RELIABILITY TESTING OF STATISTICAL PROCESS CONTROL PROCEDURES FOR MANUFACTURING WITH MULTIPLE SOURCES OF VARIATION

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(Received September 2001)

ABSTRACT

Quality inconsistencies can be caused by processes with multiple sources of variation. Therefore, the development of control charts that perform properly for both producer's and consumer's risk can be very complex. This is particularly true for real-time SPC systems that collect a great deal of data through non-contact sensing. In this paper, we demonstrate the use of a Monte Carlo simulation procedure that can be used to test SPC charts for both consumer's and producer's risk, and an experimental design procedure to analyze the results. This procedure is shown to be especially useful where design factors interact to cause high variation in a quality characteristic of a product. The approach is illustrated for a practical problem taken from the lumber manufacturing industry and demonstrates that commonly used industrial practices to control product dimensions lead to erroneous conclusions. To that end, a new mathematical approach that yields the correct results is described. The Simulation / ANOVA procedure described in this paper may have applicability in the control of many other industrial processes.

Keywords: Lumber size control, statistical process control (SPC), control charts, lumber manufacturing, simulation.

INTRODUCTION

Lumber production in a modern sawmill is a high-speed process, and dimensional variation of lumber as a result of inaccurate sawing is commonplace. There exist many sources of variation in sawing lumber, and modeling this industrial process is not a simple task. At the simplest level, two very important sources of variation must be taken into account and controlled: (1) sawing variation is the variation attributed to the movement or vibration of the work piece or saw during the cut; and (2) set works variation is the variation caused by lack of repeatability in the mechanism that sets the saw placement, or in differences between the thickness of the saw guides or spacers in fixed-position gang saws.

Control charts are statistical process control (SPC) tools that have been commonly deployed as a means for sawmills to understand and control dimensional variation. Typically, lumber is sampled once or twice per shift, which involves pulling five to ten pieces out of production and measuring their thickness and/or width (depending on the cut) in six to ten fixed locations. These data are used to compute sample means and variances, which in turn, are used in the construction of SPC charts that relate process averages and dispersion information to fixed target levels. It should be noted that setting control limits for SPC charts depends on the practitioner's ability to estimate the total variation inherent in the sawing system and to correctly identify and quantify the different sources of this variation present in the system.

Control charts are based on the idea of a statistical hypothesis test (Montgomery 2001). Consequently, their use carries an associated risk or probability of committing Type I Errors (called producer's risk), and Type II Errors (called consumer's risk). In the context of SPC, producer's risk is the probability of a false alarm, indicating the presence of assignable causes when the sawing process is, in fact, working normally. Consumer's risk is the probability that the control chart fails to detect the presence of assignable causes in a sawing process that is malfunctioning. Both incur costs to the producer for obvious reasons. Control charts are generally set up with what are called 3-sigma control limits, which result in a very small producer's risk. The 3-sigma control limits refer to the distance of +/–3 standard deviation units from the center line to the control limits. The level of consumer's risk present in a control chart depends on the magnitude of the malfunctioning sawing process. If the control limits are set up incorrectly, then the charts won't perform as expected.

Control charts have been widely used in lumber production and have been successful in controlling and reducing dimensional variation. These methods have worked in the past, as manual size measurement precluded the use of large sample sizes. However, the advancement of technology currently presents a problem with respect to the use of these SPC tools. Specifically, the trend towards non-contact laser measuring systems means that thousands of board dimensions can be measured in real-time as opposed to 100 or less using traditional sampling and handheld devices. While intuitively this richness in data would seem to yield more precise results, this does not seem to be the case. In fact, anecdotal evidence points to a higher than expected producer's risk, indicated by an alarmingly high number of "out-of-control" situations when real-time data are applied to traditional SPC tools. Mills using traditional SPC tools in a real-time context are finding that their statistical process control limits are too narrow, resulting in an excessive number of "false alarms" and putting them in the untenable position of having to manually widen their control limits to capture the expected number of out-of-control boards. Suffice it to say, this "trial and error" approach is mathematically unfounded. Traditional SPC tools are simply not robust enough to correctly process the virtually limitless data points that real-time scanning can provide.

These anecdotal results have led us to investigate the performance of traditional SPC tools applied to lumber size data with respect to producer's and consumer's risk, and to develop a new statistical foundation for lumber size control. This paper is the third in a series aimed at developing a mathematical basis for real-time size control in wood products manufacturing. In the first paper of the series, we reviewed the history and development of SPC systems used in wood processing (Maness et al. 2002). Inconsistencies were found in the way that sawing variation is estimated, and in the way that control limits for SPC charts are determined. In the second paper, we proposed a new statistical foundation for real-time SPC for lumber sizes (Maness et al. 2003). In this paper, we describe a method for comparing the reliability of SPC tools, and we use this reliability testing method to compare the performance of the new SPC system with those previously described.

Traditional approaches to size control

In a typical lumber size control system, m boards are selected randomly as they leave a machine center, and n thickness measurements are taken at random locations along the length of the board. Typically, saw or part positioning (repeatability) causes variation between boards.

Saw vibration or board movement during the cut causes size variation within each board. For this reason, it is important to partition the variation into its respective components: between- and within-board variation. Herein lies the difficulty.

When machining occurs as described above, the thickness of a piece of lumber at any point on its surface can be thought of as being a function of two random disturbance terms. The first term, β_i, is saw position, which affects each board. The second term, ϵ_{ij}, is the random sawing error, which occurs along each board during the cut. The thickness of the i^{th} board measured at point j, y_{ij}, is then given by:

$$y_{ij} = \mu + \beta_i + \epsilon_{ij} \tag{1}$$

where:

μ = target size for the sawing machine;

β_i = deviation from correct set works position for board i (causing variation between boards); and

ϵ_{ij} = saw movement or vibration error at position j on board i (causing variation within boards).

The distribution of ϵ and β can be estimated through empirical study. For this discussion, we assume the following:

$$\epsilon \sim NID(0, \sigma_w^2)$$

$$\beta \sim NID(0, \sigma_b^2)$$

We also assume that ϵ_{ij} and β_i are independent.

In our review of the literature, we found two distinct methods for estimating the standard error of the mean, and therefore the control limits (Maness et al. 2002). In addition, anecdotal evidence has shown us that these methods are commonly used in the lumber industry. For shorthand purposes, we call these two Common Lumber Industry methods CLI Method #1 and CLI Method #2.

CLI Method #1 is based on an analysis of variance (ANOVA) partitioning of the variances in a completely randomized design (CRD). The total sawing variance was obtained as the sum of the partitioned variances as shown below in Eq (2).

$$\hat{\sigma}_y^2 = \hat{\sigma}_b^2 + \hat{\sigma}_w^2 \tag{2}$$

where:

$\hat{\sigma}_y^2$ = estimate of the total variance;

$\hat{\sigma}_b^2$ = estimate of the between - board variance; and

$\hat{\sigma}_w^2$ = estimate of the within - board variance.

The standard error of the mean, which is used to determine the control limits on a standard X-Bar Chart, was found by dividing the total variance by the sample size, and taking the square root (Whitehead 1978):

$$\hat{\sigma}_{\bar{\bar{y}}_1} = \sqrt{\frac{\hat{\sigma}_y^2}{nm}} \tag{3}$$

where:

$\hat{\sigma}_{\bar{\bar{y}}_1}$ = estimate of the standard error of the mean using CLI Method # 1

m = number of boards in the sample; and

n = number of measurements taken per board

In CLI Method #2, the total standard deviation is calculated in the usual fashion from all n measurements on m boards included in the k^{th} sample (Smithies 1991):

$$\hat{\sigma}_{y_2,k} = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(y_{ijk} - \bar{\bar{y}}_k\right)}{(nm-1)}} \tag{4}$$

where:

$\hat{\sigma}_{y_2,k}$ = estimate of the standard deviation of the k^{th} sample using CLI Method # 2

y_{ijk} = thickness (or width) at the j^{th} measurement point on the i^{th} board, k^{th} sample; and

$\bar{\bar{y}}_k$ = k^{th} sample average.

To ensure that the control charts function properly, it is important to ensure that the total standard deviation represents within-sample variability only (Montgomery 2001). To obtain an estimate of the within-sample standard deviation, the average is obtained over a group of k samples taken when the process is working correctly. Since the sample standard deviation is a biased estimator of σ, the result must be divided by the statistical constant c_4 to obtain an unbiased estimator.[1] Therefore, an estimate of the total sample standard deviation using CLI Method # 2 is given in Eq. (5).

$$\hat{\sigma}_{y_2} = \frac{\sum_{k=1}^{r} \hat{\sigma}_{y_2}}{r \cdot c_4} \tag{5}$$

where:
r = the number of subgroups.

The total standard deviation is then divided by the square root of the total number of observations in the sample to get an estimate of the standard error of the mean:

1. See Montgomery (2001) section 3.2 for a further explanation of this concept and derivation of the statistical constant c_4.

TABLE 1. *Control charts based on the CLI Methods #1 and #2 and the resulting calculations of control limits*[2] *(from Maness et al. 2002).*

CLI Method #1		
Type of Control Chart	Variance Estimator	Control Limits
X Bar chart for sample average	Standard Error of the Mean $\hat{\sigma}^2_{\bar{\bar{y}}_1}$ in Eq. [3]	UCL: $\hat{\mu} + 3\hat{\sigma}_{\bar{\bar{y}}_1}$ LCL: $\hat{\mu} - 3\hat{\sigma}_{\bar{\bar{y}}_1}$
S Chart for total board variation	$\hat{\sigma}^2_{y_1}$ as in Eq. [2]	UCL: $B_4\hat{\sigma}_{y_1}$ LCL: $B_3\hat{\sigma}_{y_1}$
S Chart for within board variation	$\hat{\sigma}^2_w$ from ANOVA	UCL: $B_4\hat{\sigma}_w$ LCL: $B_3\hat{\sigma}_w$
S Chart for between board variation	$\hat{\sigma}^2_b$ from ANOVA	UCL: $B_4\hat{\sigma}_b$ LCL: $B_3\hat{\sigma}_b$
CLI Method #2		
Type of control chart	Variance estimator	Control limits
X Bar chart for sample average	Standard error of the mean $\hat{\sigma}^2_{\bar{\bar{y}}_2}$ in Eq. [6]	UCL: $\hat{\mu} + 3\hat{\sigma}_{\bar{\bar{y}}_2}$ LCL: $\hat{\mu} - 3\hat{\sigma}_{\bar{\bar{y}}_2}$
S Chart for total board variation	$\hat{\sigma}^2_{y_2}$ as in Eq. [5]	UCL: $B_4\hat{\sigma}_{y_2}$ UCL: $B_3\hat{\sigma}_{y_2}$
S Chart for within board variation	Same as CLI method #1	Same as CLI method #1
S Chart for between board variation	Variance of the board averages	Not applicable using this approach[3]

2. The statistical constants, B_3 and B_4, used in this table are commonly used in control chart applications and can be found in Montgomery (2001). They are based on confidence limits derived from the Chi-Square distribution with the appropriate corrections for bias.

3. The between board variance using CLI Method #2 is a biased estimator because it is partially composed of the within board variance. See Maness et al. (2002) for a more detailed explanation.

$$\hat{\sigma}_{\bar{\bar{y}}_2} = \frac{\hat{\sigma}_{y_2}}{\sqrt{nm}} \qquad (6)$$

The four control charts arising from CLI Methods #1 and #2 are each presented in Table 1.

In our previous studies, we detected several problems with the two CLI approaches (Maness et al. 2002, 2003). These problems arise because the two sources of variation in lumber sizes were not properly taken into account when constructing the experimental design. An approach for partitioning variation where there are several components of variability in manufacturing was introduced by Woodall and Thomas (1995). Maness et al. (2003) developed a similar approach adapted to the typical lumber SPC sampling scheme. We call this the Components of Variation (COV) approach.

Using this approach and the model previously given in Eq. (1), β and ϵ are assumed to be independent and identically distributed normal variables with zero mean. Thus, the set works position deviation distribution is $\beta \sim N(0, \sigma_b^2)$, and the saw movement distribution is $\varepsilon \sim N(0, \sigma_w^2)$. Under these assumptions, Eq. (1) becomes a random effects ANOVA model, and the properties of the random effects model can be used to construct control charts for SPC.

There are four valid control charts using this model: (1) an X-Bar Chart for subgroup means; (2) an S Chart to control within-board variation; (3) an S Chart to control between-board variation using the Satterthwaite procedure (Gaylor and Hopper 1969); and (4) a chart showing the proportion of total sawing variation accounted for by between-board variation. The four control charts and their resulting control limits are given in Table 2. The full derivation of these control limits can be found in Maness et al. (2003).

As described above, the X-Bar Chart is based on the variance of the subgroup means. However, because of the two distinct sources of variation, the variance of the subgroup means under the random effects model is more complex than

TABLE 2. *Control charts based on the COV Method and the resulting formulae for calculating control limits[4] (from Maness et al. 2003).*

COV method		
Type of control chart	Variance estimator	Control limits
X Bar chart for sample average	Standard error of the mean Mean, $\hat{\sigma}^2_{\bar{\bar{y}}_{cov}}$, in Eq.[7]	UCL: $\hat{\mu} + 3\hat{\sigma}_{\bar{\bar{y}}_{cov}}$ LCL: $\hat{\mu} - 3\hat{\sigma}_{\bar{\bar{y}}_{cov}}$
S Chart for within-board variation	same as CLI Method #1	same as CLI Method #1
S Chart for between board variation using Satterthwaite procedure	$\hat{\sigma}_b^2$ from ANOVA	UCL[5]: $\hat{\sigma}_b^2 \dfrac{\chi^2_{[1-\alpha/2;df]}}{(df)}$ LCL[5]: $\hat{\sigma}_b^2 \dfrac{\chi^2_{[\alpha/2;df]}}{(df)}$
COV Chart proportion of variation made up by between-board variation	$\dfrac{\hat{\sigma}_b^2}{\hat{\sigma}_w^2 + \hat{\sigma}_b^2}$, where $\hat{\sigma}_w^2, \hat{\sigma}_b^2$ as above	UCL: $\dfrac{F_{1-\alpha/2}\left(n\hat{\sigma}_w^2 + \hat{\sigma}_b^2\right) - \hat{\sigma}_w^2}{F_{1-\alpha/2}\left(n\hat{\sigma}_w^2 + \hat{\sigma}_b^2\right) + (n-1)\hat{\sigma}_w^2}$ LCL: $\dfrac{F_{\alpha/2}\left(n\hat{\sigma}_w^2 + \hat{\sigma}_b^2\right) - \hat{\sigma}_w^2}{F_{\alpha/2}\left(n\hat{\sigma}_w^2 + \hat{\sigma}_b^2\right) + (n-1)\hat{\sigma}_w^2}$

4. See footnote 2.

that used in either CLI approach. The true variance of the subgroup means under the random effects model for the COV approach is given by Neter et al. (1996):

$$\hat{\sigma}^2_{\bar{\bar{y}}\mathrm{cov}} = \frac{\hat{\sigma}^2_b}{m} + \frac{\hat{\sigma}^2_w}{nm} \qquad (7)$$

To our knowledge, the performance of these methods has never been empirically tested. Therefore, the main objectives of this research are:

1. to develop a methodology for estimating producer's and consumer's risk in SPC models for lumber size control;
2. to use this approach to test the reliability of the two methods used in practice under a real-time situation compared to the COV approach;
3. to recommend an approach for real-time SPC.

To meet these objectives, we use Monte Carlo simulation to test the number of out-of-control indications obtained versus the expected number based on the known theoretical properties of the underlying mathematical distribution. In addition, the robustness of each method was tested by means of sensitivity analyses on five factors: the number of measurements per board (n); the number of boards per sample (m); the board target size (μ); the between-board variation (σ^2_b); and, the within-board variation (σ^2_w).

METHODS

Simulation approach

Our purpose is to test the performance of the nine different SPC charts described above. These are: the 4 SPC charts using CLI Method #1; the two additional charts using CLI Method #2; and the three new charts using the COV Method (the within-board chart using the COV method is mathematical equivalent to the within board chart using CLI Method #1).

A Monte Carlo simulation program for evaluating the SPC charts was written in Visual Basic for Applications using a Microsoft Excel interface. The program simulated random board thicknesses using the model described in Eq. (1) with various known distributional assumptions. Control limits for the nine control charts were calculated based on the known population parameters as listed in Table 3. The simulated lumber thicknesses were then used in each of the control charts, and the number of out-of-control conditions was calculated for each situation. The results were then analyzed with Analysis of Variance to compare the performance of the nine control charts.

TABLE 3. *Experimental values used in Eq [1] for the producer's risk experiment.*

Factor		Factor levels	
	Low	Medium	High
x_1: n	10	30	50
x_2: m	10	30	50
x_3: μ	1.660″	1.680″	1.700″
x_4: σ^2_b	0.005″	0.020″	0.035″
x_5: σ^2_w	0.005″	0.020″	0.035″
Coded level	–1	0	+1

Experimental design

This simulation was replicated for each of the five factors at three different levels using a 3^k experimental design. The response variable for the experiment is the number of out-of-control (OOC) conditions signaled by the control charts. This experimental design allows us to look at the factors causing excessive OOC conditions and also to determine if there are significant interaction effects between the factors.

This factorial design results in a design matrix consisting of $3^5 = 243$ experimental runs for each of the nine control charts that we wish to test. In addition, we conducted the experiment to evaluate the performance of the chart with respect to both producer's and consumer's risk (Tables 3 and 4). Therefore, the full experiment

5. Degrees of freedom, df, are defined as:

$$df = \frac{(n\hat{\sigma}^2_b)^2}{\dfrac{MS^2_{TR}}{m-1} + \dfrac{MS^2_E}{m(n-1)}}.$$

TABLE 4. *Experimental values used in Eq [1] for the consumer's risk experiment.*

Factor		Factor levels Low	Medium	High	Base case for evaluating consumer's risk
x_1:	n	10	30	50	n/a
x_2:	m	10	30	50	n/a
x_3:	μ	1.685″	1.690″	1.695″	1.680″
x_4:	σ_b^2	0.025″	0.030″	0.035″	0.020″
x_5:	σ_w^2	0.025″	0.030″	0.035″	0.020″
Coded Level		–1	0	+1	n/a

consists of 243 runs * 9 charts * 2 types of risk, or 4374 experimental runs.

We use a regression model to present the results of the designed experiment.[6] Since our factor levels are quantitative and equally spaced, we coded the factor variables to the levels of –1, 0, and +1, which correspond to low, medium and high levels of the factors. This facilitates the analyses of the resulting regression coefficients.

In this experimental design, there are five main effects and 32 possible interaction effects. In order to reduce the complexity of the analysis, we chose to limit our analysis to the ten two-way interactions. Further, the results of a pilot study showed there were no significant interactions associated with target size (μ). Thus, we limited our analysis to the five main effects and the following six interaction effects:

$$n*m \quad n*\sigma_b^2 \quad m*\sigma_b^2 \quad n*\sigma_w^2 \quad m*\sigma_w^2 \quad \sigma_w^2*\sigma_b^2$$

The resulting regression model is:

$$\begin{aligned} y = \beta_0 &+ \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 \\ &+ \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{25} x_2 x_5 \\ &+ \beta_{34} x_3 x_4 + \beta_{35} x_3 x_5 + \varepsilon \end{aligned} \tag{8}$$

where:

y = number of out-of-control conditions (OOC's) per 1000 sample points

β_0 = model intercept (corresponds to the average number of OOC's)

β_i = regression coefficients

x_i = factors (see Table 3 for definitions)

ε = model error

We evaluated the results of each experiment in the following manner:

Compare the actual observed number of OOCs with the expected number based on the design of the chart and the chosen value of the producer's risk, α.

1. Study the main factor effects to determine their impact on the number of OOCs. If the chart is working correctly, there should be no significant impact of the main effects on producer's risk, but the appropriate factors should have an impact on consumer's risk. That is, increasing μ should have a significant impact on the performance of the X-Bar Chart, increasing σ_w^2 should have an impact on the S Chart for within-board variation and so on.
2. Study two factor interaction terms to determine their impact on the number of OOCs. There should be no significant impact on producer's risk, but the factor interaction with m and n should have an impact on consumer's risk.

The complete analysis of a control chart in this manner will allow the researcher to understand not only if the control tool is working properly, but also to diagnose why it is not working properly and to understand the conditions under which the tool will yield biased results.

6. A full description of the regression approach for analyzing the 3^k design can be found in Montgomery (1997).

Board thickness data generation

Board thickness data were generated to simulate the sawing process of a computer-controlled band saw set works. Using the model given by Eq. (1), a computer simulation program simulated a size control "sample" consisting of m boards with n measurements on each board. To accomplish this, a random number was generated for each of the m boards in the sample to simulate the saw position error (β_i). Then, n random numbers were generated to simulate the random saw movement error (ϵ_{ij}) for each measurement. Equation (1) was then used to generate the $n \times m$ thickness measurements for a given target size (μ). Both random disturbance terms were generated using the SAS function RANNOR (SAS Institute 1999), which returns a variate that is generated from a normal distribution with mean 0 and variance 1. The board thicknesses could follow any sampling distribution and the reliability testing procedure outlined in this paper would still work. However, new control charts would have to be developed that work well for non-normal data.

TABLE 5. *Comparison of calculated control limits for each of the nine charts using the population parameters for the base case for evaluating consumer's risk.*

Chart	UCL	LCL
X-Bar charts for process avg		
CLI #1	1.68283	1.67717
CLI #2	1.68281	1.67719
COV Approach	1.69114	1.66886
Charts for total variation		
CLI #1	0.03029	0.02628
CLI #2	0.03004	0.02607
Charts for between-board variation		
CLI #1	0.02809	0.01251
Satterthwaite method	0.02839	0.01225
COV method	0.67119	0.26857
Charts for within-board variation		
COV method	0.02144	0.01856

The simulation program allows the user to run R iterations, each iteration creating a simulated "sample" of m boards measured at n places along the board based on the three population parameters for the distribution of lumber thickness:

1. the lumber target size (μ);
2. the disturbance term for the overall saw movement ϵ_{ij} (σ_w^2); and
3. the disturbance term for the setworks placement β_i (σ_b^2).

For our experiment, we chose $R = 1000$, meaning that we generated 1000 sample points for each experimental run.

Determination of control limits and evaluation of chart performance

The upper and lower control limits for each of the nine SPC charts under study were calculated using the three underlying population parameters that were used to generate the board measurements. To evaluate producer's risk for a given control chart, we calculated the control limits based on the population parameters for each experimental case over the entire experimental range of the parameters in Table 3. From this, we determined the probability of a Type I error over a wide range of conditions. For a 3-sigma control chart monitoring an "in-control" situation, we expect 99.73% of the sample points to be within the upper and lower control limits based on statistical theory.

To evaluate consumer's risk, we calculated the control limits using the population parameters for an "in-control" condition. The values we used are those listed in Table 4 in the column labeled "Base Case for Evaluating Consumer's Risk." We then tested the performance of the control charts in detecting an "out-of-control" condition by generating the board measurement data using the parameters in Eq. (1) over the entire experimental range. In this situation, the low and high factor levels simulate the performance of the SPC charts in an "out-of-control" situation. The appropriate control charts were then used to detect the change both below and above the target value for each of the population parameters.

RESULTS AND DISCUSSION

Determination of control limits

To illustrate the differences in methods, the upper and lower control limits for each of the nine charts are presented in Table 5. The population parameters at the low factor levels given in Table 3 were used to calculate the control limits in Table 5. The control limits are based on $\alpha = 0.27\%$, which corresponds to 3-sigma limits on a standard control chart.

Evaluation for producer's risk

The Monte Carlo simulation procedure was executed for 1000 sample points using the 3-sigma control limits described above. Given the design of the charts and simulated normally distributed data described in the previous section, this would result in an average of 2.7 out-of-control sample points (OOCs) if the chart is working properly. We evaluated the lower and upper control limits separately, and given the symmetric nature of the underlying normal distribution, we therefore expected approximately 1.35 OOCs per simulation.

Table 6 shows the regression statistics for the first group of experimental trials, which evaluate the performance of the lower control limit of the X-Bar Chart using CLI Method 1. The regression is highly significant by the model F statistic, and the R-square indicates that 89% of the variability in the out of controls is explained by the experimental factors.

Examination of the model coefficients yields interesting results with respect to the performance of this chart. Note that the model should yield about 1.35 OOCs no matter what the factor levels are. However, it can be seen that, on average, there are 150 OOCs, and the number of OOCs increases rapidly as a function of the number of measurements per board (n), and the between-board variation (σ_b). This agrees with the theoretical analysis of the method. The inter-

TABLE 6. *Regression statistics for the lower bound of the X-Bar Chart using CLI Method #1 for producer's risk.*

Regression statistics	
Multiple R	0.945
R square	0.893
Standard error of estimate	34.130
Observations	243.000

ANOVA table

	df	SS	MS	F	Significance
Regression	11	2,250,445.1	204,585.9	175.6	<0.0001
Residual	231	269,147.1	1,165.1		
Total	242	2,519,592.2			

Model coefficients

Factor		Model coefficients	Standard error	t Stat	p-value
β_0	*Intercept*	150.45	2.19	68.71	<0.0001
β_1	n (meas)	63.25	2.68	23.59	<0.0001
β_2	m (brds)	–9.33	2.68	–3.48	0.0006
β_3	μ	1.35	2.68	0.50	0.6163
β_4	σ_b	71.26	2.68	26.57	<0.0001
β_5	σ_w	–61.52	2.68	–22.94	<0.0001
β_{12}	$n * m$	2.79	3.28	0.85	0.3970
β_{14}	$n * \sigma_b$	23.56	3.28	7.17	<0.0001
β_{24}	$m * \sigma_b$	–0.83	3.28	–0.25	0.7999
β_{15}	$n * \sigma_w$	–15.49	3.28	–4.72	<0.0001
β_{25}	$m * \sigma_b$	0.82	3.28	0.25	0.8021
β_{45}	$\sigma_w * \sigma_b$	24.59	3.28	7.49	<0.0001

action of n and both components of variation are significant as well as the interaction between the two components of variation. Since the interaction of n and σ_w is negative, this indicates that an increase in the within-board variance will mitigate increases in the number of OOCs caused by increases in n. It is also interesting to note the negative coefficient associated with the main factor effect of within-board variation. This indicates that if all other factors are held constant, the number of OOCs decreases significantly with an increase in within-board variation. Lastly, the large significant interaction between the two components of variance is also of interest because it indicates that the performance of the chart is affected by the relative ratio of between- to within-board variation.

All of these findings agree with the previous analysis of the performance of this chart relative to producer's risk on theoretical grounds (Maness et al. 2002). This provides validation for the theory and allows us to confidently interpret the coefficients for evaluating performance.

Full analysis of all nine control charts for both upper and lower control limits would result in 18 tables similar to Table 6. For brevity, we collapse the important information from these 18 tables into one matrix displayed in Table 7, which lists the model significance (based on the F test), R^2, and the model coefficients that are significant at the 0.05 level. The orthogonal nature of the input data matrix allows us to delete insignificant coefficients without having to recalculate the remaining ones.

Examination of Table 7 yields a complete picture of the performance of the charts. The model intercept is the average number of OOCs signaled per 1,000 sample points (since this is the prediction when all model inputs are at the medium level, or 0). Assuming the model is correct, we expect 1.35 OOCs per 1,000, and thus, the intercept should be close to 1.35. We also expect a low R^2, which indicates that little of the variation in the predicted number of OOCs is explained by the factors. The causes of poor chart performance are indicated by significant model coefficients. Large, significant coefficients indicate that the performance of the chart is adversely impacted (if positive) or mitigated (if negative) by that factor. The magnitude of the coefficient directly corresponds to the number of OOCs as one moves from the low factor level (–1) to the medium level (0), or from the medium to high level (+1).

Only the charts based on the COV approach and the S_w Chart yield the expected results. The performance of the S Chart for between-board variation based on the Satterthwaite formula is acceptable for the upper limit but not for the lower limit. One reason for this is that the estimate of between-board variance can be negative when within-board variation is large relative to between-board variation. However, it is more likely due to the inherent limitations of the Satterthwaite procedure. Gaylor and Hopper (1969) give guidelines for its use, noting that it is appropriate only when:

$$\frac{MS_B}{MS_e} \geq F\left[df_2, df_1, 0.975\right] * F\left[df_1, df_2, 0.50\right]$$

This condition is not satisfied when σ_b=0.005 and σ_w=0.020 or 0.035, and when σ_b=0.020, σ_w=0.020, and n=m=10. Since these cases account for 57 or the 81 experimental runs, it is not surprising that this chart does not yield the correct number of OOCs.

It is important to note that the performance of all charts based on the CLI methods is very poor. Taking into account OOCs on both the lower and upper limits, the X-Bar Charts under both CLI Methods yielded about 350 OOCs on average (130 times the expected). The results for both CLI S_t charts were similar. More than 250 OOCs occurred on average, or almost 100 times the expected number. Results for the CLI S_b charts were not as severe, where an average of 60 OOCs were reported; however, this result is still more than 20 times the expected number. In all cases, performance is severely impacted as the number of measurements per board and the between-board variation increase relative to the other factors. These charts are frequently used in the industry and should be replaced immediately.

It is also interesting to note that the performance of the upper control limit for the X-Bar Chart based on the COV approach was sensitive

TABLE 7. *Summary of the regression statistics for the all 9 control charts evaluated for producer's risk (dashes represent coefficients that are insignificant at $\alpha = 0.05$).*

Chart type	Estimation method	Control limit	Model statistics		Main effects					Interaction effects					
			p-value	R^2	intercept	meas *(n)*	brds *(m)*	σ_b	σ_w	$n*m$	$n*\sigma_b$	$m*\sigma_b$	$n*\sigma_w$	$m*\sigma_w$	$\sigma_w*\sigma_b$
	CLI #1	Lower	<0.0001	89.3%	150.45	63.25	–9.33	71.26	–61.52	—	23.56	—	–15.49	—	24.59
		Upper	<0.0001	89.0%	196.79	79.09	10.90	78.16	–65.14	—	18.02	—	–11.94	—	40.70
X-Bar	CLI #2	L	<0.0001	89.4%	152.96	62.83	–11.25	72.80	–63.01	—	23.19	—	–15.10	—	24.56
		U	<0.0001	89.0%	199.29	78.70	8.93	79.75	–66.36	—	17.81	—	–11.49	—	40.80
	COV	L	0.0108	9.9%	1.00		–0.28	—	—	—	—	0.22	—	—	—
		U	<0.0001	22.7%	2.15	—	0.41	–0.43	0.40	—	—	–0.44	—	—	–0.43
	CLI #1	L	<0.0001	90.5%	157.79	71.76	–20.45	99.70	–104.31	9.20	35.42	–13.20	–29.32	13.01	—
		U	<0.0001	90.7%	103.06	45.90	15.44	59.92	–60.02	–7.35	23.28	9.57	–17.49	–7.00	—
S_t	CLI #1	L	<0.0001	90.9%	130.44	60.85	–7.31	83.93	–90.36	—	31.25	—	–27.56	—	–10.62
	CLI #2	U	<0.0001	90.7%	126.70	54.43	7.18	73.63	–73.00	—	26.54	—	–19.39	—	—
	CLI	L	<0.0001	76.4%	40.36	–26.75	–7.52	–56.04	42.01	—	37.19	9.54	–24.15	—	–59.65
		U	<0.0001	69.1%	19.77	–16.34	—	–26.07	20.69	—	23.40	—	–17.27	—	–37.97
	Satterthwaite	L	<0.0001	66.6%	23.57	–19.80	–13.73	–32.68	25.98	—	28.69	19.62	–21 46	–13.45	–30.17
		U	0.0040	11.0%	1.40	—	–0.24	0.39	–0.25	—	—	—	—	—	—
S_b	COV	L	0.0029	11.3%	1.02	—	—	0.22	–0.25	—	—	—	—	—	0.22
		U	0.0613	7.7%	1.37	—	—	—	—	0.24	—	—	—	0.21	—
S_w	CLI	L	0.0002	13.8%	0.72	—	–0.20	—	—	0.18	—	—	—	—	0.32
		U	<0.0001	15.2%	2.12	0.58	0.33	—	—	—	—	—	0.28	—	—

to the relative ratio of the between- and within-board variation. However, the magnitude of the impact was within the expected range—even when σ_b^2 is at the high level and σ_w^2 is at the low level, the resulting number of OOCs is still below the expected level of 2.7 OOCs.

Evaluation for consumer's risk

The Monte Carlo simulation procedure was again executed for 1000 sample points using the control limits in Table 5. However, since the X-Bar, S_b, and S_t charts based on the CLI approaches failed to perform properly on producer's risk, their sensitivity regarding consumer's risk would have no meaning. Therefore, consumer's risk was evaluated only for the 2 charts based on the COV, the S_w chart, and the S_b chart derived from the Satterthwaite method.

The results of the COV based X-Bar Chart are shown in Table 8. Note that the model is highly significant (by the F test), and the high R^2 indicates that the experimental factors account for 90% of the variation in OOCs. Only three coefficients are significant at the 0.05 level: the intercept, and the coefficients for m and μ This indicates that the chart behaves as expected. As m increases, the chart will have more statistical power to detect a shift in μ. As μ increases, the chart responds by indicating 265.77 more OOCs for every factor level increase (0.005″) in the mean.

The results of the COV-based S_b chart are shown in Table 9. The model is highly significant, but the R^2 is smaller than in the previous chart, indicating that less of the variation in the OOCs can be explained by the experimental factors. Seven coefficients are significant at the 0.05 level, which is indicative of the complex behavior of the COV Chart. Of importance is the interaction term between σ_b^2 and σ_w^2. This interaction is significant because the COV measures the relative ratio of between-board variation to total variation. There-

TABLE 8. *Regression statistics for the Upper Bound of the X-Bar Chart using the COV Method for consumer's risk.*

Regression statistics

Multiple R	0.951
R square	0.904
Standard error of estimate	92.500
Observations	243.000

ANOVA table

	df	SS	MS	F	Significance
Regression	11	18,580,723.2	1,689,156.7	197.4	<0.0001
Residual	231	1,976,516.7	8,556.3		
Total	242	20,557,239.9			

Model coefficients

Factor		Model coefficients	Standard error	*t* Stat	*p*-value
β_0	*Intercept*	432.75	6.68	64.83	<0.0001
β_1	*n* (meas)	9.93	8.13	1.22	0.2230
β_2	*m* (brds)	204.31	8.13	25.14	<0.0001
β_3	μ	265.77	7.27	36.57	<0.0001
β_4	σ_b	8.60	6.68	1.29	0.1990
β_5	σ_w	2.52	6.68	0.38	0.7066
β_{12}	$n * m$	–1.70	8.90	–0.19	0.8484
β_{14}	$n * \sigma_b$	–0.08	7.71	–0.01	0.9919
β_{24}	$m * \sigma_b$	–12.66	7.71	–1.64	0.1018
β_{15}	$n * \sigma_w$	–0.56	7.71	–0.07	0.9417
β_{25}	$m * \sigma_w$	–1.55	7.71	–0.20	0.8407
β_{45}	$\sigma_w * \sigma_b$	–0.57	6.68	–0.09	0.9318

TABLE 9. *Regression statistics for the Upper Bound of the COV based S_b Chart for consumer's risk.*

Regression statistics	
Multiple R	0.720
R square	0.518
Standard error of estimate	99.300
Observations	243.000

ANOVA table

	df	SS	MS	F	Significance
Regression	11	2,453,712.5	223,064.8	22.6	<0.0001
Residual	231	2,279,904.0	9,869.7		
Total	242	4,733,616.5			

Model coefficients

Factor		Model coefficients	Standard error	*t* Stat	*p*-value
β_0	*Intercept*	58.38	7.17	8.14	<0.0001
β_1	*n* (meas)	7.76	8.73	0.89	0.3748
β_2	*m* (brds	45.22	8.73	5.18	<0.0001
β_3	μ	0.54	7.81	0.07	0.9446
β_4	σ_b	48.95	7.17	6.83	<0.0001
β_5	σ_w	–57.81	7.17	–8.06	<0.0001
β_{12}	$n * m$	–4.69	9.56	–0.49	0.6245
β_{14}	$n * \sigma_b$	7.21	8.28	0.87	0.3845
β_{24}	$m * \sigma_b$	45.74	8.28	5.52	<0.0001
β_{15}	$n * \sigma_w$	–5.36	8.28	–0.65	0.5183
β_{25}	$m * \sigma_b$	–30.11	8.28	–3.64	0.0003
β_{45}	$\sigma_w * \sigma_b$	–48.41	7.17	–6.75	<0.0001

fore, it is sensitive to a change in the factors (together or in isolation of the others) that change this ratio. Thus, this chart must be interpreted with care. Less out-of-controls result if σ_b^2 increases while σ_w^2 stays the same, but more out-of-controls result in the opposite situation.

The chart is also highly sensitive to changes in the number of boards per sample (*m*). The chart's power to detect OOCs increases with an increasing *m,* when all other factors are held constant. This is mitigated, however, by increases in σ_b^2 and decreases in σ_w^2 through the interaction terms. Practitioners should understand fully the behavior of this chart before designing a sampling plan and implementing the chart.

The results for the S_w Chart are shown in Table 10. The model is significant, but the R^2 is again smaller than in the previous chart. As expected, the coefficients associated with *n, m,* σ_w^2, and their interactions are significant. The chart's power to discriminate increases with increasing *n* and *m,* only when σ_w^2 is held constant. Increases in σ_w^2 cause more alarm indications only when *n* and *m* are constant. This chart is very sensitive, detecting even the smallest changes in σ_w^2, because its degrees of freedom are a function of $m * n$. Even when σ_w^2 is at its lowest level (and all other factors are at the medium level), the chart still sent an out of control signal 979 out of 1000 times. Practitioners should take note of this high number of OOCs when using this chart.

The results for the S_b Chart using the Satterthwaite method are shown in Table 11. The relationship between the number of OOCs and the factors is significant, and the R^2 indicates that the factors explain 69% of the variation in the OOCs. This chart is very stable, as none of the interaction effects are significant at the 0.05 level. Increases in *m* and σ_b^2 increase the chart's discriminating power. Note that changes in σ_w^2

TABLE 10. *Regression statistics for the Upper Bound of the S_w Chart for consumer's risk.*

Regression statistics	
Multiple R	0.542
R square	0.294
Standard error of estimate	64.400
Observations	243.000

ANOVA table

	df	SS	MS	F	Significance
Regression	11	398,932.3	36,266.6	8.7	<0.0001
Residual	231	959,000.3	4,151.5		
Total	242	1,357,932.6			

Model coefficients

Factor		Model coefficients	Standard error	t Stat	p-value
β_0	*Intercept*	988.43	4.65	212.56	<0.0001
β_1	n (meas)	17.24	5.66	3.05	0.0026
β_2	m (brds)	17.07	5.66	3.02	0.0028
β_3	μ	0.12	5.06	0.02	0.9815
β_4	σ_b	–0.25	4.65	–0.05	0.9572
β_5	σ_w	11.56	4.65	2.49	0.0136
β_{12}	$n * m$	33.49	6.20	5.40	<0.0001
β_{14}	$n * \sigma_b$	0.50	5.37	0.09	0.9266
β_{24}	$m * \sigma_b$	0.50	5.37	0.09	0.9259
β_{15}	$n * \sigma_w$	–17.06	5.37	–3.18	0.0017
β_{25}	$m * \sigma_w$	–16.89	5.37	–3.15	0.0019
β_{45}	$\sigma_w * \sigma_b$	0.26	4.65	0.06	0.9556

do not effect the OOC signal rate as in the case of the COV based S_b Chart.

CONCLUSIONS

This study has developed a method to study the performance of lumber size control systems based on stochastic simulation procedures using experimental design to analyze the results. The method was used to investigate two SPC methodologies commonly found in the lumber size control literature versus those developed in the general SPC literature. The COV method performed as expected under the Central Limit Theorem, providing an accurate assessment of the Type I Error. Using the methods commonly used in lumber size control, however, resulted in a higher than expected number of Type I Errors or an increased producer's risk. In our experience, these estimation methods are the ones commonly being used in industrial systems. Since these charts are especially sensitive to increases in the number of boards and number of measurements per board, this is of particular concern for real-time applications. Systems developers in particular should take note of these results.

The simulation approach was also used to study the behavior of the COV charts with respect to consumer's risk. The charts performed as expected, with more out-of-controls when the parameter being monitored shifted. The charts also responded to changes in sampling rates, with increased sensitivity for higher sampling rates. These charts not only validate the COV method, but give valuable information on the impact of parameter shifts on control charts for typical sawmill conditions.

All of the results reported rely on data simulated from a normal distribution. Had a non-symmetric distribution been chosen, such as the log-normal, results would have been different. Given the same control chart derivations and the right-skewed log normal distribution, OOCs would have been more prevalent for the upper

TABLE 11. *Regression statistics for the Upper Bound of the S_b Chart using Satterthwaite Method for consumer's risk.*

Regression statistics	
Multiple R	0.831
R square	0.690
Standard error of estimate	194.400
Observations	243.000

ANOVA table

	df	SS	MS	F	Significance
Regression	11	19,426,967.6	1,766,088.0	46.7	<0.0001
Residual	231	8,728,183.8	37,784.3		
Total	242	28,155,151.4			

Model coefficients

Factor		Model coefficients	Standard error	t Stat	p-value
β_0	*Intercept*	563.45	14.03	40.17	<0.0001
β_1	n (meas)	23.25	17.07	1.36	0.1747
β_2	m (brds)	256.92	17.07	15.05	<0.0001
β_3	μ	–0.95	15.27	–0.06	0.9504
β_4	σ_b	208.00	14.03	14.83	<0.0001
β_5	σ_w	0.86	14.03	0.06	0.9510
β_{12}	$n * m$	4.20	18.70	0.22	0.8224
β_{14}	$n * \sigma_b$	–0.02	16.20	0.00	0.9991
β_{24}	$m * \sigma_b$	31.43	16.20	1.94	0.0536
β_{15}	$n * \sigma_w$	0.47	16.20	0.03	0.9770
β_{25}	$m * \sigma_w$	–0.51	16.20	–0.03	0.9747
β_{45}	$\sigma_w * \sigma_b$	0.05	14.03	0.00	0.9970

control limit and less prevalent for the lower. This may well approximate some wood products SPC applications, such as kiln-drying where moisture content is known to be distributed log-normally. Using different distrbutional assumptions would also necessitate different derivations of limits for the COV charts as the underlying ANOVA theory relies on normally distributed data.

This paper set out to accomplish three objectives. First, we developed a method using computer simulation in a designed experiment to estimate producer's and consumer's risk in SPC charts using different distribution assumptions. We found that the simulation method can identify problem situations quickly and give useful insight into the nature of the problem. Second, we used our reliability testing method to analyze commonly used SPC methods in the lumber industry and a new proposed method called the COV approach. We found that the COV approach was stable, predictable, and reliable over a wide variety of testing situations. The common approach used in the industry performed poorly with respect to both consumer's and producer's risk. Finally, we propose the COV approach as a superior method for real-time situations.

There is a great deal of research that remains to be conducted in this growing field of study. The scanning techology used in the field is rapidly evolving and it is expected that three-dimesional approaches will replace existing methods based on lumber thicknesses and widths.

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