FAILURE MODELING: A BASIS FOR STRENGTH PREDICTION OF LUMBER

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ABSTRACT

Failure modeling of knot-containing wood members was investigated as a means to predict member tensile strength. A finite element/fracture mechanics model was developed to model the progressive fracture process observed during failure of wood members. The failure modeling process yields predicted tensile strengths for members that contain knots in the wide face. Predicted strengths compared favorably with tensile strength data measured in initial experimental tests. Predicted strengths are generated from basic engineering computation and are not derived or adjusted by any empirical factors. With further research and verification, the concepts presented hold promise for use in lumber grading and quality assurance.

Keywords: Strength prediction, lumber strength, failure modeling, mathematical models, fracture mechanics, knot effects.

INTRODUCTION

The development of new structural applications for wood and the more efficient use of wood in traditional applications require the ability to predict accurately the strength of members or components. Tensile and bending strength of structural lumber and glulam members is strongly influenced by the presence of growth characteristics such as knots, cross grain, checks, and shakes. The influence of growth characteristics has been difficult to assess because of the inherent complexity and variability of wood. As a result, lumber strength prediction methods are currently empirical and approximate.

The failure process for wood members under load and the influence of growth characteristics on this process are paramount issues in predicting lumber member strength. In most cases, knots are the most severe tensile strength reducing growth characteristic. It has been observed in tension tests of lumber that knots and their associated grain deviation caused the initiation of failure in over 80% of the tests (McGowan 1968). In addition to the large stress concentration caused by the presence of the knot, associated grain distortion also severely disrupts the stress pattern in the region surrounding the knot. Drying stresses frequently cause material separations in the form of checks and splits in the region of grain distortion. All of these factors combine to create stress conditions within a lumber member subject to load, which are difficult to assess. Furthermore, the stress conditions continually change as failure progresses throughout the member. Since ultimate

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tensile capacity often occurs at loads beyond those causing the formation of the first crack, the changing stress conditions will ultimately determine the member's tensile strength.

The investigation reported here is part of a larger study conducted at Colorado State University, which since the late 1970s has focussed on gaining an understanding and developing prediction procedures for the tensile strength behavior of lumber. The objective of this particular part of the larger effort was to develop an analytical method to quantify the strength-reducing effect of wide face knots with associated grain deviations in wood members.

An analytical approach was pursued in combination with experimental studies since determination of the complicated stress patterns by experimental means alone is extremely difficult. A finite element routine was developed that incorporates fracture considerations enabling the modeling of member strength behavior from initial load to ultimate failure. This failure modeling scheme enables the rational prediction of the tensile strength of knot-containing wood members. Empirical adjustments are not used in these predictions. Predicted strengths are compared to ultimate tensile strengths determined through experimental studies.

Finally, the strength prediction method has been reduced to an easy-to-use equation that accounts for knot size and knot position and that uses clear wood tensile strength as its basic strength parameter. Future applications of this model to lumber grading and quality assurance is clearly of significance in the development of structural uses for lumber and lumber-based products.

BACKGROUND

As early as 1923, methods were presented for sorting structural wood members into grades (Newlin and Johnson 1923). Each grade was based on the occurrence of growth characteristics. Wilson (1934) published a comprehensive guide for grading and prediction of working stresses in structural wood members. In this guide, Wilson proposed the "strength ratio" of a grade as a representation of the "remaining strength after making allowance for the maximum effect of the permitted knots, cross grain, shakes and the like" for green members.

The basic concept of the strength ratio is used today for visually graded lumber in North America, and the corresponding strength prediction method is outlined in ASTM D 245-81 (1984). Strength ratios are computed with simplified equations that assume that the strength reducing effect of a knot can be characterized by reducing the cross section of a member by the cross section occupied by the knot. A reduced section modulus or area reduction (depending on knot location) can be computed using the reduced cross section along with empirical adjustments to account for knot and member size. The simplified equations provide strength ratios for bending. Tensile strength ratios are taken as 55% of the corresponding bending strength ratios since tests have shown that the detrimental effect of knots is more severe in tension than in bending (Doyle and Markwardt 1967).

The current ASTM method accounts for knot size and location effects by rudimentary means. In using this concept, many complexities inherent to wood, such as the influence of knot-associated grain deviation, are ignored. Rather than a unified approach for wide face knots at any position, centerline knot strength ratios are computed based on an area reduction while edge knot strength ratios are computed based on a section modulus reduction. With the development of more structurally efficient and sophisticated lumber applications, a better understanding of wood failure and a more accurate means to predict lumber strength are essential. An accurate and unified analytical strength prediction procedure would be helpful in this regard.

Researchers have recognized the value of analytical procedures in assessing the strength-reducing effect of knots despite difficulties in modeling knot geometry, knot-associated grain deviation, and the lack of a verified biaxial failure theory for wood. In attempts to sidestep these obstacles, Pearson (1974) likened a knot to a transverse crack and employed linear elastic fracture mechanics theory to predict failure. Others (Green and Zerna 1968; Tang 1984) have computed stresses by modeling a knot as a hole or wood plug in a lumber member. Both approaches appear promising; however, by neglecting knot-associated grain deviation, these procedures cannot provide the basis for accurate strength prediction methods for a wide range of real cases.

Previous work by Dabholkar (1980) and the author (Cramer and Goodman 1983) included development of a finite element model that directly accounted for the knot size and location, and idealized knot-associated grain deviation. Through recognition of the similarity between fluid flow around an object and wood growth around a knot, a means to account for associated grain deviation was developed. Goodman and others (Goodman and Bodig 1980; Phillips et al. 1981) established that grain deviation around a knot could be reasonably predicted using the fluid mechanics equations describing fluid flow around an elliptical object. The "flow-grain analogy" resulted, which relates grain lines of wood around a knot to streamlines of laminar fluid flow around an elliptical object.

The first flow-grain finite element models provided stresses from which initial *local* failure could be predicted, given knowledge of the appropriate clear wood tensile strength. Unfortunately, the load corresponding to initial local failure as determined analytically was considerably below lumber strengths measured in experimental tests. Experimental investigators (Pearson 1974; Anthony 1985) also observed and verified what was observed analytically: attainment of member strength often occurs at loads in excess of those associated with initial localized fracture. This suggests that ultimate strength is not determined and thus cannot be easily predicted by a single determination of stresses associated with first local failure. Failure in lumber tension members appears to be a multistep process characterized by progressive failure, and to predict lumber strength, this process must be defined and accounted for in strength-related computations. The research reported herein centered on the development of a verified model to account for a progressive failure process and thus enables accurate predictions of the tensile strength of wood members from basic strength parameters.

THE FAILURE MODEL

The developed theoretical model accounts for a multistep failure process by performing stepwise analyses. Each analysis is performed with a finite element/fracture mechanics algorithm embodied in Program STARW (*ST*rength *A*nalysis *R*outine for *W*ood) (Cramer 1984). Once this model was verified, a single prediction equation was developed for practical use as discussed later.

Program STARW models the tension behavior of the critical segment of a wood member containing a single wide face knot from initial load to ultimate failure.

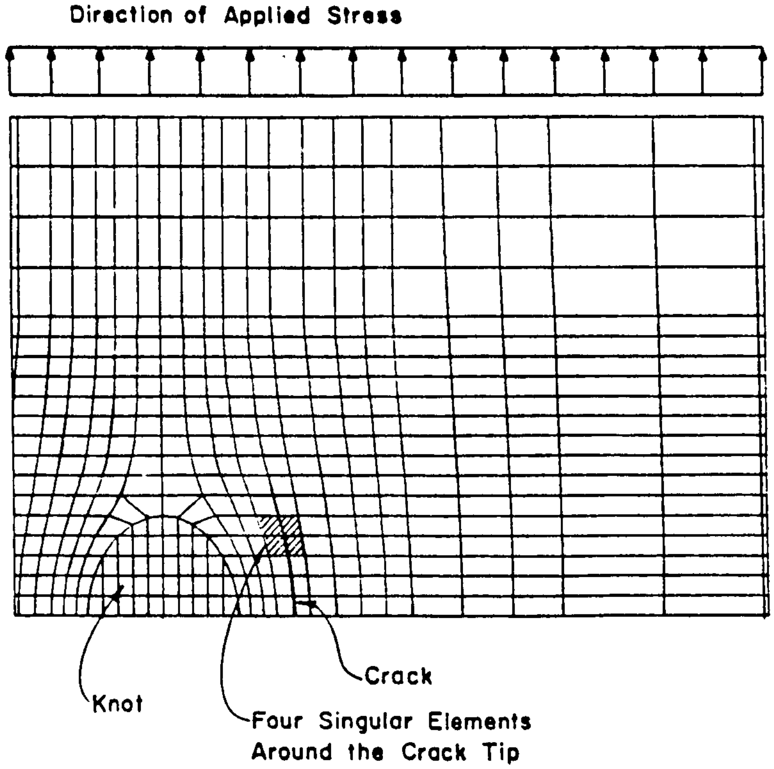

FIG. 1. Example finite element mesh including special crack tip elements.

Wide face knots of elliptical shape and most locations within the cross section can be modeled. While the study reported here has been limited to wide face knots, the method presented is capable of accounting for other growth characteristics such as general cross grain (Zandbergs 1985). With additional research, conditions such as juvenile wood and density variations could also be considered in the model.

Utilizing the flow grain analogy, a finite element mesh (refer to Fig. 1) is constructed to represent the longitudinal-tangential plane of a specimen with a knot. Cracks occurring as seasoning checks or as a result of applied loads can also be modeled. Each step of the failure process that is modeled requires the automated formation of a unique finite element representation (mesh) of the wood member that includes cracks at various locations. Each unique finite element mesh results in the solution of approximately 4,000 simultaneous equations, which yield horizontal and vertical displacements at each of 2,000 nodal points distributed throughout the mesh. Stress levels are computed from the displacements. Special computational techniques for handling the repeated large-scale analyses were developed (Cramer and Goodman 1984).

As a result of the distinct orthotropy of wood, it has been observed that cracks will form and propagate along the grain in nearly all angle-to-grain load situations (Leicester 1974; Pearson 1974). The effect of cracks are accounted for by "unzipping" the finite element mesh along the material separation coinciding with a

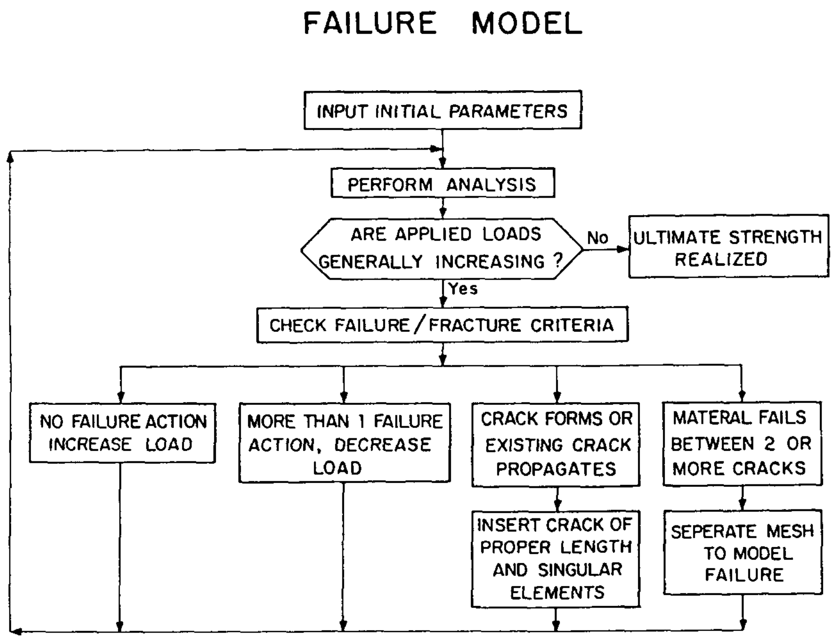

FIG. 2. Failure model for wood structural members.

flow-grain line and designating the elements surrounding the crack tip as fracture singular elements (Atluri et al. 1975). This process is done automatically upon cue form the user. A resulting model characterization of a knot-containing member is shown in Fig. 1. Symmetry is assumed about the horizontal knot centerline. Most knot sizes and locations found in structural lumber can be modeled by Program STARW. Cracks of any practical length and location along the grain can be considered. Multiple cracks can be modeled; however, crack propagation information is computed for only one crack tip per analysis.

Each analysis provides computed stresses for all elements (except the four elements immediately surrounding a crack tip) in the finite element mesh. In addition, stress intensity factors that indicate the severity of the stress field surrounding a crack tip are computed. The program searches these values and finds the most critical stress and fracture conditions. Since the computed stresses and stress intensity factors do not directly indicate failure, they must be evaluated with suitable failure criteria.

The failure theory currently employed consists of two parts: one part to evaluate element stresses and a second part to evaluate the stress intensity factors. The element stresses are evaluated with the maximum stress failure criteria, while the stress intensity factors are evaluated with Wu's fracture criterion (Wu 1967). It should be noted that the choice of a particular failure theory does not significantly alter the method of modeling failure.

The maximum stress failure theory states that fracture or local failure is assumed to occur if the computed finite element stress in either parallel-to-grain tension, perpendicular-to-grain tension, or in shear exceeds the corresponding clear wood strength determined from small clear specimen tests. Similarly, an existing crack

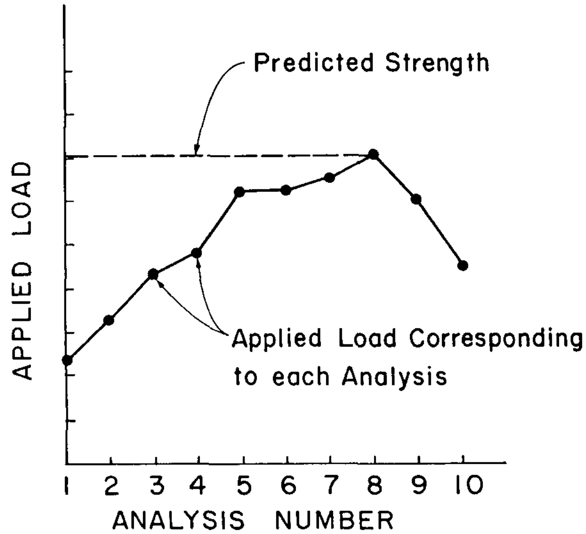

FIG. 3. Typical plot of load causing local failure vs. analysis.

is assumed to propagate if the computed stress intensity factors combine to satisfy Wu's fracture criterion (Wu 1967).

$$K_{I\text{-}TL}/K_{IC\text{-}TL} + (K_{II\text{-}TL}/K_{IIC\text{-}TL})^2 = 1.0 \tag{1}$$

where,

$K_{I\text{-}TL}$, $K_{II\text{-}TL}$ = stress intensity factors (indicators of stress surrounding the crack tip) in the TL plane, and

$K_{IC\text{-}TL}$, $K_{IIC\text{-}TL}$ = critical stress intensities (fracture toughness values) in the TL plane.

As stated earlier, member failure is considered to be a progressive process as the applied load increases. Localized failures, mainly in the form of cracks along the grain, are assumed to accumulate and interact to cause ultimate failure. This process is modeled via Program STARW by performing stepwise finite element analyses. A schematic diagram of the logic of the stepwise failure model is shown in Fig. 2. The stresses are interpreted with the failure and fracture theories as previously described. Damage, typically in the form of a crack, is inserted into the finite element mesh at the location where the stresses are interpreted as being most severe. The damage inserted acts to relieve local stress concentrations, and a subsequent analysis is performed to find the next most critical local stress condition. Analyses are performed repeatedly; each step models a situation that has more fracture than the preceding step. The load causing local failure is recorded, and the load sequence typically appears as shown in Fig. 3. Eventually, the idealized member will sustain damage such that a further increase in load cannot be achieved. At that point, the ultimate strength is reached, and the capacity of the idealized member is defined as the maximum attained load.

Use of the failure model in conjunction with experimental tests has provided a means to study proposed wood failure processes and qualitatively describe the

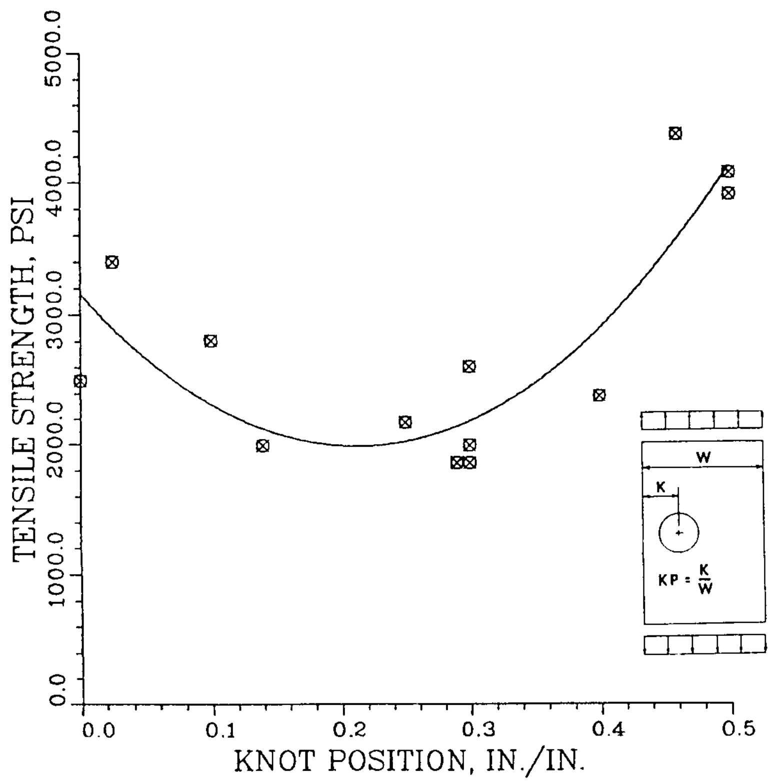

FIG. 4. Parabolic fit for tension test data for specimens with knots of similar size and variable position (KP) (Anthony 1985).

tensile behavior of knot-containing members. In a practical sense, however, such capabilities are of academic interest. A detailed discussion of knot-containing member behavior can be found in Cramer (1984). The developed failure modeling procedure most importantly provides a means to quantify lumber member strength for a variety of knot geometries and material properties. An accurate means to predict wood member strength by rational computation is of utmost practical interest.

STRENGTH PREDICTION AND EXPERIMENTAL VERIFICATION

The tensile strength of a wood member as influenced by knot size and position within the cross section was investigated through the use of the failure model. To demonstrate the capabilities of the model and to provide a limited verification of its capabilities, knot sizes, knot locations, and wood species were chosen to correspond to available experimental data. The experimental data were collected both in association with development of the failure model and from the literature.

The effect of knots on tensile strength was examined experimentally by Anthony (1985). Anthony tested a group of ¼- by 4- by 14-inch Douglas-fir specimens, each of which contained a single knot of various size and location. Of the 23 specimens tested, 13 possessed knots of approximately equal size with transverse

TABLE 1. *Properties used in the strength prediction verification.*

Property	Clear wood value	Knot wood value	Source
Elastic			
E_L	1,820,000 psi	—	Bodig and Goodman 1973
E_T	128,000 psi	—	Bodig and Goodman 1973
ν_{LT}	0.42	—	Bodig and Goodman 1973
G_{LT}	112,000 psi	—	Bodig and Goodman 1973
E_R	—	50,000 psi	Pugel 1980
E_T	—	50,000 psi	Pugel 1980
ν_{RT}	—	0.47	Pugel 1980
G_{RT}	—	38,000 psi	Pugel 1980
Strength			
$\sigma_{\parallel ult.}$	15,000 psi	—	Anthony 1985
$\sigma_{\perp ult.}$	400 psi	—	Anthony 1985
$\sigma_{shear\ ult.}$	1,390 psi	—	Anthony 1985
Fracture			
$K_{IC\text{-}TL}$	310 psi $\sqrt{in.}$	—	Petterson and Bodig 1983
$K_{IIC\text{-}TL}$	870 psi $\sqrt{in.}$	—	Anthony 1985

diameters (refers to horizontal knot diameter in Fig. 1) of 1.65 to 1.85 inches and a longitudinal diameter (refers to vertical knot diameter in Fig. 1) of approximately 2.0 inches. Knot diameters and knot location were determined by averaging the two faces of the ¼-inch-thick specimens.

The collected strengths are shown versus knot position (KP) in Fig. 4. Knot position was quantified as the minimum transverse distance (minimum horizontal distance in Fig. 4) from the knot center to either member edge, divided by the transverse width of the member. A parabolic relationship for tensile strength versus knot position was found to best represent the test data as shown in Fig. 4.

Six failure analyses were conducted with Program STARW for comparison with the parabolic representation of the test data collected by Anthony. Thirteen elastic and strength properties are required in the input of the model. The needed wood properties were determined from measurements or from relationships with specific gravity and moisture content. The properties as determined are listed in Table 1. The material properties listed and the median knot size were employed to predict the strength for specimens with six different knot positions varying from edge (KP = 0.0 in Fig. 5) to center knot (KP = 0.5). As indicated in Fig. 5, reasonable agreement exists between the parabolic representation of the test data and the predicted values. Overall, the STARW predictions averaged 8.5% less than the parabolic representation of the test strengths, and thus the strength prediction capability of Program STARW was initially shown to be reasonably accurate.

ASTM D 245 predicted strengths are shown in Fig. 5 for comparison. Computation of tensile strength ratios for off-centerline knots by the ASTM provisions is not entirely clear. Provision 4.1.1 of ASTM D 245 (ASTM 1984) states that all wide face knots are treated as either edge knots or centerline knots for simplicity. Provisions 5.3.4.8 and 5.3.5.5 state that linear interpolation of strength ratios between edge and centerline knots is permitted. However, each of these provisions begins with "Where the grade is used for single-span bending only." It is not clear

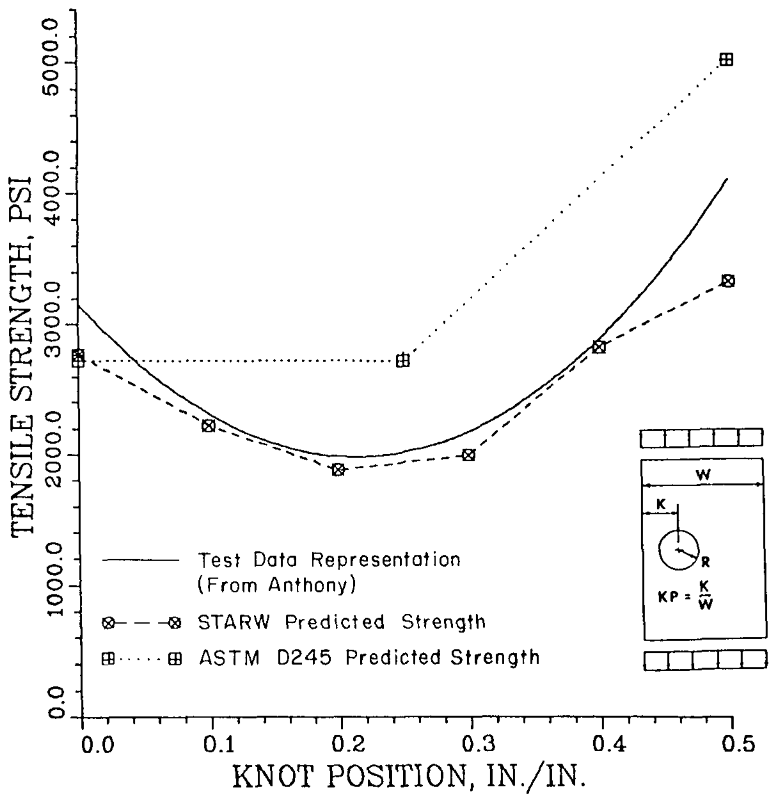

FIG. 5. Comparison of tensile strength obtained from tests (Anthony 1985) to predicted strengths as determined with Program STARW and ASTM D 245-81 (1984) for different knot positions (KP).

if these provisions apply to tension. One interpretation of the ASTM provisions for tension is given in Fig. 5.

Additional verification of the strength prediction capability of Program STARW was sought through examination of experimental data available in the literature. Data collected by Kunesh and Johnson (1972) were used to investigate the effect of knot size. They presented test strengths for nominal 2- by 8-inch Douglas-fir lumber approximately 8 feet in length, which contained center knots and edge knots. By their definitions, the two knot locations identified actually represented many knot locations since in their study an edge knot was considered to be any knot in the outer quarter of the member, and a center knot was any knot located in the middle half of the member. Unfortunately, Kunesh and Johnson (1972) did not measure any clear wood properties other than specific gravity and moisture content. This lack of measured data makes a rigorous comparison of predicted and measured strengths impossible. The mean specific gravity and moisture content recorded, however, coincided closely with the corresponding values from Anthony's data. As a result, the clear wood properties as determined by Anthony were assumed to be a reasonable estimation of the corresponding properties of the lumber tested by Kunesh and Johnson.

Six knot radius to member width ratios were studied for edge knot members

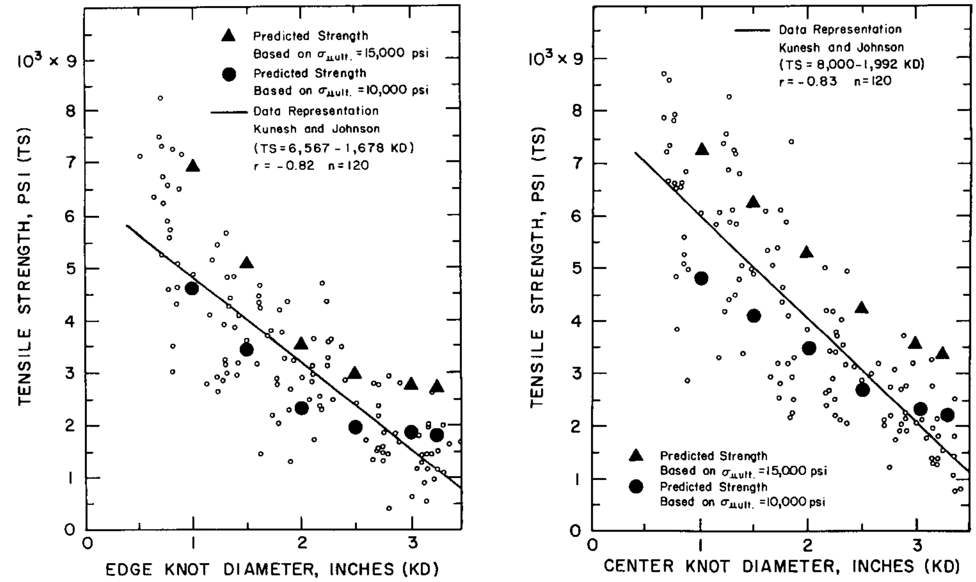

FIG. 6. Comparison of lumber tensile strengths measured from tests (Kunesh and Johnson 1972) to predicted strengths as determined with Program STARW.

and center knot members. Since Kunesh and Johnson did not measure clear wood strength properties, member strength predictions were computed for two possible values of clear wood tensile strength parallel-to-grain. This basic clear wood property appears to influence ultimate tensile strength more than any other clear wood property. Values of 10,000 psi and 15,000 psi were employed for tensile strength parallel to grain. The two values were chosen to attempt to bracket the range of likely clear wood strengths. The resulting range of predicted lumber strengths for edge knots is shown with the test results of Kunesh and Johnson in Fig. 6. The predicted strengths lie within the observed variation of the test data. The model does seem to predict a limiting minimum strength as knot size becomes large. This is possibly caused by simplifying assumptions concerning the condition of the knot, and further research with larger knot sizes is warranted. In general, the predicted values follow the trend of the test data.

In a similar manner, the effect of center knots of different sizes was explored using Program STARW. Again, a range of possible strengths was bracketed by considering predicted member strengths based on clear wood parallel to grain tensile strengths of 10,000 and 15,000 psi. Figure 6 shows the failure model predicted strengths superimposed upon the center knot member test results of Kunesh and Johnson. While firm conclusions cannot be drawn because of the uncertainty of the clear wood strength values for the lumber members tested, the predicted values follow the general trend of the test data.

SIMPLIFIED STRENGTH PREDICTION BASED ON FAILURE MODELING

The major goals of this study were realized with the initial verification of the strength prediction capability of Program STARW. The failure model involving intensive use of Program STARW, however, lacks the pragmatic, easy-to-use nature necessary to enable its intensive use. As a result, additional investigation was directed to the initial development of a single, general prediction equation.

Knot size, knot position, and clear wood parallel-to-grain tensile strength were identified as the parameters largely controlling the strength of knot-containing members. The effects of knot position and knot size as determined by the STARW model were characterized and combined in mathematical equations. To aid in this regard, all predicted strengths were reduced to strength ratios that are a function of knot position and knot size. (The strength ratios yield the predicted member tensile strength when multiplied by the clear wood tensile parallel-to-grain strength. Thus the strength ratio quantifies the strength-reducing effect of the knot.) Parabolic relationships were used to express the variation in strength for knot size and knot position. These parabolic equations were combined to form a single equation to predict the tensile strength of a Douglas-fir member containing a knot of any practical size and position. The proposed strength prediction equation is of the following form (Cramer 1984):

$$\begin{aligned} TS = (\sigma_{\|ult.}) \times [&(0.59 - 1.73(R/W)) \\ &- 0.60(0.5 - KP) + 1.17(0.5 - KP)^2] \\ &\times [1.0 + (1.0 - 2.0KP) \times (0.57 - 11.1(R/W) \\ &+ 36.0(R/W)^2)] \end{aligned} \tag{2}$$

where

TS = predicted tensile strength of the knot-containing member,
$\sigma_{\|ult.}$ = clear wood tensile strength parallel-to-grain,
R/W = ratio of knot radius to member width, $0.05 \le R/W \le 0.25$,
KP = knot position, defined as the minimum transverse distance from the knot centerline to the member edge divided by the member width, $0.0 \le KP \le 0.5$.

The equation for predicted strength ratio is shown graphically in Fig. 7 for six knot size R/W ratios. The parabolic trend observed for R/W = 0.22 and verified by Anthony's experiments has been generalized to the other knot sizes. The generated data points shown in Fig. 7 were used to derive the prediction equation.

Further development and verification of the prediction equation may be needed before it can be employed in practical application. However, the development of this equation demonstrates that the strength prediction capability of the complex failure modeling process can be capsulized in a relatively easy-to-use form. The proposed equation can be easily employed with a programmable hand-held calculator or incorporated into a microprocessor to enable others to make comparisons with the work presented herein.

SUMMARY AND CONCLUSIONS

The goal of this research was to formulate a rational method to quantify the strength-reducing effect of wide face knots and their associated grain deviation in wood members. The research goal was achieved through the development of a finite element/fracture mechanics-oriented model embodied in Program STARW. The progressive failure process observed in experimental tests of knot-containing

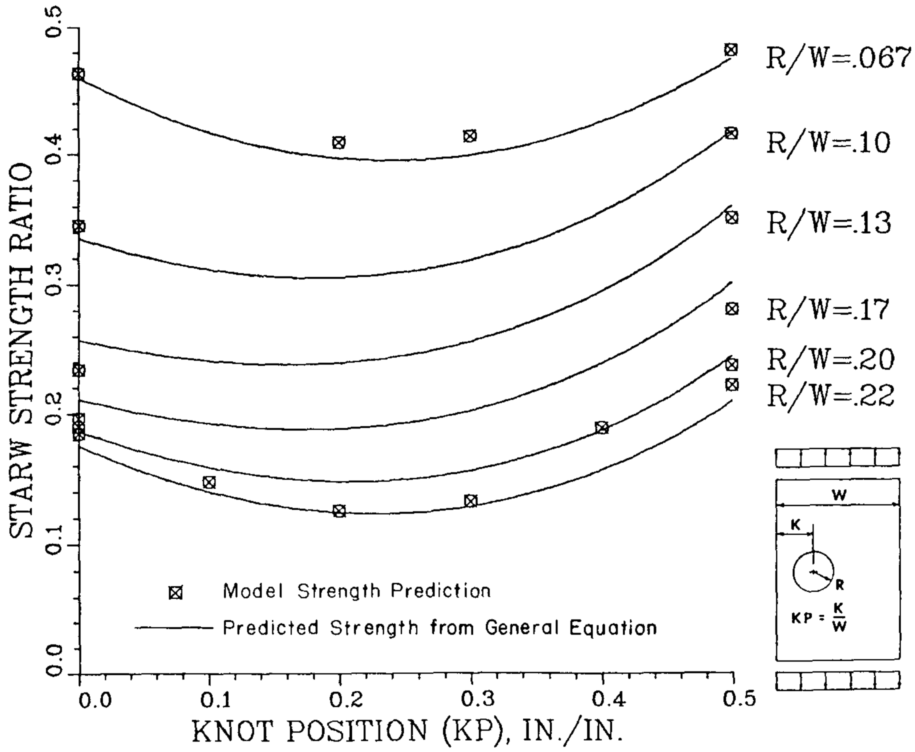

FIG. 7. Predicted STARW strength ratios and prediction equation curves for tension members containing knots of variable size and location.

members can be modeled with stepwise application of Program STARW. The failure modeling process enables the study of the behavior of wood members containing wide face knots from initial load to ultimate failure and, most importantly, yields a prediction of member strength.

Predicted strengths as determined by Program STARW have been compared and initially verified with experimentally determined strengths from Anthony (1985). Though only a limited set of strength data was collected, initial comparison with the predicted strengths proved favorable. Further verification was sought by comparing lumber tensile strength data collected by Kunesh and Johnson (1972) with Program STARW predicted strengths. Since needed material properties were not available with these data, only a qualitative comparison could be made. Despite the unknown properties, a reasonable comparison is shown between the predicted and measured strengths.

Further investigation was directed toward translation of the strength prediction capability of the complex finite element/fracture mechanics model to an easy-to-use general equation. A strength prediction equation was developed by identifying the main factors influencing strength and characterizing their effect from trends observed through use of the failure model. Others may use the developed equation to obtain the net results of the complex finite element/fracture mechanics model (within the equation limitations) in a quick and easy way.

Failure modeling, as performed with the STARW model and represented in the developed prediction equation, provides a sound technical basis for strength prediction of knot-containing wood members. Predictions can be made with knowledge of member geometry and properties. Empirical adjustments that are usually tied to a particular data set are not used in this process and are not needed to obtain reasonable predictions of member strength.

Before widespread application of the concepts presented herein can occur, further research that incorporates other types of growth characteristics and extensive verification with commercially available material are needed. With more research and verification, the strength prediction capability developed and initially verified can be applied in automated and improved grading methods. Such improvements might consist of incorporating the strength prediction equation into a microprocessor-based grading procedure that could include optical scanners and other nondestructive evaluations of wood properties. Potential applications of the strength prediction capability also exist in the areas of quality control of wood components and in improved design/analysis of the many structures which utilize knot-containing wood.

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