independent studies, Stamm (1964) arrived at a similar conclusion as Zhurkov. We may therefore assume that $E_T = E_b = E$, and Eq. (11) simplifies to

$$\frac{df}{dt} = \exp\left(-\frac{E}{RT}\right)[-\psi - f\omega_b \exp(+\beta\varphi(t))] \tag{12}$$

Substituting the term $\varphi(t)$ by Eq. (8) and expressing $T = T(t)$,

$$\frac{df}{dt} = \exp\left(-\frac{E}{RT(t)}\right)\left[-\psi - f\omega_b \exp\left(+\beta\frac{\tau(t)}{f}\right)\right] \tag{13}$$

Eq. (13) can be generalized as a four-parameter—a_1, a_2, a_3, and a_4—model for bond breakage as a result of both stress and pyrolysis:

$$\frac{df}{dt} = \exp\left(-\frac{a_1}{T(t)}\right)\left[-a_2 - f a_3 \exp\left(+a_4\frac{\tau(t)}{f}\right)\right] \tag{14}$$

Note that these parameters must be positive real numbers greater than zero. Furthermore, we assume

$$f|_{t=0} = \frac{\sigma_0}{\sigma^*} \tag{15}$$

where σ_0 is the short-term strength and σ^* represents the ideal strength of the material. Thus f can have a maximum value of 1. The failure criterion is $f = 0$. If we treat $1 - f = \alpha$, where α is a damage parameter, then Eq. (14) is equivalent to a damage accumulation model of the following form:

$$\frac{d\alpha}{dt} = \exp\left(-\frac{a_1}{T(t)}\right) \times \times \left[a_2 + (1-\alpha)a_3\exp\left(a_4\frac{\tau(t)}{(1-\alpha)}\right)\right] \quad (16)$$

where a_1, a_2, a_3, and a_4 are model parameters. The initial value of α is given by

$$\alpha\big|_{t=0} = 1 - \frac{\sigma_0}{\sigma^*} \quad (17)$$

and failure is governed by the criterion

$$\alpha\big|_{t=t_f} = 1 \quad (18)$$

Key model characteristics.—In order for a damage-accumulation function to describe lumber strength accurately, the function must possess the ability to describe how "damage" is accumulated in lumber under ramp-loading at room temperature. Researchers in this field have generally agreed that damage should initially accumulate very slowly, with most of the damage occurring near the end when the applied load nearly reaches the ultimate load. This phenomenon explains why we do not generally observe significant rate-of-loading effects at room temperature at rates such as those used in our study. The damage-rate function, with its exponential terms, indeed possesses this characteristic.

The other key characteristic involves σ^* which was treated as a dependent on σ_0. The relationship

$$\sigma^* = \exp\left(\frac{0.463}{\sigma_0}\right)(-0.218\sigma_0 + 508.0) \quad (19)$$

was developed to enforce the consistency requirement that when stress history $\tau(t) = k_0 \cdot t$ is entered into the model, the failure time $t_f = \sigma_0/k_0$ should be predicted if σ_0 is the strength determined in a standard ramp-load test with rate of loading k_0.

Failures outside exposed region.—About 20% of the specimens of the room-temperature groups failed outside the exposed region. The results of these outside failures were

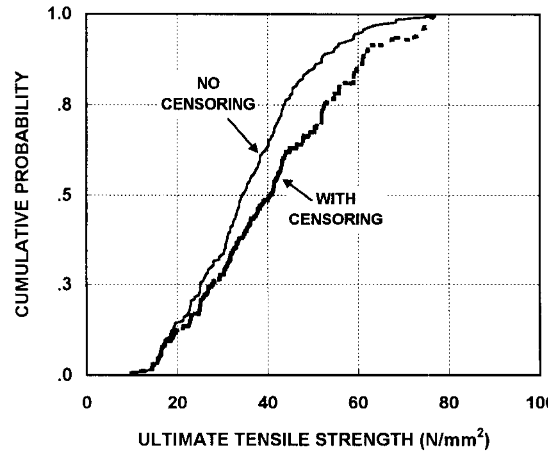

FIG. 3. Cumulative frequency distributions of sample containing outside failures.

"censored"—not discarded—according to survival probabilities (Bury 1975) since it was ascertained that the exposed region survived the observed load. For a sample contributing censored information,

$$F(x') = 1 - \prod_{i=1}^{k} (n - i)/(n - i + 1) \quad (20)$$

where $F(x')$ is the nonparametric cumulative distribution of x', k is defined by $x_k \leq x' \leq x_{k+1}$, n is the original sample size, i is the order of a failed specimen in the complete sample including the censored failure loads. Figure 3 shows how the CDF of the room-temperature groups tested at the loading rate of 1.85 kN/s is shifted by the censoring. This is intuitively correct since the failure strength of the center 2,440-mm portion ought to be higher than that of the full length between the tension grips (4,267 mm) because a longer piece of lumber is more likely to process a weaker "link" (Madsen and Buchanan 1986).

Temperature histories.—Temperature histories were expressed in terms of the mean temperatures calculated from measured temperature data using this equation:

$$T = \frac{2T_C + T_S}{3} \quad (21)$$

where T_C is the temperature at the center of the member and T_S is the exposure temperature. The individually calculated mean temperature histories were then averaged by each temperature of exposure. The final result was assumed to be the temperature history of the members for each exposure temperature. The following function fits the final calculated mean temperature histories using t and T_S as input:

$$T - T_0 = [P_0(T_S - T_0) + P_1 t]\left[1 - \exp\left(-\frac{Kt}{P_0}\right)\right] \quad \text{for } T_S > T_0$$

$$T - T_0 = 0 \quad \text{for } T_S = T_0 \qquad (22)$$

where T_0 is the room temperature (K), T_S the temperature of exposure (K), t the time (s), $P_0 = 0.66514\ K^{-1}$, $P_1 = 0.011514\ K{\cdot}s^{-1}$, $K = 0.002242\ s^{-1}$, and $T_0 = 293$ K. With this model, the initial rate of increase of the mean temperature is given by $K\cdot(T_S - T_0)$. The model approaches the linear model given by $P_0\cdot(T_s - T_0) + P_1\cdot t$ as time t is increased. Both are reasonable expectations. The fit of this model to the test data is illustrated in Fig. 4.

Stress histories.—The stress history of the experiment was given by

$$\tau(t) = \frac{k}{A}(t - t_0) \quad \text{for } t > t_0$$

$$\tau(t) = 0 \quad \text{for } t \leq t_0 \qquad (23)$$

where $\tau(t)$ is the stress history, A the cross-section area of the member, k the rate of loading and t_0 the time at which the tension test began (see Fig. 1).

Method of fitting.—The set of coefficients which "best" fits the test data was found by minimizing the normalized sum of square of the differences between predicted t_f from the model and observed t_f from the experiments. These differences were evaluated at N arbitrarily chosen percentiles of the t_f distributions. The normalized sum of square of the differences (Ω) was given by

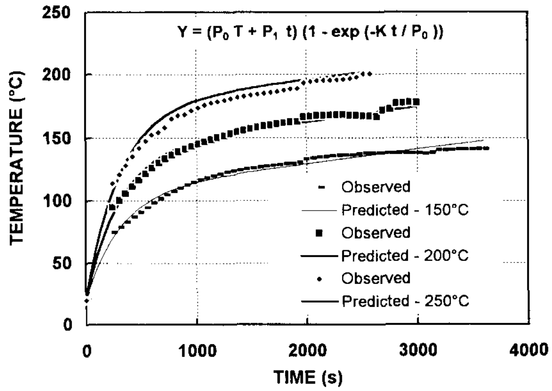

FIG. 4. Temperature histories predicted by Eq. (22) in comparison with observed data.

$$\Omega = \sum_{k=1}^{N} \sum_{i} \sum_{j} \left(1.0 - \frac{(\hat{t}_f)_{i,j}}{(t_f)_{i,j}}\right)^2 \qquad (24)$$

where $(\hat{t}_f)_{i,j}$ and $(t_f)_{i,j}$ were, respectively, the predicted and experimental k^{th} percentile t_f values for the i^{th} temperature of exposure and j^{th} rate of loading. The minimum of Eq. (24) is found by the *Polak-Ribiere* algorithm (Press et al. 1992). The inputs to the minimization algorithm are short-term strength σ_0, and temperature and stress histories. Given values of the coefficients—σ^*, a_1, a_2, a_3, and a_4, a sample of t_f is simulated in accordance with Eq. (16) for each temperature-stress history. The nonparametric t_f percentile values for each temperature-stress history are then determined, and entered into Eq. (24) as the predicted values. The observed percentile values are the nonparametric values determined from the distribution of the experimental data.

The short-term strength σ_0 is generated from the nonparametric distribution of the actual room temperature test results to which the censoring (for location of failure) has been applied. First, a random number r between 0 and 1 is generated. Given this value, the value of σ_0 is interpreted by the following formulae:

$$\sigma_0 = (\sigma_0)_i + [(\sigma_0)_{i+1} - (\sigma_0)_i] \times \times \left[\frac{r - P[(\sigma_0)_i]}{P[(\sigma_0)_{i+1}] - P[(\sigma_0)_i]}\right] \qquad (25)$$

where $(\sigma_0)_i$ and $(\sigma_0)_{i+1}$ are the two ordered values such that $P[(\sigma_0)_i] \leq r \leq P[(\sigma_0)_{i+1}]$, and $P(\cdot)$ is the nonparametric percentile value.

Results of model fitting

The fitted parameters are a_1 = 16,400 K, a_2 = 1.0×10^{10} S^{-1}, a_3 = 1.8×10^7 s^{-1}, and a_4 = 0.065 mm^2N^{-1}. Figure 5 shows the model's predicted t_f expressed as cumulative frequencies, compared with the nonparametric distributions of the experimental t_f data. The model fits the observed data very well overall, and exceptionally well at 200 and 250°C. Also, the model was able to predict the decrease in variance of the t_f with increasing temperature. The model does exhibit some difficulty in fitting the data at 150°C, particularly for the slowest rate of loading (0.067 kN/s). The reason the fit at 150°C is less accurate may be explained by the fact that at this exposure temperature, the mean maximum temperature of the specimens was only 125°C. At this temperature, the degradation process predominantly involved dehydration or hydrolyzation of the hemicellulose fraction. These processes are different from the pyrolytic processes associated with temperatures at 200°C or higher.

Relevance with published data.—According to Eq. (13) and Eq. (14), $a_1 = E/R$, $a_2 = \psi$, $a_3 = \omega_b$, and $a_4 = \beta$, where E, as defined previously, is the activation energy for bond breaking and reforming, R is the universal gas constant, ω_b is the frequency of jump motion of "elements" with respect to the bond breaking process, ψ is the preexponential constant in the Arrenhius equation for weight loss, and β is a positive quantity that modifies the energy barrier as a consequence of an applied stress in the kinetic strength model. Since $a_1 = E/R$ = 16,400 K, substituting R = 1.986 cal/mole/K we obtain E = 32,570 cal/mole. This value is close to the value—29,500 cal/mole—reported by Stamm (1964, Table 18-6) for coniferous wood. Values reviewed by Roberts (1970) ranged from 25,000 to 35,000 cal/mole for small specimens (sizes in the order of 10 mm). Schaffer (1973) calculated a value for ω_b of 0.784×10^6 s^{-1} for dry Douglas-fir versus our value of 1.8×10^7 s^{-1}. Roberts (1970) reported values between 6×10^7 to 7.5×10^8 s^{-1}.

Schaffer (1973) quoted a value for β of 2.87×10^{-7} $mm^2 \cdot N^{-1}$ for clear, dry Douglas-fir. Our value is 0.065 $mm^2 \cdot N^{-1}$ as given by a_4. Since this variable is a modifier of the energy barrier impeding molecular motion, it is not surprising that our value is substantially higher because the tested material contains defects and moisture. Defects and moisture should theoretically make it easier for the molecules to jump the barrier, since these are strength-reducing factors.

When the value of ψ of 1.0×10^{10} s^{-1} and the value of E/R of 16,400 K are entered into Eq. (10), then,

$$\left(\frac{df}{dt}\right)_T = -\psi \exp\left(-\frac{E_T}{RT}\right) = $$
$$= -1.0 \times 10^{10} \exp\left(-\frac{16{,}400}{T}\right) \quad (26)$$

This expression denotes the rate of degradation due to pyrolysis alone. Given f at $t = 0$ is approximately 0.1, as implied from the ratio of the mean short-term strength to the mean ideal strength, the time at which f becomes 0 is approximately 3.2 hours if T = 473 K (200°C). That is, all the bonds would be broken as a result of the pyrolysis at 3.2 hours if the whole element's temperature is maintained at 473 K. Similar calculation gives 27 s at T = 573 K (300°C), which is not unreasonable given that 573 K is the nominal charring temperature of wood. In reality, the time to failure will be shorter because of degradation due to stress or other strength-reducing factors.

Predictions—Constant-load data.—The accuracy of the model was evaluated against a sample of t_f results obtained under a separately conducted experiment. The specimens had been similarly sampled and were subjected to an exposure temperature of 250°C (523 K) and a constant load of 40 kN applied at all times. In predicting the t_f of this sample, a total of 500 t_f values were generated using the model, based on the experimental temperature and stress conditions. The input short-term

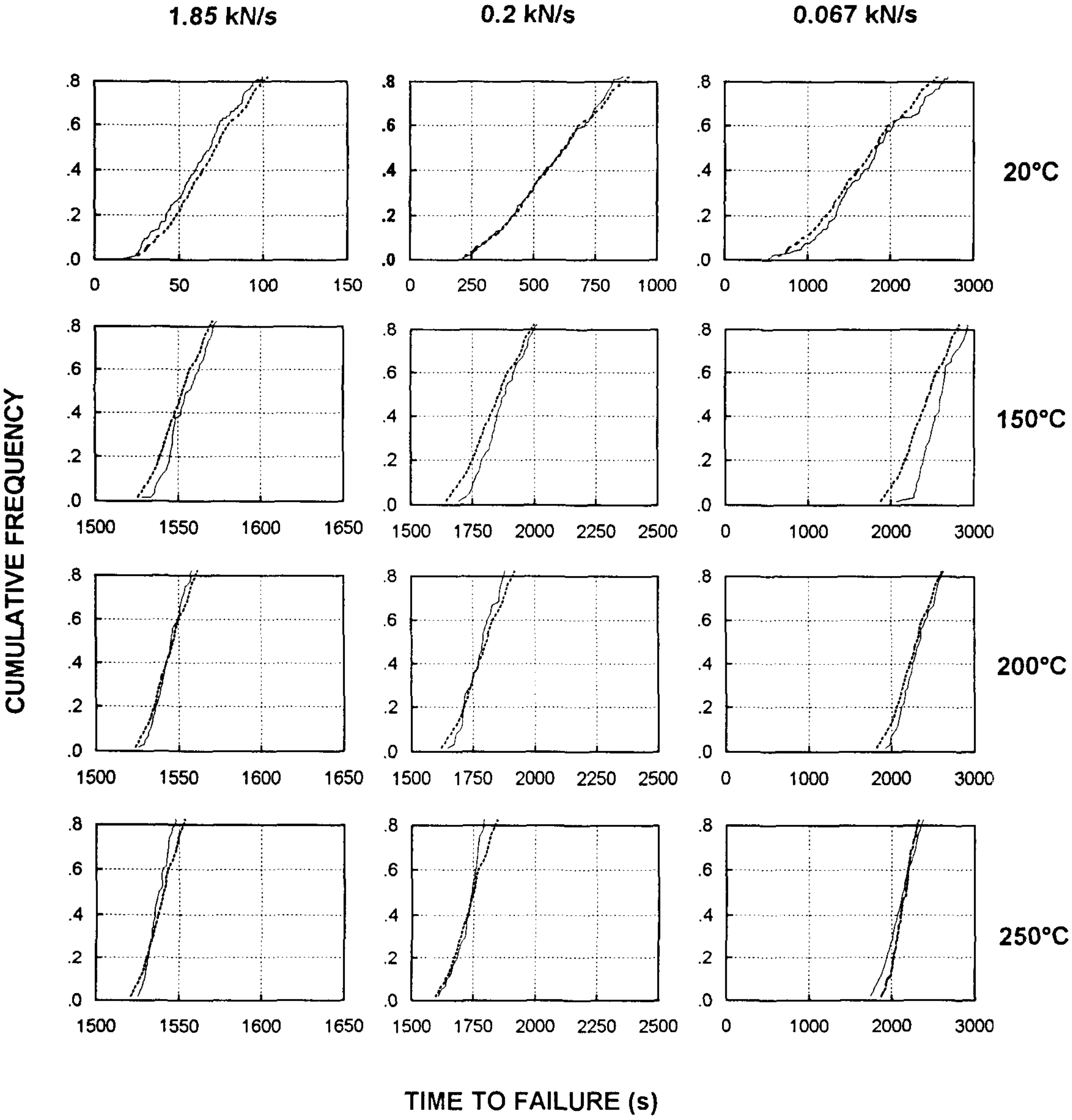

FIG. 5. Fitting of the model to experimental data (note: solid lines represent observed data and dotted lines represent predictions; rates of loading are given along the top margin and exposure temperatures along the right margin.

strength σ_0 were generated using Eq. (25). These predicted t_f values are plotted in Fig. 6 as a nonparametric cumulative distribution with the observed t_f. The agreement is reasonably good over the full range of σ_0 values evaluated. In general, the predicted values are conservative as compared to the observed values at small percentiles. The mid-range percentile values are predicted exceptionally well. The conditions of the test simulate the performance of a tension member stressed to nearly the 5th percentile characteristic value while the member is in contact with a hot surface at temperatures near charring. Such a situation is

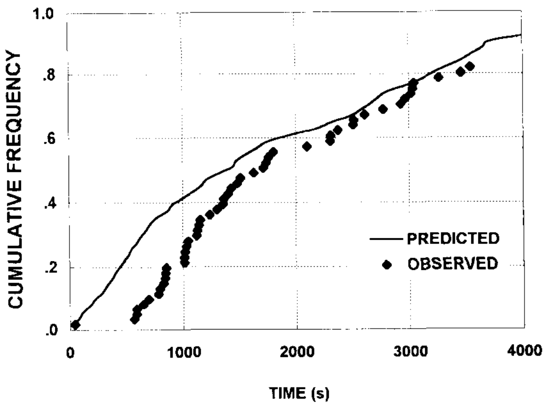

FIG. 6. Predicted time to failure of constant-load data.

probable in a concealed space, protected by wallboard with fire approaching from the outside.

TABLE 3. *Procedures to determine the reliability indexes of different grades of structural lumber.*

Step	Procedure
(1)	For each grade, five thousand σ_0 values are generated from the corresponding Weibull distribution. The stress history is $\tau(t)$ = 8.26 N/mm^2 for 2400f-2.0E and 4.56 N/mm^2 for 1650f-1.5E.
(2)	Each strength value σ_0 is entered into the model to calculate a t_f value. This process is repeated for all σ_0 values. The calculated t_f values are then sorted into an ascending order. From the ordered values the nonparametric cumulative frequency distribution is determined.
(3)	The probabilities of failure P_f at t = 60, 120, 600, 1,200, 1,800, 2,700 and 3,600 seconds are the cumulative frequency values at those durations.
(4)	The reliability index is calculated from the probability of failure: $B = \Phi^{-1}(1 - P_f)$.
(5)	Steps (2) through (4) are repeated for different constant temperatures C ranging from 423 to 523 K (150 to 250°C).
(6)	Steps (1) through (5) are repeated for another grade.

RELIABILITY ANALYSIS

The model was applied to calculate member reliability as affected by lumber grade. The sensitivities of member reliability to variation in both temperature and stress were also examined. The grade effect was analyzed based on properties published for MSR grades 2400f-2.0E and 1650f-1.5E. These two grades are used extensively in engineered wood systems. Generally, 2400f-2.0E is a premium grade comparable to Visual Grade *Select Structural*. 1650f-1.5E is a lower grade comparable to Visual Grade *No. 2*. The premium grade has a characteristic tension strength value approximately twice that of the latter.

Grade effect.—The complete analytical procedures are detailed in Table 3. Only the input short-term strength was treated as a random variable. The random strength value was generated using known Weibull distribution parameters for these two grades of lumber according to (Bury 1975):

$$x_i = \sigma\left[\ln\left(\frac{1}{1 - r_i}\right)\right]^{1/\lambda} \tag{27}$$

where r_i is a randomly generated number between 0 and 1, x_i is the corresponding randomly generated short-term strength value, and σ and λ are the Weibull parameters. The member stress was assumed to be constant at ⅓ of the 5th percentile of short-term strength distribution. The stress histories defined were $\tau(t)$ = 8.26 or 4.56 N/mm^2, respectively, for the two grades. The reliability index β, defined as $\beta = \Phi^{-1}(1 - P_f)$, where Φ is the standard *Normal* cumulative function, was calculated for a constant temperature history of $T(t) = C$ where C ranged from 423 to 523 K (150° to 250°C).

Figure 7 shows the reliability index β plotted against temperature for various durations between 60 s and 3,600 s. The significant decreasing trend of β with increasing temperature or duration is certainly expected. Also not surprisingly, MSR grade 1650f-1.5E shows a lower β value than 2400f-2.0E over the whole range of temperatures or durations evaluated. The differences, however, appear to be insignificant in regions of low probability of failure (β values > 1.0) but increase substantially in regions of high probabilities of failure (β values < 1.0). Since in design we generally aim for a low probability of failure, the differences in β between the two grades are therefore in-

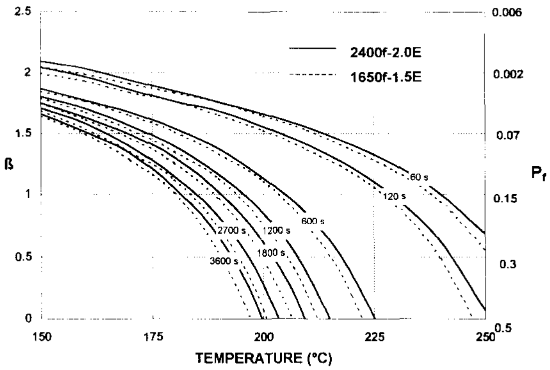

FIG. 7. Reliability indices of structural lumber as a function of material temperature and duration for the MSR grades of 2400f-2.0E and 1650f-1.5E.

significant. Such a spread of β values is not uncommon among similar structural designs according to current code specifications.

When the lines drawn in Fig. 7 are extrapolated to 20°C, they converge to a single β value because duration has no effect at or near room temperature. This value is approximately 2.5 based on calculations at 20°C. It should be noted that the value as calculated depends on the stress or stress history entered into the reliability analysis.

In general, reliability-based designs can make use of reliability plots such as Fig. 7. Supposing that Fig. 7 represents a reliability plot of a tension member exposed to a constant-temperature history and that the designer is required to ensure that the probability of failure of the member is no more than 5% ($\beta \approx 1.6$) at 2,700 s from the beginning of the heat exposure. From Fig. 7, the designer looks up the mean temperature corresponding to $\beta = 1.6$ on the reliability curve for duration = 2,700 s. This temperature is approximately 165°C. This temperature of the member, on average, then should not be exceeded over the duration of 2,700 s. The designer can achieve this either by specifying an appropriate protective membrane or some form of protective coating, or by requiring a larger member cross-sectional size. The above example should not be regarded as an example of typical reliability-based design of a wood member for fire safety, as this design approach has not yet been implemented in fire safety engineering. In actual reliability analyses, the stress history is determined by the charring rate, the dead and occupancy loads assumed to be initially on the members, and assumptions about how these loads degenerate over time during a fire. In general, reliability-based design needs a reliability plot expressing β as a function of time measured from the beginning of a fire. The damage accumulation model developed can be used to produce this plot given proper inputs for the temperature and charring history, and the loads on the member.

Temperature effect.—The sensitivity of β to temperature variations depends on the sum effect of many variables including species, density, permeability, and moisture content of wood, gain orientation with respect to the direction of heat-flow, exposure condition, and specimen dimensions. Since these variables are unlikely to be related to each other, their sum effect on temperature can be assumed to be a *Normal* distribution (Bury 1975). Randomly generated temperature histories were obtained from

$$T(t) = C_i \tag{28}$$

where

$$C_i = \mu\,(1 + \Phi^{-1}(r_i)\gamma) \tag{29}$$

μ and γ are the mean and coefficient of variation of temperature, r_i is a randomly generated number between 0 and 1, and $\Phi^{-1}(\cdot)$ is the inverse of the standard cumulative *Normal* distribution function. The value of γ was varied from 0.0 to 0.05 at an increment of 0.01. Given μ = 150°C and $\gamma = 0.03$, 95% of C_i would fall between 150 ± 9°C. Such a range of temperatures is not unexpected among different tests of same exposure.

The stress in the member was fixed at ⅓ of the 5th percentile of the grade's short-term strength distribution. The procedures used to generate the t_f distributions were similar to those listed in Table 3, except this time C_i and σ_0 were randomly generated variables. The results are shown in Fig. 8 in which the reli-

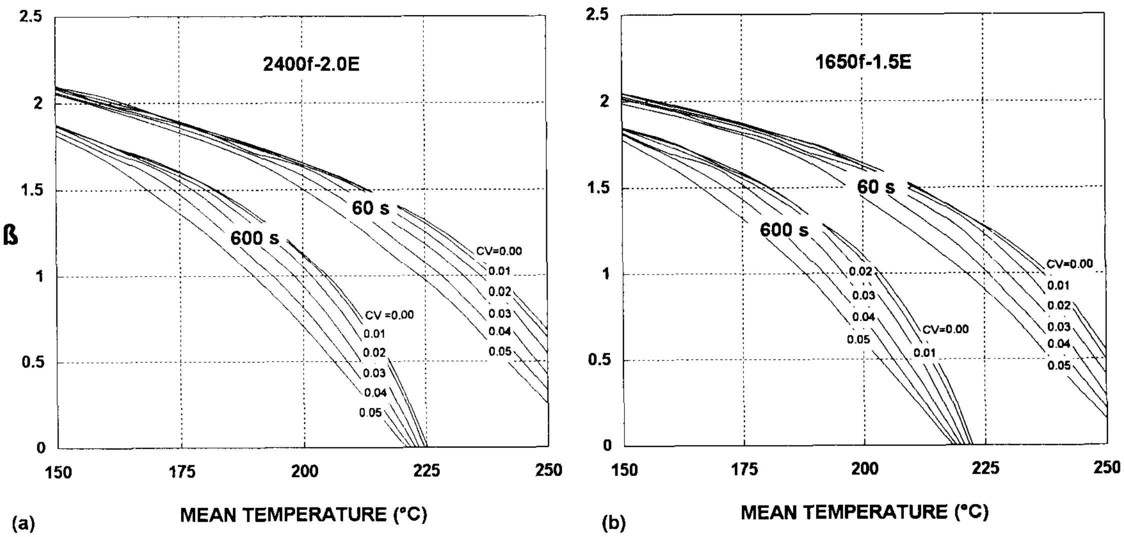

FIG. 8. Effect of temperature variability on reliability indexes of structural lumber (a) 2400f-2.0E and (b) 1650f-1.5E.

ability index β for the durations 60 s and 600 s are illustrated. The results are shown for different γ values and for each of the two grades: (a) 2400f-2.0E and (b) 1650f-1.5E. As indicated, the reliability index β is quite sensitive to temperature variations and higher variabilities in temperature lead to lower β values. This is not surprising since the effect of temperature on t_f is prominent. Also, the effect of temperature variations is unaffected by grade.

Stress effect.—The sensitivity of β to stress variation was analyzed assuming the distribution of this variable is also *Normal*. The analytical procedures are similar to those used for the analysis of the temperature effect. The randomly generated stress history was given by

$$\tau(t) = q_i \qquad (30)$$

where

$$q_i = \mu \,(1 + \Phi^{-1}(r_i)\,\gamma) \qquad (31)$$

μ and γ are the mean and coefficient of variation of stress, and r_i is a randomly generated number between 0 and 1. The mean stress μ in the member was also fixed at ⅓ of the 5th percentile of the grade's short-term strength distribution. The value of γ was varied from 0.0 to 0.15 at selected increments. Temperature was maintained as a constant parameter. The results, plotted in Fig. 9, show that the reliability index β is generally insensitive to variations in stress.

CONCLUSIONS AND RECOMMENDATIONS

The effect of exposure to temperatures up to 250°C and the load-history effects at these elevated temperatures can be modeled using a damage accumulation approach. The model should be able to explain the damage accumulation process of wood members under ramp-loading in that damage should accumulate insignificantly for a major portion of the loading history. A damage accumulation model based on a kinetic strength model coupled with a pyrolytic relationship—based on an Arrhenius function of temperature—was developed, and found to explain the experimental data reasonably well.

The reliability of lumber at temperatures up to 250°C appears not significantly affected by grade of lumber. The differences in the predicted reliability between two grades, calculated under the assumption that the load applied in each grade is constant and equal to ⅓ of the 5th percentile grade strength, are expected to be insignificant as far as current de-

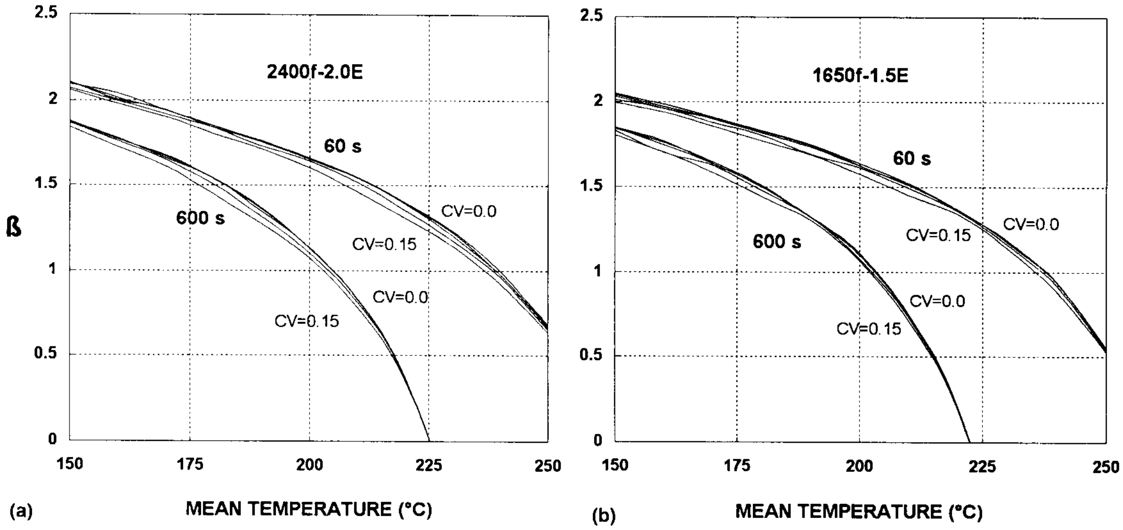

FIG. 9. Effect of stress variability on reliability indexes of structural lumber (a) 2400f-2.0E and (b) 1650f-1.5E.

sign practices are concerned. The reliability as calculated is sensitive to variability in temperature, but insensitive to variability in the constant stress. Since reliability calculations are sensitive to temperature, factors having a significant impact on temperature calculations should be carefully examined and accurately characterized. Models for prediction of heat transfer in wood-based building systems should be vigorously scrutinized as well.

The model may be improved: first, the model may be modified to treat materials having uneven temperature distribution as an assembly of elements with different local temperatures. Under this scheme each element can then be analyzed for the damage incurred, as a function of its temperature and stress history. The linear damage accumulation rule still applies regarding the criterion for failure of each individual element and of the whole assembly. With this approach, the model can account for the effect of thickness through differences in temperature distribution. Second, the model itself may be improved to better fit data up to 150°C, to account for the fact that at or below this temperature the degradation process is due mostly to dehydration or hydrolysis rather then to pyrolysis. Third, the model could be extended to consider compression behavior. When a member is axially loaded in compression and exposed to fire from one side, the load becomes eccentric with respect to the major axis of the member during fire. The "fire-side" fibers are in compression but also exposed to much higher temperatures. The compression effect is expected to dominate the ultimate load-carrying capacity.

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