NONLINEAR RACKING ANALYSIS OF NAILED WALLS

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ABSTRACT

An energy formation of nonlinear analysis of racking resistance of nailed walls is presented. The analysis is dependent on the nonlinear load-slip relationship of nailed connections. The method permits the calculation of the racking force associated with a given horizontal displacement and uses as input data panel geometry, number and spacing of nails, and load distortion relationship of a single connection.

Two 4 ft × 8 ft panels with two thicknesses of ½ in. and ⅝ in. plywood are analyzed using linear and nonlinear methods, and results of horizontal displacement versus racking force are displayed graphically. It is shown that while the two methods converge at ultimate loads, true representation of wall behavior at low levels of displacement is best described through nonlinear analysis. Good comparison is obtained with a finite element approach.

Keywords: Fasteners, framed structures, nails, shear forces, shear strength, sheathing, structural analysis, studs, walls.

INTRODUCTION

Racking resistance of nailed walls is defined as "the ability of walls to resist in-plane shear forces induced by lateral wind and earthquake loads, such walls are commonly known as shear walls."

When a structure is acted upon by wind or other horizontal forces, the major resistance to these loads is provided by the end walls parallel to the direction of wind. These in-plane shear forces are called racking forces. When these walls are made of wood (frame and sheathing), most of these forces would be supported by the nail connections holding the sheathing to the frame.

Previous research has related the overall racking strength of walls to properties of a single nail connection in shear. The most significant property of such connections is their load-slip relationship in shear. Experimental tests indicate that there is distinct nonlinearity in this relationship which causes the wall to exhibit a nonlinear behavior.

To predict racking strength of walls, many researchers used various linear approximations of the load-slip curves of nailed connections. Good examples of such approximations are the tangent and second moduli. These approximations resulted in discrepancies in the overall wall behavior at various levels of loading. In some instances they predicted the racking strength for low levels of lateral displacements.

Studies using linear analysis concluded that even though analytic results compared to experimental results, they consistently overestimated the racking force

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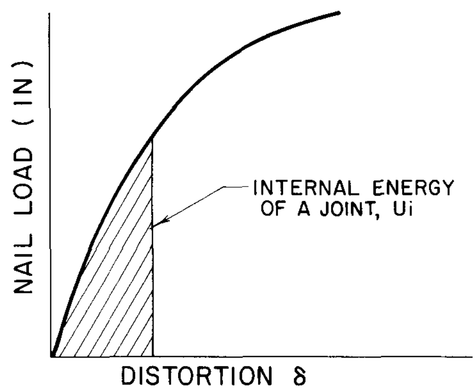

FIG. 1. Load-slip relationship.

at high levels of displacements. These studies attributed this difference to the nonlinearity of connections. While linear analysis simplifies computations, it fails to provide a true description of wall behavior at various levels of displacements. This limitation is overcome through nonlinear analysis.

This paper presents a theoretical formulation for predicting the nonlinear load-displacement relationship of nailed walls. The method utilizes a nonlinear formulation and an energy approach to predict the racking force associated with a given wall displacement.

The method presented here is compared with the finite element approach of Cheung and Itani (1983), which has been extensively verified using experimental data of several walls. Since the two methods compared well, no attempt has been made to present experimental data in this paper. The intent of this study is to present the theoretical approach and demonstrate its use through sample examples.

LITERATURE REVIEW

There have been several studies dealing with racking resistance of nailed walls. Kuenzi in unpublished research developed a method of design for racking resistance. This was later reported by Potter (1968). Similar studies were developed independently by Burgess (1976).

Tuomi and Gromala (1977) presented a study in which they evaluated the racking strength of walls with let-in corner bracing. They also studied the effect of various sheathing materials subjected to racking forces.

Empirical equations for relating wall racking resistance to lateral nail strength have been developed by several researchers. They based their studies in similar nailing patterns on 4 ft × 8 ft sheets of fiberboard sheathing. They accurately predicted racking strength within the limits of test data. However, no theoretical models were developed to consider variables in panel geometry or nailing pattern.

Tuomi and McCutcheon (1978) developed a study for the computation of racking strength of light frame nailed walls. They used procedures similar to those reported by Tuomi and Gromala (1977). Good correlation between ultimate actual loads and developed theory was obtained.

Walker (1978) used the force equilibrium method to obtain the results presented

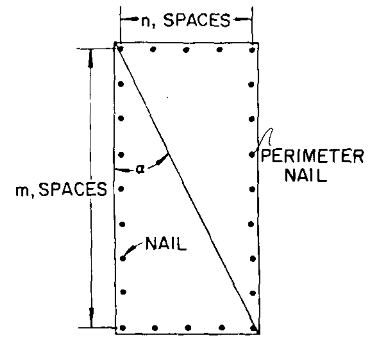

FIG. 2. Original panel.

by Kuenzi for symmetrical nail patterns. In 1979, he presented a general review of available methods of racking resistance analysis and provided simplified formulas suitable for design purposes.

Gromala and Price (1980) conducted a series of tests using full-size racking panels and small panels to calculate the racking strength of walls sheathed with structural flakeboards made from southern species.

Itani et al. (1982) presented a methodology for calculating the racking performance of sheathed wood-stud walls. They considered wall configurations that include openings resulting from doors and windows.

Easley et al. (1982) presented a study for the analysis of wood-framed shear walls with sheathing attached by discrete fasteners. The study was based on experimental observations of the displacements and deformations of individual wall panels. Formulas are developed for the determination of sheathing fastener forces, the linear stiffness, and the nonlinear shear load-strain behavior of a wall. Eight full-scale 8 ft × 12 ft (2.438 m × 3.658 m) plywood sheathed walls were tested under static loading. Good agreement was obtained between results from these formulas and wall tests.

Cheung and Itani (1983) presented a mathematical finite element model for predicting the static and dynamic behavior of nailed shear walls. The method employs a fastener compliance element derived from the nonlinear load-slip properties of nailed connections. The finite element method showed excellent comparisons with several experimental results.

THEORY

The formulation of this problem builds on the study presented by Tuomi and McCutcheon (1978), in which they developed equations for evaluating racking resistance of a panel assuming linear load-slip relationship for a single nail connection in shear.

Nonlinear analysis

In this study the following assumptions and limitations are imposed:

1. The load-slip relationship of a single nail connection is nonlinear. Such a relationship is established from experimental studies (Fig. 1).

2. The frame distorts as a parallelogram, while the sheathing material retains

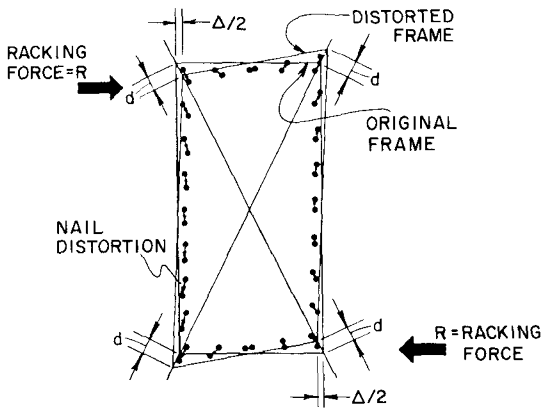

FIG. 3. Distortion under load.

its initial rectangular shape (Fig. 3). The edges of the sheathing are unrestrained and free to displace without interference from adjacent sheets or foundations.

3. The sheathing is applied vertically to the frame and is continuous from the top to the bottom of the frame.
4. Nails are evenly and symmetrically spaced. The numbers of nails along the top and bottom edges and along the left and right edges of the sheathing are equal.
5. Loading is considered to be monotonically increasing and is commonly known as a "quasi-static" loading.
6. Distortions and deflections are small.
7. The racking force is applied as shown in Fig. 3, i.e., in a direction parallel to the frame. No vertical loading is considered.
8. The strain energy resulting from bending of frame and stresses in sheathing is neglected. This is a reasonable assumption since most of the resistance in such walls is provided by connectors.

Evaluation of internal energy

Consider a single panel fastened around its perimeter to the frame (Fig. 2). It is assumed that the four corner nails distort along the diagonal lines of the sheathing (Fig. 3). This has been observed to be true by using double-headed nails in laboratory tests.

Using the assumption of small displacements and denoting the total slip of any nail by δ and the corner nail distortions by d, the following equations were derived by Tuomi and McCutcheon (1978):

Top and bottom edges:

$$\delta = d\left[\sin^2\alpha + \left(\frac{2i}{l} - 1\right)^2 \cos^2\alpha\right]^{1/2} \tag{1}$$

Left and right edges:

$$\delta = d\left[\cos^2\alpha + \left(\frac{2j}{m} - 1\right)^2 \sin^2\alpha\right]^{1/2} \tag{2}$$

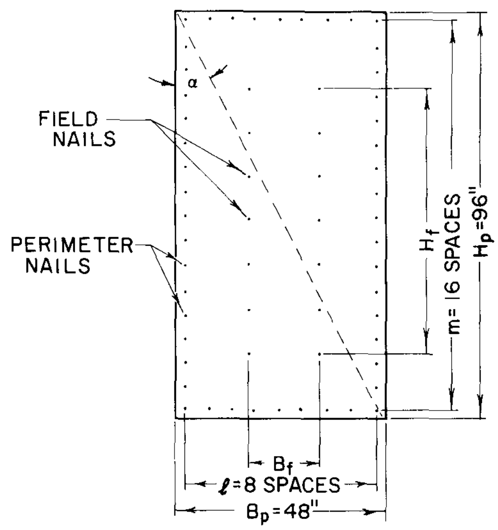

FIG. 4. 4 ft × 8 ft wall panel.

where,

α = angle between diagonal and vertical edges of sheathing (Fig. 2)
l and m = number of spaces between nails along edges of sheathing (Fig. 4)
i and j = fastener number along horizontal and vertical edges, respectively

Another relationship between the horizontal displacement that a racking force witnesses and the slip of the corner nail d can be obtained using the geometry of Fig. 3.

$$d = \frac{\Delta}{2 \sin \alpha} \tag{3}$$

Thus the nail slip δ given in Eqs. (1) and (2) is a function of the corner nail-slip, d, which is in turn a function of the frame displacement Δ.

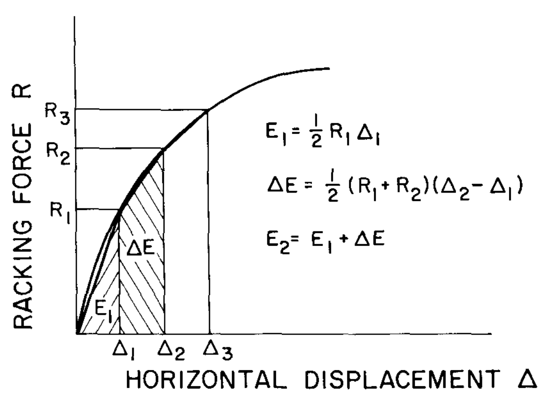

FIG. 5. Racking force vs. horizontal displacement.

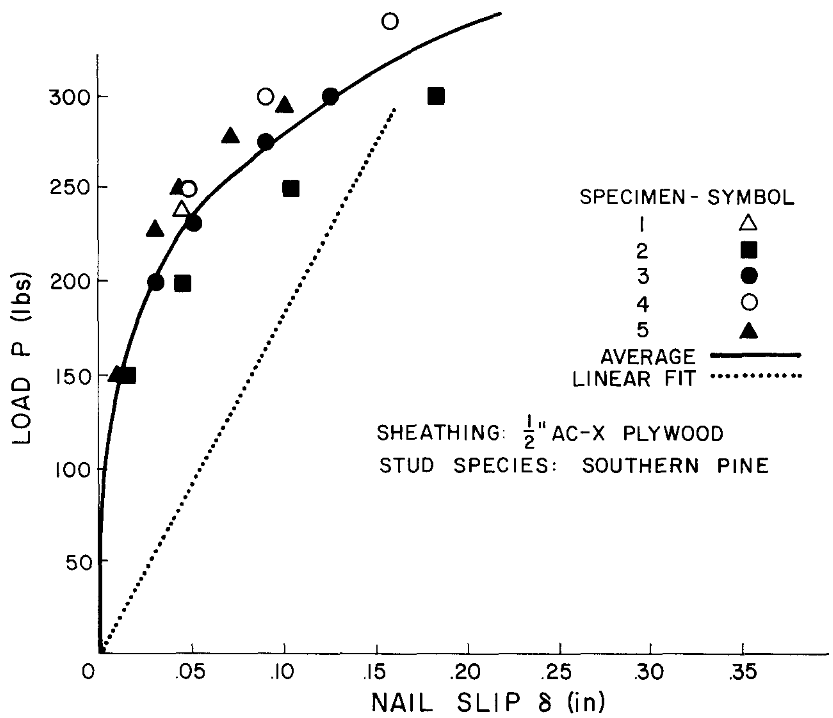

FIG. 6. Load-slip curve for ½ in. plywood.

A typical wall includes interior nails commonly referred to as "field nails" (Fig. 4). Assuming that these nails follow the same distortion pattern as the perimeter nails, the distortion δ' of any nail will be:

For top and bottom rows of field nails:

$$\delta' = d\left[a^2\sin^2\alpha + b^2\left(\frac{2i}{l} - 1\right)^2\cos^2\alpha\right]^{1/2} \tag{4}$$

For left and right columns of field nails:

$$\delta' = d\left[b^2\cos^2\alpha + a^2\left(\frac{2j}{m} - 1\right)^2\sin^2\alpha\right]^{1/2} \tag{5}$$

where,

$$a = \frac{\text{field height}}{\text{perimeter height}} = \frac{H_f}{H_p} \qquad \text{(Fig. 4)}$$

$$b = \frac{\text{field width}}{\text{perimeter width}} = \frac{B_f}{B_p} \qquad \text{(Fig. 4)}$$

Thus, for a given horizontal displacement Δ, the distortion of each connection is known. Knowing the load-slip curve of the connection (Fig. 1), the internal energy u_i is evaluated as the area under the curve (Fig. 1).

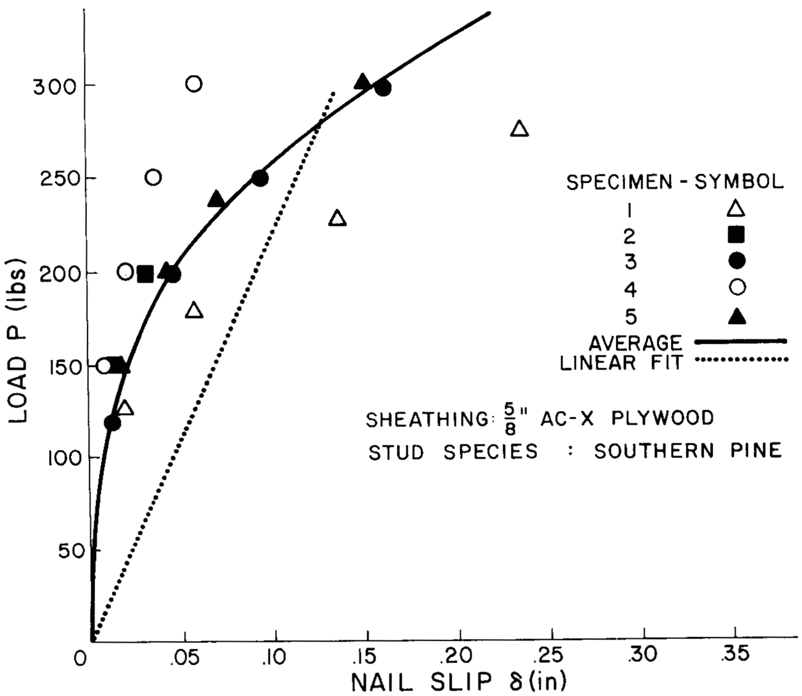

FIG. 7. Load-slip curve for ⅝ in. AC-X plywood.

$$u_i = \int_{\delta} F_i \, d\delta_i \tag{6}$$

The total internal energy U of all connections is given by:

$$U = \sum_{i=1}^{n} \int_{\delta} F_i \, d\delta_i \tag{7}$$

where,

n = number of nails connecting the sheathing to the frame.
F_i = the shear force acting on a nailed connection.

Computation of racking force:
The external energy E is expressed as:

$$E = \int_0^{\Delta} R \, d\Delta \tag{8}$$

where,

Δ = horizontal displacement
R = racking force

The relationship between the racking force and the horizontal displacement is nonlinear and is approximated by linear segments within small intervals of Δ (Fig. 5).

For a given horizontal displacement Δ, the corresponding racking force R is computed by equating the internal energy as given by Eq. (7) to the external work given by Eq. (8). An incremental approach of the horizontal displacement Δ is followed (Fig. 5).

For $\Delta = \Delta_1$ and assuming Δ_1 is small:

$$E_1 = \tfrac{1}{2}\Delta_1 R_1 \tag{9}$$

Equating the internal energy U_1 to that of the external energy E_1, the corresponding R_1 is obtained.

$$R_1 = \frac{2}{\Delta_1} U_1 \tag{10}$$

To find R_2 associated with Δ_2, refer to Fig. 5.

$$U_2 = U_1 + \frac{(R_1 + R_2)}{2}(\Delta_2 - \Delta_1) \tag{11}$$

Thus R_2 is computed from Eq. (11) as:

$$R_2 = \frac{2(U_2 - U_1)}{\Delta_2 - \Delta_1} - R_1 \tag{12}$$

A general form of Eq. (12) is given as:

$$R_{i+1} = \frac{2(U_{i+1} - U_i)}{(\Delta_{i+1} - \Delta_i)} - R_i \tag{13}$$

where,

$i = 1, 2, 3, 4, \ldots n$

A computer program was developed in Fortran IV to compute the racking force R_i associated with various displacements Δ_i. The input data to this program consist of experimental relationships of load-distortion to connection. Because of available computer software on the WSU system, a cubic spline function was fitted to experimental data of load-slip of a nailed connection and the internal energy is evaluated using Simpson's rule. Knowing the internal energy, the program proceeds to evaluate the racking force R in accordance with Eq. (13).

SAMPLE ANALYSIS AND RESULTS

A 4 ft × 8 ft panel (Fig. 4) was analyzed using the method presented in the previous section. The sheathing was ½ in. and ⅝ in. AC-X plywood nailed to the frame with 8d common galvanized nails. Perimeter and field nails were considered to be spaced at 6 in. and 12 in., respectively. The frame was made of southern pine.

The method presented here requires that the load-slip relationship of the connectors be provided. This was accomplished by performing a single shear nail

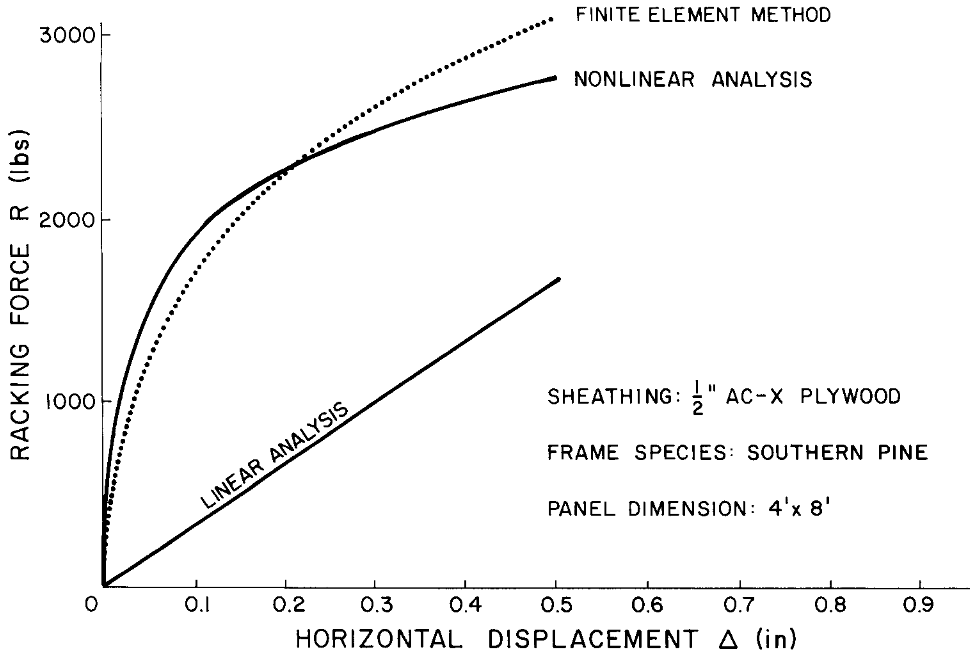

FIG. 8. Theoretical racking force vs. horizontal displacement for 4 ft × 8 ft panel with ⅝ in. plywood sheathing.

slip test on two groups of specimens. Group I consists of ½ in. sheathing, while group II consists of ⅝ in. sheathing using 8d common galvanized nails. Five specimens of each group were tested. The test used was similar to that of the standard ASTM D1761 and to that utilized by Atherton et al. (1980) of Oregon State University.

Results for the two groups are shown in Figs. 6 and 7. The average values for the five specimens within each group were used to establish the racking force versus displacement.

The linear solution consists of fitting the load-slip data of connection by a straight line passing through the origin using the method of least squares. These lines are shown in the data of Figs. 6 and 7 and represented by the following equations:

$$P = 1806\ S \quad \text{(for 1/2 in. plywood)} \tag{14}$$
$$P = 2167\ S \quad \text{(for 5/8 in. plywood)} \tag{15}$$

where,

P = load acting on connection (lb)
S = slip between sheathing and frame (in.)

Selected points from Eqs. (14) and (15) are used as input data into the developed computer program and the resulting racking forces versus displacements, and are shown in Figs. 8 and 9.

To demonstrate the validity of the analysis performed by the method presented

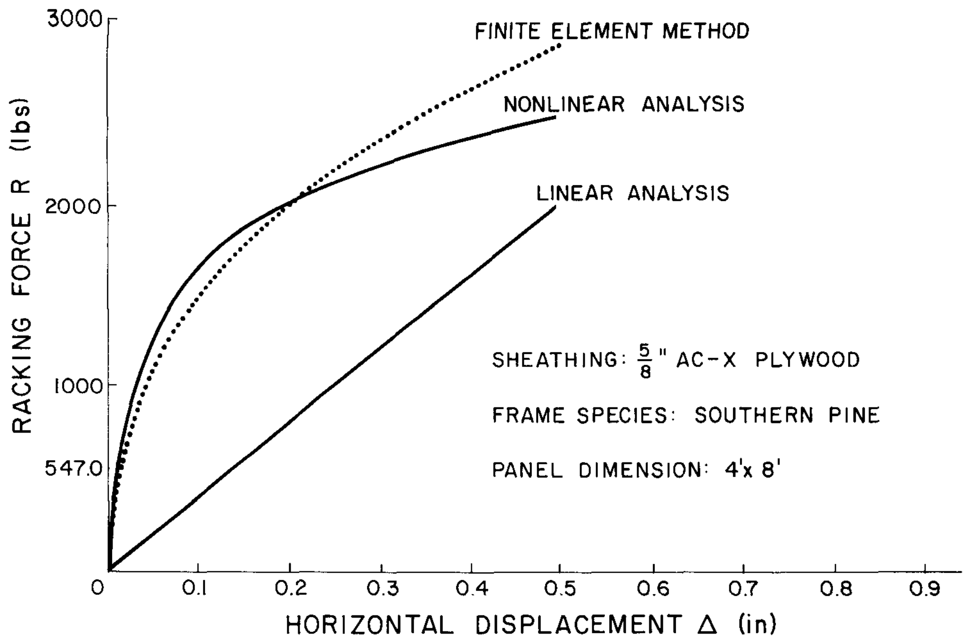

FIG. 9. Theoretical racking force vs. horizontal displacement for 4 ft × 8 ft panel with ⅝ in. plywood sheathing.

and to verify the logic and computational process of the developed computer program, results for the ½ in. plywood sheathing are compared to the linear solution provided by Tuomi and McCutcheon (1978).

For the panel shown in Fig. 4, Tuomi and McCutcheon (1978) showed that the racking force R is given by:

$$R = 8.35r \tag{16}$$

where,

r = lateral nail force associated with a given slip (lb)

Based on their assumption of r = 285 lb, Eq. (16) results in panel resistance of 2,380 lb.

Assuming a corner nail force of 285 lb, Eq. (14) gives a slip of 0.158 in. and consequently Eq. (3) yields a horizontal displacement of 0.706 in. For this displacement, the linear analysis of Fig. 8 shows a corresponding racking force of 2,355 lb. This compares well with the 2,380 lb. obtained by Tuomi and McCutcheon (1978). This is expected since for the linear case the two methods are identical with the difference resulting from fitting segments of load-slip data by cubic polynomials.

A word of caution regarding the comparison of the two linear methods needs to be made here. In their solution, Tuomi and McCutcheon used for purposes of illustration an ultimate connection load of 285 lb. While the value was arbitrarily selected, it was assumed to lie on the least square linear regression used in this paper.

TABLE 1. *Load-slip data for nailed joint with 8d common galvanized nail.*

Load (lb)	½ in. plywood: slip as determined by testing (in.)	⅝ in. plywood: slip as determined by testing (in.)
50	0.0012	0.0012
100	0.0034	0.0054
150	0.0117	0.0192
200	0.0290	0.045
250	0.0638	0.0892
300	0.130	0.156
350	0.246	0.178
400	—	0.342

The nonlinear method outlined in this paper uses the actual experimental load-slip data of a connection as input into the developed computer program. For the ½ in. and ⅝ in. plywood panels studied, the average slip at 50-lb load increments of the five specimens (Figs. 6 and 7) is used for establishing the load-slip relationship. Table 1 shows the input data used in this method.

Results of nonlinear analysis are displayed in Figs. 8 and 9 and compared to the finite element method presented by Cheung and Itani (1983). Good comparison is obtained over the range of horizontal displacements shown. The difference is attributed to the functional relationship used in fitting the load-slip data of Table 1. The method presented here uses a cubic spline function, while that of Cheung and Itani utilizes an exponential fit.

From these figures, it is noted that while linear and nonlinear methods of analysis converge for large magnitudes of horizontal displacement, they show considerable discrepancies in predicting the panel racking force at low levels of displacements. The difference between the linear and nonlinear analysis is attributed to the fact that the linear method underestimates the internal energy of connections. This underestimation is most pronounced in nails far removed from panel corners.

CONCLUSION

A theoretical study for nonlinear analysis of racking resistance of nailed walls is presented. The method utilizes the principle of minimum potential energy as the basis for its development. A computer program is developed for carrying out the complex computations associated with such analysis.

It is shown that in order to obtain a true representation of the racking force versus displacement, nonlinear analysis is needed.

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