# POTENTIAL FAILURE OF A DECAYED TREE UNDER WIND LOADING

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ABSTRACT

Trees with decayed wood that are subject to moderate winds often collapse and cause property damage or injury and death to people. The purpose of this paper is to describe a decision-making aid to help identify a tree that may fail in the forest or be a potential hazard in the city. A tree may fail when the probability of radial shear cracks developing for a given wind load is sufficiently high.

Mathematical models are used to estimate the constant wind force on trees and to evaluate the cracking and collapse mechanisms under this loading. The physical dimensions are used to determine the wind force or drag on the tree, and the amount of decay in the tree is used to determine its ability to resist this load. Owing to uncertainties associated with accurately measuring and modeling a decayed tree, estimating the wind load, and specifying the wood strength of a tree species, reliability analysis is used to assess the potential risk of failure. Coupling this information with meteorological data for the largest wind speed value expected at the tree site and the topography of the tree site completes the analysis of potential failure. Case studies of balsam fir trees with the same exterior diameters but with different dimensions of decay columns, tree weights, tree heights, and wind speed conditions are analyzed and compared.

*Keywords:* Balsam fir, cracking, collapse, decay, failure, forecasting, reliability analysis, wind.

## INTRODUCTION

Destructive field testing of balsam fir (*Abies balsamea*) with root and butt decay caused by *Armillaria mellea* showed that the amount of decay was the most significant factor in predicting cracking and collapse failure forces on trees subject to static forces. As a result, mathematical models were developed to simulate the cracking and collapse mechanisms of a balsam fir under constant horizontal loading. The mechanism consists of two sequential events. First, a pair of radial cracks were observed on either side of the decay column of the main stem. Second, tree collapse occurred when the bending stress equaled the modulus of rupture of the tree. The tree resistance to cracking and collapse was dependent upon the external dimensions of the tree and dimensions of the decay column. The drag or wind forces on a tree have been determined in wind tunnel tests. These tests have shown

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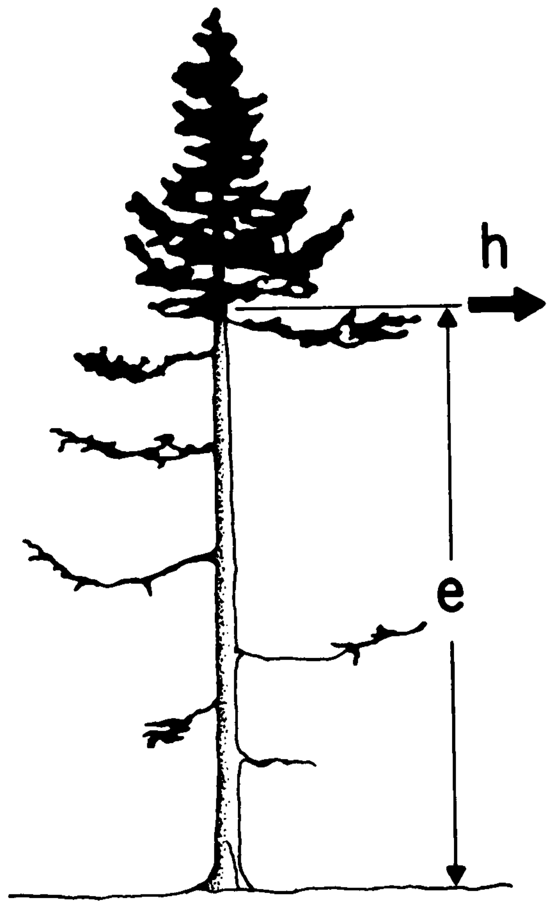


FIG. 1. The wind force diagram.

that the magnitude of the wind force is dependent upon the weight of the tree and the wind speed (Fraser 1962).

The mathematical models that are used to describe the cracking-collapse mechanism and to estimate the wind load are combined in an analysis procedure. Owing to the uncertainties associated with estimating model input variables of wind loading and tree strength and inherent variability of physical properties of wood, reliability analysis is used. The rationale behind the use of reliability analysis is described and its practical application is demonstrated with case studies.

## CRACKING-COLLAPSE MECHANISMS

Analysis of destructive field test data of balsam fir trees leads to the development of a failure theory (Peters et al. 1984). The tree is assumed to be loaded with a wind pressure load that can be represented as a resultant force h acting at a distance e above ground (Fig. 1). Since a typical balsam fir tree is susceptible to decay at the base (root rot), cracking and bending failures are expected in this region.

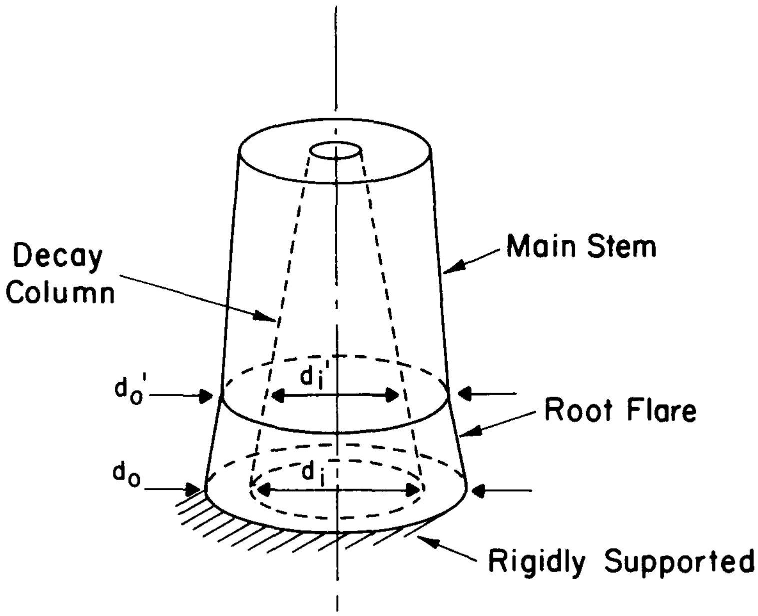


FIG. 2. The tree model.

*The failure model*

The tree is assumed to act as a tapered cantilever beam subject to a constant horizontal force h that is rigidly supported at the base (Fig. 1). Since cracking and bending failures are assumed to occur at the base, the tree is modeled as a tapered cylinder with a conical-shaped decay column (Fig. 2). The wood in the decay column is assumed to have zero strength. The wood in the main stem and root flare regions is assumed to be sound and of equal strength.

The wind force will cause the tree to bend. Since the tree is assumed to act as a cantilever beam, bending and shear stresses are developed. For simplicity, the result of bending will be represented as two internal forces, tension (T) and compression (C) forces (Fig. 3). Shear failure is assumed to be predominant in the phase that leads to radial cracking on either side of the decay column. The T and C forces act in opposite directions, tending to cause the tree segments on either side of the maximum shear plane to slide past one another. The maximum shear stress is assumed to occur on the plane of maximum shear stress (Fig. 3). The point of maximum shear stress on the shear plane is assumed to develop at point A, at the edge of the decay column at the base of the tree. From symmetry, the shear stresses at points A are equal. When the wind force h is sufficient, the shear stress at points A will equal the critical shear strength of the wood parallel to the grain. Cracks will develop at points A and propagate upward into the main stem and outward towards points B. If the wind load is sufficient to cause cracking to initiate at points A, it is assumed that cracking will proceed upward into the main stem and outward to the exterior edge of the tree. The net result is that the tree no longer reacts as a single cantilever beam, but as two cantilever beams. The cracking phase is assumed to be complete and the collapse phase begins.

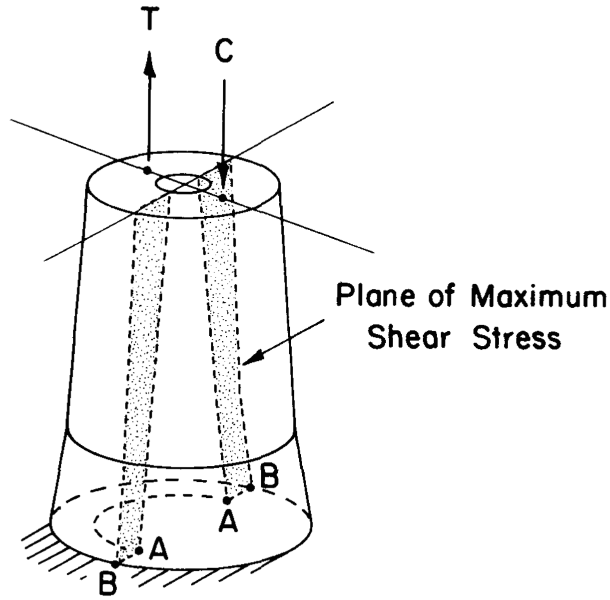


FIG. 3. The cracking phase.

Since the main stem is assumed to behave as two cantilever beam segments, the main stem of the tree behaves as two half hollow cylinders (Fig. 4). During the collapse phase, there is a redistribution of internal forces within each half hollow cylinder. The internal tension and compression forces are shown simply as T/2 and C/2 for a half tree segment because the two half cylinders are assumed to be equally strong. The bending strength of these segments is assumed to control during the collapse phase. When the wind force is sufficient, the tension and compression stresses caused by forces T/2 and C/2 equal the ultimate tension and compression strengths of the wood. Wood fibers will fail and the tree collapses. The collapse mechanism is expected to initiate above the root flare.

The hypothesis presented here is supported by visual observation during destructive field testing and analytical data gathered during these tests. The following is a mathematical model description of the failure theory.

*Analytical models*

The critical load to cause radial cracking $h_c$ is estimated with the following mathematical model:

$$h_c = \frac{\tau}{2.089} \frac{(d_o^4 - d_i^4)}{(2.25d_o^2 + d_i^2)} \qquad (1)$$

where $d_o$ = inside bark tree diameter at the base, $d_i$ = diameter of the decay column at the tree base, and $\tau$ = critical shear strength. The model assumes that the tree responds as a hollow cylindrical or tubular section under shear loading. Radial cracks that are present prior to loading are assumed not to affect the determination of $h_c$; therefore, they are not considered in this analysis. Further-

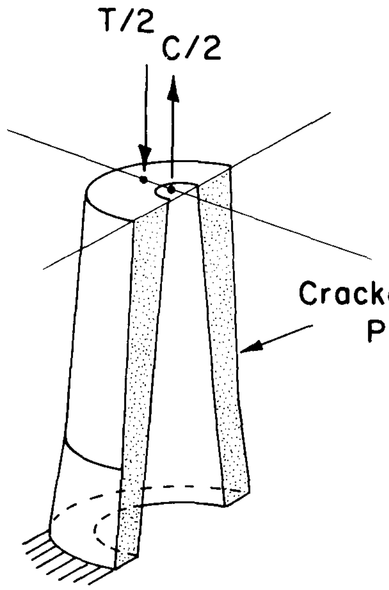


FIG. 4. The collapse phase.

more, it was found in our field and laboratory tests (Peters et al. 1984) that the average critical shear strength of 239 N/m² (39 psi) is significantly lower than the shear strength of 4,600 N/m² (668 psi) for sound wood samples. The coefficient of variation $\Delta_\tau$ was estimated to be 0.125.

The critical load to cause tree collapse, the ultimate horizontal load $h_u$ that may be placed on the tree, is

$$h_u = \frac{2\sigma I}{ec} \tag{2}$$

where I = moment of inertia of a half cylindrical section, c = critical distance between the neutral axis and extreme fiber to cause maximum bending stress in a half hollow cylindrical section, $\sigma$ = modulus of rupture of green balsam fir, and e = moment arm or distance between resultant wind force and tree base. Since it is assumed that the radial cracking has occurred, a one half hollow cylindrical section is used for the collapse mechanism because the effective area for bending resistance is reduced to two half sections. In addition, it is assumed that the bending caused by the horizontal wind force is equally distributed between the two half hollow cylindrical sections. The equations of the moment of inertia I and critical distance c for a half hollow cylindrical section are respectively

$$I = \frac{\pi}{128}(d_o'^4 - d_i'^4) - \frac{1}{18\pi}\frac{(d_o'^3 - d_i'^3)^2}{(d_o'^2 - d_i'^2)} \tag{4}$$

$$c = \frac{d_o'}{2} - \frac{2}{3\pi}\frac{(d_o'^3 - d_i'^3)}{(d_o'^2 - d_i'^2)} \tag{5}$$

where $d_o'$ and $d_i'$ are the outside tree diameter and diameter of the decay column above the root flare.

## WIND FORCES

Fraser (1962) conducted wind tunnel studies to determine the drag force of various tree specimens under constant wind speed. He found that wind speed v (knots) and tree weight w (pounds) are the significant factors in determining the resultant wind force. For coniferous trees, the regression equation is

$$h = 1.441v + 0.029vw - 0.328w + 7.426 \tag{6}$$

with a standard error estimate of 17.4 lb. The dry weight $w_s$ in pounds of the above-ground components of a balsam fir tree (Tritton and Hornbeck 1982) is estimated with the regression equation

$$w_s = 1.81(\text{dbh})^{2.4} \tag{7}$$

where dbh = tree diameter at breast height (1.37 m above ground) in inches. The total weight is equal to

$$w = w_s(1 + m) \tag{8}$$

where m is the moisture content (expressed as a decimal) of the tree. The average moment arm $\mu_e$ is estimated to be

$$\mu_e = 0.65L \tag{9}$$

where L = height of the tree. The moment arm estimates ranged from 0.52L to 0.84L (Peters et al. 1984).

## RELIABILITY ANALYSIS

A tree is defined to be safe if the tree is sufficiently strong to resist a wind load without cracking. Otherwise, it is classified as hazardous and is considered to be potentially dangerous.

$$\text{Safe:} \quad h < h_c \tag{10a}$$

$$\text{Hazard:} \quad h \geq h_c \tag{10b}$$

The variables h and $h_c$ will be treated as random variables H and $H_c$, respectively, because of the associated uncertainties found in nature and estimating the input variables of Eqs. (1) and (6). The difference in the magnitudes of wind load critical load is defined to be the margin of safety for cracking or

$$M_c = H - H_c \tag{11}$$

If a tree is safe $H < H_c$, then $M_c < 0$. Similarly, if a tree is hazard $H \geq H_c$, then $M_c \geq 0$. The probability that a tree is safe or hazard is expressed as

$$\text{Safe:} \quad P[M_c < 0] \tag{12a}$$

$$\text{Hazard:} \quad P[M_c \geq 0] \tag{12b}$$

Likewise, the following classification is used for tree collapse.

$$\text{No collapse:} \quad h < h_u \tag{13a}$$

$$\text{Collapse:} \quad h \geq h_u \tag{13b}$$

The variables h and $h_u$ will be treated as random variables, H and $H_u$, respectively. The margin of safety for collapse is defined as

$$M_u = H - H_u \tag{14}$$

The collapse, $H_u \geq 0$, must be preceded by cracking, $M_c \geq 0$. Thus, the probability of no collapse and collapse are given by the conditional probabilities

$$\text{No collapse:} \quad P[M_u < 0 | M_c \geq 0] \tag{15a}$$

$$\text{Collapse:} \quad P[M_u \geq 0 | M_c \geq 0] \tag{15b}$$

The assumptions and derivations of the classification Eqs. (12a), (12b), (15a), and (15b) are given in the Appendix.

## EXTREME WIND SPEEDS

In the evaluation of the sources of uncertainty, it was assumed that the wind speed is known. As a result, the probabilities of tree cracking or tree collapse for a given wind speed v may be expressed as conditional probabilities. Using the classification probabilities of Eqs. (14) and (15), they are rewritten as

$$\text{Hazard:} \quad P[M_c \geq 0 | V = v] \tag{16a}$$

$$\text{Collapse:} \quad P[M_u \geq 0 | (M_c \geq 0) \cap (V = v)] \tag{16b}$$

Statistical analyses of extreme wind speed data at 141 locations in the United States indicate that the Type I and Type II probability distributions of largest values most adequately describe the yearly occurrence of these extreme winds. The probability that a tree under evaluation will crack or collapse in any single year is estimated as

Hazard:

$$P[M_c \geq 0] = \int_0^\infty P[M_c \geq 0 | V = v] f_V(v)\, dv \tag{17a}$$

Collapse:

$$P[M_u \geq 0 | M_c \geq 0] = \int_0^\infty P[M_u \geq 0 | (M_c \geq 0) \cap (V = v)] f_V(v)\, dv \tag{17b}$$

where $f_V(v)$ is the probability density function of a Type I or Type II distribution of largest wind speed values.

### *Extreme wind speed distribution*

The selection of $f_V(v)$ will be dependent upon the meteorological and terrain conditions at the tree site. For purposes of this evaluation, the extreme wind speeds recorded at airport locations will be used (Simiu et al. 1979). These wind speeds represent recordings taken in open terrain; thus they must be corrected to account for local surface roughness conditions. The wind-speed power law relationship for open country; wooded areas, small towns or suburbs; and central areas of large cities will be used to correct for local effects (Hart 1982; International Conference of Building Officials 1979).

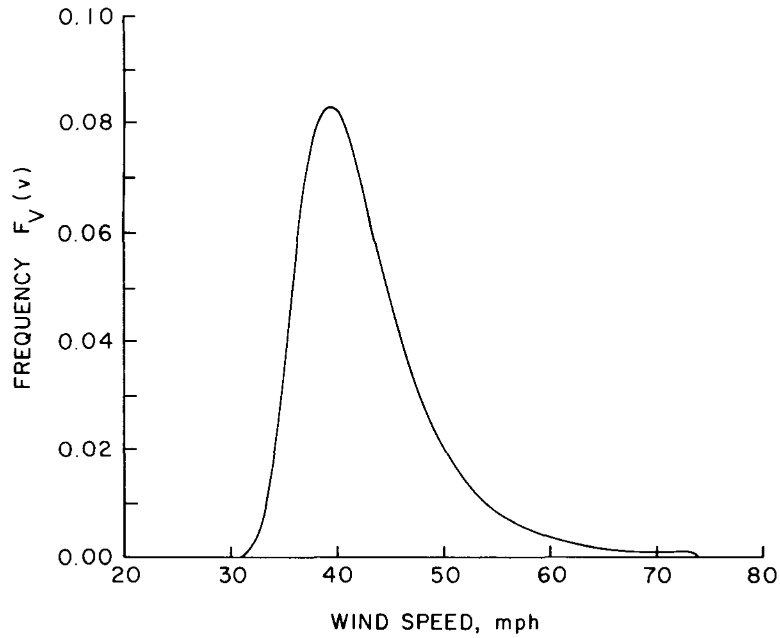


FIG. 5. Probability distribution of extreme wind speeds in Concord, New Hampshire.

### *The likelihood of tree collapse*

Tree collapse occurs only if the applied load H is sufficient to exceed the tree cracking resistance load $H_c$ and the ultimate load $H_u$. The probability $P[M_c \geq 0]$ is the likelihood that only radial cracking will occur in any year. Similarly, $P[M_u \geq 0 | M_c \geq 0]$ is the likelihood of tree collapse in any year, given that the tree is already cracked. The cracking may have occurred from an earlier extreme wind condition, or possibly, both cracking and collapse have occurred from the same extreme wind condition. The probability that a tree will collapse because of the same extreme wind is the joint probability that both cracking and collapse occur or

$$P[(M_u \geq 0) \cap (M_c \geq 0)] = P[M_u \geq 0 | M_c \geq 0] \cdot P[M_c \geq 0] \quad (18)$$

It is possible that the critical cracking resistance load is equal to or greater than the ultimate load or $h_c \geq h_u$. Whether the critical cracking resistance $h_c$ is less than, equal to, or greater than the ultimate load $h_u$ will depend upon the configuration and extent of internal decay. When $h_c \geq h_u$, it is assumed that the wind load will be sufficient to cause cracking and collapse to occur simultaneously. There is no reserve strength in the tree after cracking occurs. In terms of probability, the conditional probability,

$$P[M_u \geq 0 | M_c \geq 0] = 1.0 \quad (19)$$

is assumed to be equal to one.

## DISCUSSION

In order to vividly describe the classification scheme, numerical results of the reliability analyses for several case studies are presented. In case 1, a tree with

TABLE 1. *Sources of measurement error.**

| Model variable X | Range estimates $\epsilon = x_u - \mu_x = \lvert x_l - \mu_x \rvert$ | Coefficient of variation $\delta_x$ |
|---|---|---|
| $D_o$ | 3.42 cm (0.125 in.) | 0.012 |
| $D_o'$ | 3.42 cm (0.125 in.) | 0.012 |
| $D_i$ | 3.42 cm (0.125 in.) | 0.012 |
| $D_i'$ | 3.42 cm (0.125 in.) | 0.012 |
| $D_{bh}$ | 3.42 cm (0.125 in.) | 0.012 |
| E | 0.2L | 0.154 |

* The uniform probability distribution is assumed for all model parameters except E where

$$\delta_x = \frac{1}{\sqrt{3}}\left(\frac{x_u - x_\ell}{x_u + x_\ell}\right) = \frac{\epsilon}{\sqrt{3}\bar{x}}$$

with

$x_u$ = upper range of x, and $x_l$ = lower range of x, and $\mu_x$ = mean of x.

The model parameter E is assumed to be a normal probability distribution where

$$\mu_e = 0.65L$$

$$\delta_e = \frac{1}{2}\left(\frac{x_x - x_\ell}{x_u + x_\ell}\right)$$

with the upper and lower ranges of E assumed to be within two standard deviations of the mean $\mu_e$ of $x_u = 0.85L$ and $x_l = 0.45L$.

the following dimensions was analyzed: $d_o = 6$ in.; $d_i = 4.6$ in.; $d_o' = 5.7$ in.; $d_i' = 3.3$ in.; dbh = 4.8 in., and L = 360 in. The decay column for the tree samples used in the destruction sampling showed the following average relationships: $d_i' = 0.72d_i$; and $d_o' = 0.95d_o$. A balsam fir tree of these dimensions is representative of the average tree subject to destruction testing (Peters et al. 1984). It will be assumed that the tree will be subject to extreme wind speed recorded at the airport in Concord, New Hampshire. The mean extreme wind speed and coefficient of variation is 42.9 mph and 0.195, respectively. A Type II probability distribution for largest value with a tail length parameter of 9 will be used (Simiu et al. 1979) and is shown in Fig. 5. The sources of measurement error, inherent variability, and model uncertainty assumed in the analysis are recorded in Tables 1, 2, and 3.

The probability of tree cracking and collapse for a given wind speed v is presented as the conditional probabilities $P[M_c \geq 0 \mid V = v]$ and $P[M_u \geq 0 \mid (M_c \geq 0) \cap (V = v)]$, respectively. The relationships for case 1 are shown in Fig. 6. Owing to the greater certainty associated with model inputs for cracking mechanism as compared to ultimate load mechanism, a more sharply defined increase is observed for $P[M_c \geq 0 \mid V = v]$ than for $P[M_u \geq 0 \mid (M_c \geq 0) \cap (V = v)]$. The sources of uncertainty for H, $H_c$, and $H_u$ as well as other model variables are given in Table 4. The major source of uncertainty in estimating $H_u$ is the moment arm estimate of 3. The use of Eqs. (17a), (17b), and (18) gives the likelihoods of tree

TABLE 2. *Sources of inherent variability.**

| Model variable X | Mean $\mu_x$ | Coefficient of variation $\Delta_x$ |
|---|---|---|
| Υ | 239 N/m² (39 psi) | 0.125 |
| Σ | 38.61 × 10⁶ N/m² (5,600 psi) | 0.125 |
| M | 0.70 | 0.082 |

* Υ and Σ are assumed to have a normal probability distribution. M is assumed to have a uniform probability distribution with a range of values from 0.60 to 0.80. See footnote of Table 1 for equations for estimating $\Delta_m$.