# EVALUATING WESTERN HEMLOCK STEM CHARACTERISTICS IN TERMS <br> OF LUMBER VALUE 

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#### Abstract

Of those stem characteristics studied, taper had the greatest influence on value for trees of a particular diameter. This effect acts primarily through change in volume with change in height at a fixed diameter. A relatively small component of this value difference results from the geometric effect of taper on sawing recovery of logs with differing top and butt diameters. Consideration should be given to determining the heritability of this factor and the establishment of acceptable taper limits in plus-tree selection. The other stem characteristics of sweep, crook, and eccentricity have a statistically significant but relatively minor influence on tree value where value is based on green volume recovery. It has not been possible to assess the effect on product value of compression wood associated with these stem characteristics. Based on the volume recovery effects, only the most extreme conditions of sweep may have a sufficient negative effect on value to warrant consideration as a selection criterion. Natural variation in stem eccentricity and major-axis rotation does not have an important negative effect on tree value. Similar to sweep, significant reductions in stem value resulting from crook appear, at least under optimized bucking conditions, to occur only under the most severe crook conditions. These value reductions amount to about $1 \%$ at the extreme levels of crook. "Rule-of-thumb" bucking results in a general decrease in product value of about $3 \%$, but the differential with magnitude of crook is still about $1 \%$.


Keywords: Western hemlock (Thuja occidentalis (Raf.) Sarg.), stem characteristics, lumber value, taper, eccentricity, sweep, crook, SAWSIM.

INTRODUCTION/BACKGROUND
Tree-improvement programs, such as that administered by the Coastal Tree Improvement Council (CTIC) in British Columbia, are expected to make an important contribution to maximizing future forest productivity. If the criterion of productivity is to be based on maximum product value rather than cubic volume alone, it is necessary to determine the relative influence of various stem characteristics on product yield. Such information will permit the tree breeder to decide what tree characteristics to consider as potential criteria in the selection of parent trees for a breeding program.

In the western hemlock parent tree selection program of the B.C. Forest Service, a number of stem characteristics are included in the essential selection criteria. Specifically, the stem must be straight with no repeated crooks or double sweeps.

Slight lean or sweep in one direction is permitted if found elsewhere in the stand. The problem with applying these criteria is their lack of quantification. One logical basis for such quantification is the effect of such stem characteristics on product recovery or value. The question of heritability of stem characteristics must be answered, but knowledge of those stem characteristics that exert an important influence on product value should simplify the heritability investigation. In addition to the stem characteristics of sweep and crook, interest has been expressed also in quantifying the effects of stem eccentricity and taper on stem values, thus permitting the establishment of rejection criteria for current or future parent-tree candidates. In this study emphasis is placed on the devaluation effect of the selected stem characteristics from the obvious major effects of height and diameter.

A number of studies have shown that sweep in logs reduces lumber yields (Siimes 1962; Dobie 1964; Brown and Miller 1975; Karlsson and Sederholm 1978; Dobie and Middleton 1980). Likewise, the adverse effect of severe taper has been demonstrated (Dobie 1964) as well as its interaction with sweep (Brown and Miller 1975). Stem eccentricity has been recognized and quantified (Walters and Kozak 1964; Williamson 1975; Kärkkäinen 1975; Kellogg and Barber 1981), but the study of its effect on lumber recovery is very limited (Asikainen and Panhelainen 1970).

The purpose of this study was to evaluate the effect of variations in the selected stem characteristics on recoverable product value. To do this effectively, one requires data that cover the range and combination of factors under study in a reasonably complete manner; such data are, in fact, very difficult to obtain economically. Accordingly, it was decided to construct tree profiles with the combinations and levels of factors dictated by a well-defined design. These, although artificial, would be based on actual data. Thus, field measurements of sweep, crook, and stem eccentricity were made in second-growth stands of western hemlock in order to evaluate the expected variation in these characteristics for which insufficient information was available. Tree-length log descriptions were generated which represented selected levels of the stem characteristics over the range of variation in each as observed in natural stands. Product value was determined for these generated logs by means of a sawmill simulation program (SAWSIM) ${ }^{1}$ which incorporates an optimized bucking option.

## EXPERIMENTAL PROCEDURES

Materials
The field measurements of sweep, crook, height, diameter, and stem eccentricity were made on 87 second-growth western hemlock trees from four study areas within the Coastal Western Hemlock Zone as designated by Krajina (1959). Details on the study areas and general characteristics of the study trees have been described previously (Kellogg and Barber 1981).

The purpose of these measurements was to determine the degree of variation in these trees from the concept of linear, upright, tapering stems with circular cross sections. Two longitudinal profiles of each stem were measured with a theodolyte along perpendicular sight lines ( X and Y ). Measurements of displace-

[^0]ment of the stem from the vertical axis $(\mathbb{Z})$ passing through the center of the tree at stump height ( 30 cm ) were made at $2-\mathrm{m}$ intervals along the stem to a top diameter of 15 cm . Tree height also was determined with the theodolyte. Trees were then felled and stem eccentricity and major-axis rotation measurements were made as previously reported (Kellogg and Barber 1981).
Site-index curves and taper-prediction equations were available for western hemlock from the B.C. Ministry of Forests. Since the variability in stem height and diameter is dependent on site index, a medium site index of 100 was selected as representative of a high proportion of hemlock sites on Vancouver Island. Anticipated rotation age for these sites is approximately 70 yr. Average height for that site index and age is 28 m and might be expected to vary from 19 m to 37 m . Breast-height diameter (DBH) might be expected to vary from 20 to 60 cm and average 40 cm . For any combination of tree height and breast-height diameter, the taper equations available from the Ministry of Forests were used to generate appropriate height-diameter profiles.

On the basis of the earlier analysis of the eccentricity data (Kellogg and Barber 1981), the average eccentricity ratio for the two Vancouver Island sites (0.915) was selected as representative. The average standard deviation of the eccentricities for these sites is 0.0234 , and thus the average eccentricity ratio might be expected to vary from 0.87 to 0.96 .

The angular rotation of the major cross-sectional axis over a $5-\mathrm{m}$ stem length was positively related to average stem eccentricity according to the following linear relationship:

$$
\begin{equation*}
Y=-323.34+387.3 \mathrm{X} \tag{1}
\end{equation*}
$$

where $Y=$ angular rotation (degrees) $/ 5 \mathrm{~m}$ stem length and $\mathrm{X}=$ average stem eccentricity.
Since the variation about this line appeared reasonably homogeneous, the different levels of angular rotation of the major cross-sectional axis can be conveniently expressed as a series of parallel lines of rotation on eccentricity that covers a range of $\pm 2$ standard deviations from the mean line. The slopes of the lines are thus fixed at 387.3, and the intercepts range from -278.9 to 367.8 .
Sweep profiles were determined from the stem-displacement measurements. Stem-displacement values were determined at height intervals equivalent to $5 \%$ of the tree height. Values for the X and Y displacement were averaged separately for all trees over the 5 th percentile height intervals to arrive at an average $X$ and Y profile. The range of displacement data at each height interval was analyzed, and the extremes were arbitrarily taken as $\pm 2$ standard deviations from the mean. The averaging process used produced smooth sweep curves essentially devoid of crook. The resulting mean sweep profile and those for the sweep extremes are shown in Fig. 1 for the $X$ profile. The sweep curves for the $Y$ profile are virtually identical.

The analysis for crook was carried out on the original displacement data for each tree. Four crook sizes were defined that represented a deviation from linearity of $1,5,10$ and 20 cm at the center of a $5-\mathrm{m} \log$ length. For each of the four sizes of crook, a critical angle was determined that represented the angular change in direction of the line connecting successive points in the height-displacement profile necessary to create that size of crook in a $5-\mathrm{m}$ length. Analyses for each size of crook were run separately on both $X$ and $Y$ profiles of each tree within a site.


Fig. 1. Examples of sweep profiles used in describing tree length western hemlock logs.

The number and location of crooks of each size were determined. Summaries of this information were used to define low and high frequency levels of crook for each crook size. For $1-, 5$ and $10-\mathrm{cm}$ crooks, a low frequency level of crook for a site was chosen such that almost all trees had that number of crooks. A high level of crook was chosen such that a few ( 2 to 4 ) trees had that number of crooks. Few trees had more than two $20-\mathrm{cm}$ crooks. Low and high levels for this crook size differed only in the placement of the crooks. The low level positioned the two crooks at 20 and $45 \%$ of tree height, whereas the high level located them at 10 and $15 \%$ of tree height. For most tree heights, this placed both within the first $5-\mathrm{m}$ butt log length. Crook size and location information is shown in Table 1.

## Levels of the factors

The choice of the levels of the factors for investigating the relationship between a dependent variable, in this case product value, and several independent variables (taper, sweep, crook, and eccentricity) presents a number of problems. The greatest of these, as the number of independent variables increases, is the impossibility of experimentally studying all combinations of variables.

Table 1. Crook size, frequency, and location, used in the study.

| Crook <br> size, cm | Crook <br> level | Crook <br> level <br> desig- <br> nation | No. of <br> crooks <br> tree | Crook location, <br> $\%$ of height |
| :---: | :---: | :---: | :---: | :--- |
| 1 | Low | 1 | 6 | $10,20,35,45,50,60$ |
|  | High | 2 | 12 | $5,10,20,25,30,35,40,45$, |
| 5 | Low | 3 |  | $50,55,60,65$ |
|  | High | 4 | 2 | 20,45 |
| 10 | Low | 5 | 7 | $10,20,30,40,45,55,60$ |
|  | High | 6 | 2 | 20,45 |
| 20 | Low | 7 | 4 | $15,20,40,50$ |
|  | High | 8 | 2 | 20,45 |

The experimenter may select combinations of levels of the independent variables that will enable the approximation of a functional relationship for the response surface by fitting a Taylor series expansion up to third-order terms by the method of least squares and will have the property of rotatability. This property, first advanced by Box and Hunter (1957), is that the variances of estimates of the response made from the least-squares estimates of the Taylor series are constant on circles, spheres, or hyperspheres about the center of the design. Thus, a rotatable design can be rotated through any angle around its center and the variances of responses estimated from it would remain the same.
A standard rotatable design with four independent variables developed by Draper (1960) was selected for this study. This design consists of 96 points plus any number, $\mathrm{n}_{0}$, of center points. Thirteen levels of each variable are defined as follows: $0, \pm a_{1}, \pm \sqrt{2 a}, \pm 2 a_{1}, \pm a_{2}, \pm \sqrt{2 a^{2}}, \pm 2 a^{2} ;$ where $48\left(a_{1}{ }^{2}+a_{2}{ }^{2}\right) / N=1$. In our case, with a single center point, $\mathrm{N}=97$. In rotatable designs, this sort of scaling is customary for convenience. The scaled range of the design can be related to the actual range of interest for any variable by means of a linear transformation. Values for $a_{1}$ and $a_{2}$ are not unique so that they must be selected to provide the most appropriate distribution of the 13 data points over the range of the variable. In the absence of prior knowledge concerning the nature of the response surface, a value of $a_{1}=0.5$ was selected, which produced a relatively uniform distribution of data points over the variable range. Given that

$$
a_{2}=\left[2+\frac{n_{0}}{48}-a_{1}^{2}\right]^{1 / 2} \quad \text { and } \quad a_{1} \neq a_{2},
$$

it follows that when $\mathrm{n}_{0}=1, \mathrm{a}_{2}=1.3307$ and $2 \mathrm{a}_{2}$, the limit of the range, $=2.6614$. Accordingly, with a range of breast height diameters (DBH) of 20 to 60 cm , the following transformation would occur:

$$
\begin{array}{lll}
20 & \rightarrow & -2.6614 \\
40 & \rightarrow & 0 \\
60 & \rightarrow & +2.6614
\end{array}
$$

The design points for DBH (positive values) are then

| 0 | $\mathrm{a}_{1}$ | $\sqrt{2 \mathrm{a}_{1}}$ | $2 \mathrm{a}_{1}$ | $\mathrm{a}_{2}$ | $\sqrt{2 \mathrm{a}_{2}}$ | $2 \mathrm{a}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 40.00 | 43.69 | 45.22 | 47.39 | 50.00 | 54.14 | 60.00 |

The independent variables of height, breast-height diameter, sweep, and eccentricity were scaled in this manner. The 13 levels of sweep curves were derived by scaling the range of stem-displacement values at each fifth percentile height position.

Once the 13 levels of stem eccentricity ratio were determined, an average angular rotation of the cross-sectional axis could be calculated from the previously mentioned regression line. Indeed, while angular rotation of the cross-sectional axis could be linked to stem eccentricity, it also was possible to examine its separate effect as an independent variable by deriving 13 levels of regression lines expressing the variation in this property about the mean angular rotation-stem eccentricity relationship.
A computer program was developed that generated a three-dimensional description of the 97 whole-tree logs in the selected design.

## Computer runs to determine tree values

The SAWSIM program, developed by H. A. Leach and Company Ltd. of Vancouver, B.C., was used to determine the values of trees of various diameters, heights, eccentricities, and degrees of crook and sweep. To do this, the program bucks each tree into many alternative combinations of sawlog lengths, determines the total of the sawlog values for each combination, and selects the combination giving the highest total value. A dynamic programming technique is used to ensure that the highest valued combination is found without evaluating unnecessary alternatives. SAWSIM can also be used to generate single value solutions for specific bucking patterns. At the time these analyses were carried out, SAWSIM was available only in Imperial units of measure. For this reason, the descriptive material that follows will be expressed in those units.

Sawlog values are determined by simulating the sawing of the logs to lumber. The following is a summary of the assumptions that were made for this study:

Prior to sawing, logs were rotated about their axes to a "horns down" position, with the "belly" of the crook or sweep upwards. It was assumed that a 4-in., 6-in., 8 -in. or 10 -in. center cant was cut from the log, centered about a "least-squares" line through the centers of the $\log$ on a flat bed, i.e., split taper. As many $2-\mathrm{in}$. side boards were cut from either side of the cant as possible. The cant was then assumed to be turned so that the "horns" were against a straight edge and then sawn into $2-\mathrm{in}$. boards parallel to the straight edge. Three positions were tried for the opening face in the cant, at $0.625 \mathrm{in} ., 0.875 \mathrm{in}$. and 1.125 in . from the straight edge. The one giving maximum value was chosen. The side boards and pieces from the center cant were directed to a chipper edger, if necessary, for edging and/ or reduction of thickness to 1 in ., and all pieces were trimmed to shorter lumber lengths, if necessary, to meet No. 2 and Better wane specifications for boards wider than nominal 4 in , and Standard and Better for $2 \times 4$ boards. The board edger was assumed to have a splitter saw to allow two pieces of lumber to be developed at the same time from each piece. Although several small-log sawmill systems can carry out this sequence of processing events, the system envisioned was that of a quad-band or four-saw scrag mill as illustrated in Fig. 2.

The choice of center cant size depended on the small-end diameters of logs according to Table 2.

The value of each sawlog was determined by totalling the value of the lumber, chips and sawdust, planer shavings, and dry trimmings produced, less the cost of sawing the log. The sawing cost was determined by assuming a feed speed of 180 $\mathrm{ft} / \mathrm{min}$ plus a delay time of 3 sec for each $\log$, and charging for time at the rate of $\$ 1,600.00$ /hour plus $\$ 12 / \mathrm{MFBM}$. This rate includes an allowance for the capital and operating costs of the mill.

Lumber prices used in this study are shown in Table 3. These are relative prices derived from values reported in Random Lengths in 1979 and represent composite values for typical lumber grade proportions. These prices reflect the $\$ 12 / \mathrm{MFBM}$ reduction, which is part of the processing costs.

The lumber prices also reflect the demand for these items in current construction practice. The price of $14-20 \mathrm{ft}, 2 \times 8$ lumber was given the same premium as the $2 \times 4$ lumber of the same lengths on the assumption that these would be split in the planer. This means that a $14-20 \mathrm{ft} \log$ length was favored for logs of sufficient diameter to produce $2 \times 8$ or $2 \times 10$ lumber, whereas $22-\mathrm{ft}$ and $24-\mathrm{ft}$ lengths were favored for smaller-diameter logs.


Fig. 2. Diagram of sawmill system used by SAWSIM to process logs in this study.

Many of the optimum solutions resulted in bucking for a length of top log which left a remaining $\log$ length of less than 8 ft . This material would be chipped. Prices of residues were as follows: chips $\$ 50 /$ cunit; sawdust, planer shavings, and dry trimmings $\$ 12 /$ cunit; roundwood, short logs, and residual pieces $\$ 50 /$ cunit. All are based on the volume of solid wood in the log prior to chipping, planing, or sawing.

It should be noted that this analysis of product value is based only on volume recovery. The stem characteristics of eccentricity, sweep, and crook will undoubtedly be related to the amount of compression wood present in the lumber. Compression wood generally will result in drying degrade. The degree to which this will affect product value has not been quantified and therefore could not be considered in our analysis.

## Form of analyses

Four separate analyses were carried out in this study.
Analysis 1 considered the effect on tree value of variations in taper, stem eccentricity, and sweep. Since taper is a function of height and breast-height

Table 2. Choice of center cant size (in.) selected depending on log small-end diameter and length.

| Diameter <br> range <br> (in) | Nominal log length (ft) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |  |
| $\leqq 7.0$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |  |
| $7.0 \leqq 8.0$ | 4 | - | 6 | 4 | 4 | 6 | 6 | 6 | 6 |  |
| $>8.0 \leqq 9.0$ | 4 | - | 6 | 4 | 4 | 4 | 4 | 6 | 6 |  |
| $>9.0 \leqq 10.0$ | 6 | - | 6 | 6 | 6 | 6 | 6 | 6 | 6 |  |
|  |  |  |  |  | or 8 |  |  |  |  |  |
| $>10.0 \leqq 11.0$ | 6 | - | 6 | - | 6 | 6 | 6 | 6 | 6 |  |
| $>11.0$ | - | - | 10 | 10 | 10 | 10 | 10 | 10 | 10 |  |

Table 3. Lumber prices, $\$ / M F B M$.

| Lumber <br> size | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 8 | Lumber length ( ft$)$ |  |  |  |  |  |  |  |  |
|  | 108 | 113 | 128 | 118 | 138 | 138 | 138 | 118 | 120 |  |
| $1 \times 6$ | 108 | 113 | 128 | 118 | 138 | 138 | 138 | 118 | 120 |  |
| $2 \times 4$ | 213 | 222 | 236 | 245 | 276 | 267 | 267 | 231 | 236 |  |
| $2 \times 6$ | 213 | 200 | 258 | 245 | 276 | 276 | 276 | 290 | 294 |  |
| $2 \times 8$ | 166 | 166 | 251 | 245 | 276 | 267 | 267 | 270 | 274 |  |
| $2 \times 10$ | 173 | 173 | 295 | 300 | 295 | 295 | 295 | 286 | 290 |  |

diameter ( DBH ), the four independent variables were height, diameter, eccentricity, and sweep. Ninety-seven tree-length logs were generated for this analysis. Each exhibited the selected combinations of 13 levels of the four independent variables according to the design. In this analysis, major-axis rotation was held at the mean values corresponding to the selected levels of eccentricity.

Analysis 2 considered the effect on tree value of taper, stem eccentricity, and rotation of the major cross-sectional axis. Thus, the four independent variables were height, DBH, eccentricity, and major-axis rotation. In all trees, the sweep profiles were held at the mean values, and no crooks were introduced.

Analysis 3 differed from the previous design. In this case the intent was to hold diameter and eccentricity fixed at their mean values and use a reduced design to examine the effects on tree value of variation in height and sweep at each of the eight combinations of size and frequency of crook. The full design contained 17 combinations of height and sweep at the mean values of height and eccentricity. It is these combinations that form the reduced design. All deviations in stem profile due to crook were confined to one of the two profiles. A total of $(8 \times 17)$ 136 tree-length logs were generated with the desired profiles.

Analysis 4 was carried out to determine the effect that optimized bucking had on reducing the negative effects on total tree value of certain stem characteristics. That is, the analysis was designed to determine whether, with the normal application of "rule-of-thumb" log bucking at the sawmill, the negative effects of taper, sweep, and crook would be more pronounced.
"Rule-of-thumb" bucking solutions were determined for the 136 trees analyzed with optimized bucking in Analysis 3. To do this, tree-length log profiles were computer-plotted to provide a visual impression of the tree form. In addition, it was recognized that, in order to obtain a valid comparison, the manual bucking requires that the sawyer have an assumption of sawlog values similar to that used by the SAWSIM analysis. In order to impose this assumption on the selection of bucking solutions, the results of the optimized bucking in Analysis 3 were examined to establish the frequency of production of logs relative to their smallend diameter and length. This analysis developed what might be considered equivalent to the sawyer's "experience."

On the basis of this "experience," the following "rule-of-thumb" bucking policy was developed: top all trees at 6-in. diameter inside bark (DIB). Buck 12-20 ft nominal lengths starting at the large end, until DIB is about 11 in . Then cut 24ft lengths as possible until the diameter is 7.0 in . Below 7.0 in . cut $16-20 \mathrm{ft}$ lengths where possible. Override this policy as necessary to buck out severe crooks.

Bucking patterns for all trees were determined by applying this policy to the information available on diameter profiles and the visual impression created by the plotted tree length log profiles.

The resulting logs from the 136 trees were then processed through SAWSIM with a single-value solution obtained for each tree based on the bucking pattern determined by applying the policy described above.

## RESULTS AND DISCUSSION

Analysis 1
In this analysis, the effects on tree value of taper, stem eccentricity, and sweep were examined. Rotation of the major cross-sectional axis was held at the mean value associated with the selected level of stem eccentricity. The four independent variables were thus height (H), breast height diameter (DBH), eccentricity (E), and sweep (S).

A general third-order polynomial was fitted to the total values. The sum of squares carried by the parameters and the transformed equation are available from the authors. The following equation describes the response surface with the variables expressed in terms of the original units. All terms used in the transformed equation are statistically significant. Deviations between computed (actual) and fitted values range from $-\$ 3.62$ to $+\$ 2.71$.

$$
\begin{align*}
\mathrm{Y}= & -561.815-2.907 \mathrm{H}-4.127 \mathrm{DBH}+1357.952 \mathrm{E} \\
& -0.0184 \mathrm{~S}+0.0623(\mathrm{DBH})^{2}-728.836 \mathrm{E}^{2} \\
& -0.00210 \mathrm{~S}^{2}+0.107 \mathrm{H}(\mathrm{DBH})+0.00304 \mathrm{HS} \\
& -0.000384(\mathrm{DBH})^{3}+0.00000547 \mathrm{~S}^{2}+0.00218 \mathrm{H}(\mathrm{DBH})^{2} \tag{2}
\end{align*}
$$

where:

$$
\begin{aligned}
\mathrm{Y} & =\text { total product value, } \$ \\
\mathrm{H} & =\text { tree height } \mathrm{m} \\
\text { DBH } & =\text { breast height diameter (inside bark), } \mathrm{cm} \\
\mathrm{E} & =\text { eccentricity ratio } \\
\mathrm{S} & =\text { sweep, deviation of stem center from vertical at } 50 \% \text { height, } \mathrm{cm} .
\end{aligned}
$$

Figure 3 illustrates curves generated from this equation relating total value to DBH. The curves represent the relationship for trees of three different heights and with average and maximum sweep profiles. Comparing trees of an average sweep profile at an average DBH of 40 cm , the total product value of a tree of average height ( 28 m ) is $\$ 120.64$. At the upper limit of the height range ( 37 m ), the value rises to $\$ 166.74$, whereas at the lower limit ( 19 m ) it decreases to $\$ 74.54$. This value differential with taper increases with increasing diameter. Clearly, the taper factor has an economically, as well as statistically, significant effect on product value. The effect of taper on tree volume is not separated from the strictly geometric consideration involved in sawing logs with differing top and butt diameters. An attempt was made to separate these effects by analyzing thirteen trees in which height and diameter varied, but eccentricity and sweep were held at the mean levels. Taper of each tree was expressed as the ratio of DBH (inside bark)


Fig. 3. Relationship between total tree value and diameter (DBH) for trees of three heights and average, and extreme sweep profiles.
in cm to the total tree height in m . Tree value per $\mathrm{m}^{3}$ of merchantable volume is plotted against taper for these trees in Fig. 4. A polynomial of the form

$$
V=a+b_{1} D+c_{1} D^{2}+b_{2} T
$$

where:
$\mathrm{a}, \mathrm{b}_{1}, \mathrm{c}_{1}, \mathrm{~b}_{2}$ are coefficients
$\mathrm{D}=\mathrm{DBH}$ (inside bark), cm
$\mathrm{T}=$ taper ratio
was fitted to this data. The following equation:

$$
\begin{equation*}
\mathrm{V}=3.289+3.8334 \mathrm{D}-0.03174 \mathrm{D}^{2}-13.168 \mathrm{~T} \tag{3}
\end{equation*}
$$

was derived. As can be seen in Fig. 4, this expression fits the data points fairly well. The average absolute deviation is about $\$ 1.40$, with a coefficient of determination ( $\mathrm{R}^{2}$ ) of 0.942 . For a $40-\mathrm{cm}$ diameter tree, the merchantable volume varies from $0.949 \mathrm{~m}^{3}$ at tree height of 19 m to $1.841 \mathrm{~m}^{3}$ at a height of 37 m . While the total tree value differs by $\$ 92.20$, the value per unit of merchantable volume changes by only $\$ 10.91$. For a tree of average diameter $(40 \mathrm{~cm})$, the tree of greatest expected height ( 37 m ) is worth $\$ 47.99(39.8 \%)$ more than a tree of average height $(28 \mathrm{~m})$, but the component of this increase, due to the geometric effect on sawing recovery, is only $\$ 4.47(3.7 \%)$. On the basis of the limited data points for other tree diameters, it appears that the effect of taper on value per unit volume increases slightly as diameter decreases.

The differential in the value resulting from a comparison of trees of average diameter and maximum values of sweep is less marked than the total effect of taper, but still notable. For trees of average height ( 28 m ), the tree with maximum sweep is worth $\$ 11.09(9.2 \%)$ less than that of the tree with average sweep.

## Analysis 2

In this analysis, the effects on tree value of taper, stem eccentricity, and rotation of the major cross-sectional axis were examined. Sweep profiles were held at their


FIG. 4. Relationship between tree value per unit of merchantable volume and taper for different tree diameters.
average values. The four independent variables thus were height $(\mathrm{H})$, breast-height diameter (DBH), eccentricity (E), and major-axis rotation (R).

Once again, a general third-order polynomial was fitted to the total values. The sum of squares carried by the parameters and the transformed equation are available from the authors. The following equation describes the response surface with the variables expressed in terms of their original units. All terms are statistically significant. Deviations between computed (actual) and fitted values range from $-\$ 2.60$ to $+\$ 2.06$.

$$
\begin{align*}
\mathrm{Y}= & 188.759-21.755 \mathrm{H}-4.309 \mathrm{DBH}-421.290 \mathrm{E} \\
& +3.362 \mathrm{R}+0.078(\mathrm{DBH})^{2}+314.522 \mathrm{E}^{2}+0.0052 \mathrm{R}^{2} \\
& +0.0866 \mathrm{H}(\mathrm{DBH})+0.00234 \mathrm{H}(\mathrm{DBH})^{2}-0.000186 \mathrm{R}^{2} \mathrm{H} \\
& -27.968 \mathrm{E}^{2} \mathrm{H}-0.121 \mathrm{HR}+0.144 \mathrm{REH}+46.698 \mathrm{EH} \\
& -4.0280 \mathrm{RE}-0.00056(\mathrm{DBH})^{3} \tag{4}
\end{align*}
$$

where:
$\mathrm{Y}=$ total product value, $\$$
$\mathrm{H}=$ tree height, m
DBH $=$ breast height diameter (inside bark), cm
$\mathrm{E}=$ eccentricity ratio
$\mathrm{R}=$ major axis rotation, degrees $/ 5 \mathrm{~m}$ log length.

Table 4. Ratio of tree values with and without crook using optimized bucking.

| Height ${ }^{1}$ sweep nations | Crook level designation ${ }^{2}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Average |
| AG | 1.0000 | 0.9947 | 0.9974 | 1.0034 | 0.9925 | 0.9825 | 0.9921 | 0.9602 | 0.9904 |
| BB | 1.0010 | 1.0000 | 1.0075 | 1.0026 | 0.9936 | 1.0075 | 0.9808 | 0.9939 | 0.9984 |
| BL | 1.0060 | 1.0060 | 1.0133 | 1.0084 | 1.0022 | 0.9980 | 0.9946 | 0.9909 | 1.0024 |
| DG | 0.9998 | 0.9998 | 0.9922 | 0.9940 | 1.0051 | 0.9995 | 0.9962 | 0.9757 | 0.9953 |
| EE | 1.0028 | 1.0039 | 0.9944 | 0.9986 | 1.0053 | 0.9920 | 0.9921 | 1.0024 | 0.9989 |
| EI | 0.9938 | 0.9994 | 0.9994 | 0.9748 | 0.9858 | 0.9774 | 0.9923 | 1.0022 | 0.9906 |
| GA | 0.9936 | 0.9962 | 0.9943 | 0.9944 | 0.9836 | 0.9778 | 0.9951 | 0.9958 | 0.9914 |
| GD | 1.0014 | 0.9934 | 1.0021 | 0.9946 | 1.0020 | 1.0065 | 1.0017 | 1.0008 | 1.0003 |
| GG | 1.0004 | 1.0001 | 1.0023 | 0.9980 | 1.0025 | 0.9991 | 0.9997 | 0.9916 | 0.9992 |
| GJ | 0.9992 | 0.9975 | 0.9932 | 0.9963 | 0.9913 | 0.9906 | 0.9904 | 0.9936 | 0.9940 |
| GM | 0.9999 | 0.9905 | 1.0057 | 1.0082 | 1.0075 | 1.0031 | 0.9965 | 1.0035 | 1.0019 |
| IE | 1.0006 | 0.9907 | 0.9953 | 0.9887 | 0.9986 | 0.9944 | 0.9934 | 0.9691 | 0.9914 |
| II | 0.9976 | 0.9970 | 0.9928 | 0.9943 | 0.9950 | 1.0022 | 0.9895 | 0.9926 | 0.9951 |
| JG | 0.9981 | 0.9981 | 0.9872 | 0.9931 | 0.9924 | 0.9876 | 0.9819 | 1.0017 | 0.9925 |
| LB | 1.0018 | 0.9993 | 0.9979 | 0.9982 | 0.9895 | 1.0006 | 0.9982 | 0.9933 | 0.9974 |
| LL | 1.0075 | 0.9950 | 1.0124 | 1.0053 | 0.9915 | 0.9920 | 0.9984 | 0.9884 | 0.9988 |
| MG | $\underline{0.9999}$ | $\underline{1.0017}$ | $\underline{0.9880}$ | $\underline{0.9891}$ | 0.9955 | 0.9884 | $\underline{0.9828}$ | 0.9794 | 0.9906 |
| Avg. | 1.0002 | 0.9978 | 0.9986 | 0.9966 | 0.9961 | 0.9941 | 0.9927 | 0.9903 | 0.9958 |

${ }^{1}$ The 13 levels of height and sweep are designated by letters $A-M$. A and $M$ represent minimum, and maximum levels, while $G$ is the mean level.
${ }^{2}$ Crook levels are defined in Table 1.

The additional information available from this analysis is the joint effect of stem eccentricity and rotation of the major cross-sectional axis. The effect of eccentricity can be assessed by differentiating Eq. (4) relative to E. Mathematically, this is easier to deal with in the transformed form and results in the following expression:

$$
\frac{\mathrm{dY}}{\mathrm{dE}}=0.495-0.266 \mathrm{E}
$$

therefore,

$$
\frac{\mathrm{dY}}{\mathrm{dE}}>0 \text { if } 0.495>0.266 \mathrm{E}
$$

i.e., if $\mathrm{E}<0.495 / 0.266=1.861$

For any height and diameter combination, the total product value is maximal for a transformed eccentricity ratio of 1.861 , which is equivalent to an actual eccentricity ratio of 0.946 . It would appear that the out-of-roundness is beneficial up to a point, beyond which it may be limited by the associated rotation of the major cross-sectional axis. This effect may be more easily visualized if one considers a hypothetical log with a rectangular cross-section. The advantage of this cross-sectional shape, relative to lumber yield, would obviously be lost if the ends of a rectangular log were angularly displaced from each other. The same type of effect may be taking place in our results.

For a tree of average height, diameter and sweep, the product value recovered is $\$ 122.16$ at the eccentricity ratio of 0.946 . This value decreases to $\$ 122.07$ at the upper eccentricity ratio limit of 0.96 and $\$ 119.39$ at the lower limit of 0.87 .


Fig. 5. Crook/No Crook value ratio for various degrees of crook under both optimized and "rule-of-thumb" bucking conditions.

Analysis 3
The effect of crook was evaluated by a comparison of the value of trees with crook and the same tree without crook. Table 4 lists the ratio of these two values for the 17 selected combinations of height and sweep and the 8 crook levels. Within any one height-sweep combination, there is no clear pattern of the effect of crook, but overall, the pattern is one of decreasing value with increasing crook magnitude. For a given crook magnitude, there is a slightly lower value for the greater crook frequency condition. The upper curve in Fig. 5 shows the decrease in value ratio with increasing magnitude and frequency of crook. However, even in the most extreme condition considered, the average loss, under optimized bucking conditions, is only about $1 \%$.

## Analysis 4

A comparison of the effect of crook on product value under "rule-of-thumb" bucking with that under optimized bucking conditions is the objective of this analysis. The ratios of total product values for trees with and without crook under "rule-of-thumb" bucking are shown in Table 5. Overall there is a reduction in value of about $3 \%$. There appears to be a downward trend in value ratio with increasing magnitude of crook similar to that found in Analysis 3, albeit somewhat more erratic (Fig. 5). The general reduction in value due to "rule-of-thumb" bucking appears to be independent of the level of crook, but not the taper-sweep combination. Here the reduction, averaged over crook levels, ranges from $1.25 \%$ for a tree of average taper and sweep profile to $5.52 \%$ for a tree with the most severe taper condition and an average sweep profile.

SUMMARY AND CONCLUSIONS
The stem characteristics of sweep, crook, taper, and eccentricity can not, of course, be studied in isolation from height and diameter which, through their effect on determining volume, clearly must be the major contributors to tree value. Emphasis has been placed on the devaluation effects on product volume recovery

Table 5. Ratio of tree values with and without crook using "rule of thumb" bucking for trees with crook.

| $\overline{\text { Height }{ }^{1}}$ sweep combinations | Crook level designation ${ }^{2}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Average |
| AG | 0.9510 | 0.9485 | 0.9560 | 0.9599 | 0.9390 | 0.9338 | 0.9312 | 0.9394 | 0.9448 |
| BB | 0.9689 | 0.9689 | 0.9597 | 0.9618 | 0.9696 | 0.9648 | 0.9627 | 0.9508 | 0.9634 |
| BL | 0.9791 | 0.9789 | 0.9878 | 0.9836 | 0.9643 | 0.9671 | 0.9654 | 0.9603 | 0.9733 |
| DG | 0.9706 | 0.9811 | 0.9667 | 0.9756 | 0.9790 | 0.9977 | 0.9750 | 0.9587 | 0.9756 |
| EE | 0.9605 | 0.9727 | 0.9599 | 0.9765 | 0.9599 | 0.9553 | 0.9507 | 0.9698 | 0.9632 |
| EI | 0.9757 | 0.9710 | 0.9624 | 0.9501 | 0.9439 | 0.9487 | 0.9405 | 0.9749 | 0.9584 |
| GA | 0.9873 | 0.9867 | 0.9826 | 0.9881 | 0.9589 | 0.9525 | 0.9530 | 0.9908 | 0.9750 |
| GD | 0.9681 | 0.9805 | 0.9884 | 0.9821 | 0.9896 | 0.9912 | 0.9912 | 9.9695 | 0.9826 |
| GG | 0.9917 | 0.9924 | 0.9937 | 0.9775 | 1.0012 | 0.9800 | 0.9845 | 0.9789 | 0.9875 |
| GJ | 0.9774 | 0.9776 | 0.9753 | 0.9753 | 0.9741 | 0.9770 | 0.9693 | 0.9544 | 0.9726 |
| GM | 0.9666 | 0.9762 | 0.9441 | 0.9614 | 0.9405 | 0.9747 | 0.9415 | 0.9713 | 0.9595 |
| IE | 0.9630 | 0.9686 | 0.9568 | 0.9528 | 0.9617 | 0.9476 | 0.9649 | 0.9369 | 0.9565 |
| II | 0.9872 | 0.9826 | 0.9775 | 0.9893 | 0.9764 | 0.9550 | 0.9560 | 0.9764 | 0.9750 |
| JG | 0.9685 | 0.9852 | 0.9669 | 0.9755 | 0.9709 | 0.9658 | 0.9620 | 0.9894 | 0.9730 |
| LB | 0.9643 | 0.9671 | 0.9678 | 0.9574 | 0.9657 | 0.9736 | 0.9652 | 0.9644 | 0.9657 |
| LL | 0.9662 | 0.9684 | 0.9691 | 0.9587 | 0.9670 | 0.9749 | 0.9664 | 0.9657 | 0.9670 |
| MG | 0.9558 | $\underline{0.9564}$ | $\underline{0.9505}$ | 0.9592 | $\underline{0.9581}$ | $\underline{0.9577}$ | 0.9535 | $\underline{0.9342}$ | $\underline{0.9534}$ |
| Avg. | 0.9707 | 0.9743 | 0.9685 | 0.9697 | 0.9659 | 0.9657 | 0.9609 | 0.9639 | 0.9714 |

${ }^{1}$ The 13 levels of height and sweep are designated by letters $A-M$. A and $M$ represent minimum, and maximum levels, while $G$ is the mean level.
${ }^{2}$ Crook levels are defined in Table 1.
of the selected characteristics rather than the obvious major contributions of height and diameter to value.

This study suggests that the stem characteristic of taper has the greatest influence on value for trees of a particular diameter. This effect acts primarily through change in volume with change in height at a fixed diameter. A relatively small component of this value difference results from the geometric effect of taper on sawing recovery of logs with differing top and butt diameters. Consideration should be given to determining the heritability of this factor and to the establishment of acceptable taper limits in plus-tree selection criteria. The other stem characteristics of sweep, crook, and eccentricity have a statistically significant, but relatively minor, influence on tree value. Only the most extreme conditions of sweep may have a sufficient negative effect on value to warrant consideration as a selection criterion. Natural variation in stem eccentricity and major-axis rotation does not have an important negative effect on tree value. Similar to sweep, significant reductions in stem value resulting from crook appear, at least under optimizedbucking conditions, to occur only under the most severe crook conditions. These value reductions amount to about $1 \%$ at the extreme levels of crook. "Rule-ofthumb" bucking results in a general decrease in product value of about $3 \%$, but the differential with magnitude of crook is still about $1 \%$.

If taper should prove to be a heritable trait, genetic gains in that characteristic have the potential of resulting in a considerable increase in the total product value. With a total hem-fir lumber value in excess of $\$ 500$ million for 1980 in coastal British Columbia, even a small percentage gain in value may be a justifiable goal of a tree-improvement program.

The results of this work should be valid for other coniferous species, provided that the variation in stem characteristics and the price structures used are similar to that exhibited by western hemlock in south coastal British Columbia.

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[^0]:    ${ }^{1}$ SAWSIM is a computer program that simulates log bucking and sawing. It was developed by H . A. Leach and Company Ltd., Vancouver, B.C.

