EFFECT OF SIZE ON TENSION PERPENDICULAR-TO-GRAIN STRENGTH OF DOUGLAS-FIR

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(Received 15 May 1974)

ABSTRACT

The strength of wood in tension perpendicular-to-grain has been studied by several authors and found to depend on specimen geometry. In this paper, the weakest-link concept has been applied to predict the relationship between specimen volume and load-carrying capacity for Douglas-fir specimens loaded in uniform tension perpendicular-to-grain. The theory allowed the prediction that logarithm of maximum strength should decrease linearly with logarithm of volume. Experimental data taken from the literature were used to evaluate the theoretical model and agreement was found to be high ($R^2 \geqslant 0.85$). Average strength of a unit volume is approximately 460 psi, whereas the predicted strength of a 10- × 10- × 20-inch specimen (2000 inches^3) is approximately 100 psi. The magnitude of the size effect may depend on the quality of material in the specimens, but certainly any rational development of working stresses for tension perpendicular-to-grain must consider effects of specimen (or structural component) size.

Additional keywords: Pseudotsuga menziesii, size effects, tension, Weibull distribution, strength, duration of load, glued-laminated beams, pitched-tapered beams, design of structures.

SYMBOLS

- B risk of rupture
- D beam depth
- F cumulative distribution function giving probability of failure
- f frequency distribution
- g_1, g_2 function of volume
- k shape parameter of Weibull distribution
- n an integer
- P probability indicator
- S cumulative distribution function giving probability of survival
- s size effect parameter
- V volume
- X a generalized strength value
- x stress parameter
- x_l a lower limit on strength (location parameter of Weibull distribution)
- x_o scale parameter of Weibull distribution
- β stress-volume coefficient
- Γ gamma function
- ψ a variable
- ξ stress distribution coefficient
- σ a generalized stress
- σ_o scale parameter
- ρ a parameter

INTRODUCTION

The relationship between structure load-carrying capacity and the size, shape, and stress distribution within members has been documented for many materials, including wood (Weibull 1939a,b; Pierce 1926; Tucker 1927, 1941; Frankel 1948; Epstein 1948; Bohannen 1966; Johnson 1971; Leicester 1973; Keenan and Selby 1973; Schniewind and Lyon 1973). The size, shape, and stress distribution effects observed in materials are a manifestation of material strength as defined classically. In the classical theory of strength, as embodied in the maximum-stress theory, for example, it is assumed that strength is controlled by a combination of stress components, with failure occurring when this generalized stress reaches a maximum value. This strength concept makes use of the mean strength obtained from a number of geometrically similar tests as the measure of material strength. The implications of natural variability observed in tests of similar specimens are often neglected and it is this variability that gives rise to various "statistical" effects that influence load-carrying capacity (Weibull 1952).

The normal scatter in material properties has been attributed to a natural statistical distribution of flaws characteristic of the material, with strength being controlled by the size of the critical flaw in the critically loaded volume. Since the flaw distribution is a material characteristic, the resulting measured strength values will exhibit a characteristic statistical variation. Strength, which cannot be completely represented by the single average value normally presented, can be more completely characterized by a cumulative distribution function. Effects of size, shape, and stress distribution on ultimate load-carrying capacity have been studied in detail by several authors (Weibull 1939a,b; Tucker 1927, 1941; Epstein 1948; Bohannen 1966). In addition, a vast amount of statistical literature exists that is directly applicable to the study of size effects. Recently, Johnson (1971) published a detailed discussion of the theoretical statistical foundations of size-effects theories as part of a dissertation on concepts of size, safety, and economical structural design. The economical design of a structure was shown to be dependent on the distribution functions of strength and applied loads, as well as the costs of the structure and anticipated costs of failure and personal injury. In this paper only the effects of material variability on strength will be reviewed. The statistical concepts employed by Weibull (1939a,b) will be used to quantify size effects in tension perpendicular-to-grain for Douglas-fir [*Pseudotsuga menziesii* (Mirb.) Franco].

Motivation for the study of size effects presented in this paper was based on the need to develop fundamental strength data for use in design of curved beams and, in particular, pitched-tapered beams. The purpose of this paper is to (1) present the experimental information that suggests a size effect exists in tension tests perpendicular to grain, and (2) present a theoretical model consistent with the experimental observations.

EFFECT OF SIZE, SHAPE, AND STRESS DISTRIBUTION ON STRENGTH

Theoretical concepts

To simplify the discussion, it is assumed that all variability in load-carrying capacity is due to natural material variability. Johnson (1971) has discussed in more general terms the effects of statistical variation of loads and strength. A complete evaluation of risk of failure would necessarily require a thorough knowledge of statistical variation of load quantities, but in the following sections loads will be considered deterministic.

Weibull (1939a) presented the first theories capable of quantifying effects of stress distribution and volume on strength of materials. The weakest-link concept, previously used by Pierce (1926) and Tucker (1927), was fundamental to the development. By considering the strength of a material to be analogous to the strength of a chain, Weibull showed how strength of rods would be a function of length as follows:

Assume that strengths of specimens of unit (or elementary) volume are represented by a cumulative distribution function. The distribution function of strength is denoted by $F(x)$, where $F(x) = P(X \leqslant x)$ and x and X are the generalized stress and strength components. The frequency distribution $f(x)$, is obtained from the cumulative distribution according to $f(x) = dF(x)/dx$. Given this definition of strength, we want to predict the behavior of a structure containing n unit (or elementary) volumes.

At this stage it is necessary to make some assumptions about material fracture. For these purposes Johnson (1971) considered three material types:

1. Perfectly brittle—materials in which total failure occurs when fracture occurs at the weakest point;
2. Perfectly plastic—failure by ductile yielding; failure occurs when maximum load capacity of any cross section is exceeded;
3. Brittle materials that do not follow

the weakest-link concept; maximum load-bearing capacity does not necessarily coincide with fracture of weakest element.

Weibull (1939a,b) developed the theory to explain effects of volume and stress distribution on strength of perfectly brittle materials. Using the chain analogy, the effect of the number of elementary volumes can be calculated using the cumulative distribution function for the elementary volumes. F(x) gives the probability that strength is less than or equal to x; therefore $1 - F(x)$ gives the probability of strength being greater than x. The probability that a chain of n links will have strength greater than or equal to x is given by:

$$1 - F_n(x) = [1 - F(x)]^n = S_n \qquad (1)$$

where F_n is the probability distribution for chains of n links, and S_n is the survival probability.

Taking logarithms:

$$\ln S_n = B = n \ln[1 - F(x)]. \qquad (2)$$

If the elementary volumes have unit volume, then the cumulative distribution function has the form $F_v = 1 - \exp(B)$ where $B = V ln\ [1 - F(x)]$. Generally the stress distribution within the body varies with position and the contribution to B from each elementary volume dV is given by $dB = n(x)dv$ where $n(x) = ln\ [1 - F(x)]$. Therefore dB is a function of position and the total value of B is given by:

$$B = \int n(x)dv \qquad (3)$$

Therefore, we observe that the cumulative distribution function for materials that follow the weakest-link concept is of the exponential type. To this point we have considered the effect of number of elements on strength when elements are assumed to act in series. In this case, it is obvious that strength of the weakest element controls strength of the structure. Weibull also argued that the same weakest-link model applied for elements in parallel. Certainly it is possible to conceive of structures where this is not the case—for example, structures composed of parallel strands where a single-component failure redistributes load to the remaining components. Such materials would not fall in the class of perfectly brittle materials.

Pierce (1926), one of the first scientists to study the statistical nature of strength and size effects, recognized the relationship between size effects and the general statistical problem of determining the distribution of extreme values of a sample taken from a parent distribution. Johnson (1971) discussed the distribution of extreme values in detail as they related to size, shape, and stress distribution effects. Using very general arguments, it is possible to show that three types of exponential distribution are possible, which are called the Type I, II, and III extreme-value distributions. The Type III distribution function is given by (Gumbel 1958)[1]

$$P(X \le x) = F(x) = 1 - \exp[-(\rho(x - x_l))^k] \qquad (4)$$

where ρ is a scale factor and x_l an arbitrary lower limit of possible values. This distribution is identical to that chosen by Weibull (1939a). It appears that Weibull's choice of the form of B (Eq. 2) was purely expedient, in that it provided a mathematically simple distribution function. The specialization of B according to

$$B = \int n(x)dv = \int[(x - x_l)/x_0]^k\, dv \qquad (5)$$

produces a cumulative distribution function of general form:

$$F(x) = 1 - \exp[-\int((x - x_l)/x_0)^k\, dv] . \qquad (6)$$

If we assume a uniformly loaded volume, then

$$F(x) = 1 - \exp[-((x - x_l)/x_0)^k\, V] \qquad (7)$$

which is identical in form to the Type II

[1] This cumulative distribution function has been defined as Type II by Johnson (1971) which appears to contradict conventional notation.

extreme-value distribution. The coefficient of variation (cv) is given by (Johnson 1971)

$$cv = \frac{[(\Gamma(1+2/k)/(\Gamma(1+1/k))^2)-1]^{1/2}}{[(x_\ell V^{1/k}/m_1 - x_\ell)+1]} \quad (8)$$

where m_1 = mean strength at a reference volume V_1 and $\Gamma(x)$ is the gamma function.

Therefore, when $x_\ell > 0$ the coefficient of variation decreases as V increases; however cv remains constant if $x_\ell = 0$ (i.e. two-parameter Weibull).

Using the concepts presented above, Weibull was able to show effects of volume and stress distribution on strength. The previous applications of size-effects theory to studies of wood strength have relied on applications of the two-parameter Weibull distribution (Bohannen 1966; Leicester 1973). This choice appears rational, particularly for studying tensile strength perpendicular-to-grain and additionally affords benefits by reducing the complexity of the analysis.

Effect of volume variation

To evaluate the predicted change in strength with volume in geometrically similar specimens with similar loading, Eq. 7 is employed. Assume that the cumulative distribution function, F(x), has been determined for a specimen of volume V_1. Then according to Eq. 7 (assuming $x_\ell = 0$).

$$F(x) = 1 - \exp[(-x/x_0)^k V_1] \quad (9)$$

and the values of k and x_0 are obtained by fitting the experimental data.

The strength of specimens of volume V_2 is obtained by considering any fixed value of $F(x) = 0.5$; then

$$\exp[-(x_1/x_0)^k V_1] = \exp[-(x_2/x_0)^k V_2] \quad (10)$$

and

$$x_1^k V_1 = x_2^k V_2 . \quad (11)$$

Therefore

$$x_2 = x_1(V_1/V_2)^{1/k} . \quad (12)$$

Combined volume and stress distribution variation

Effects of different stress distributions and volumes in specimens can be assessed by generalizing the method developed to study volume effects alone. The integral to be evaluated in all cases is (from Eq. 6):

$$\int (x/x_0)^k \, dv \quad (13)$$

which has, in the general case, a value that can be expressed in the form

$$\int_V (x/x_0)^k \, dv = (x_{max}/x_0)^k \psi V \quad (14)$$

where V = specimen volume, ψ a constant depending on the stress distribution and the shape parameter (for a uniform stress distribution $\psi = 1$). From Eq. 1, define a survival probability S and using Eq. 14:

$$S = 1 - F(x) = \exp[-(x_{max}/x_0)^k \psi V] . \quad (15)$$

Evaluation of Eq. 14 for simple bending yields

$$\int (x/x_0)^k \, dv = (\sigma_B/\sigma_0)^k V_B/2(k+1)^2 \quad (16)$$

where σ_B = maximum bending stress, V_B = volume of bending specimen, σ_0 and k = scale and shape parameters, respectively.

The strength of an equivalently loaded volume in pure tension can be calculated according to:

$$\int (x/x_0)^k \, dv = (\sigma_T/\sigma_0)^k V_T = (\sigma_B/\sigma_0)^k V_B/2(k+1)^2 . \quad (17)$$

Therefore

$$\sigma_T/\sigma_B = [V_B/(2 V_T(k+1)^2)]^{1/k} \quad (18)$$

and in general the effects of stress and volume differences are given by:

$$\sigma_1/\sigma_2 = [(\int g_2(V)^k \, dv)/(\int g_1(V)^k \, dv)]^{1/k} . \quad (19)$$

It can be shown that the combined effect of differences in stress distribution and volume can be reduced to a change in equivalent volume. Comparing the predicted strengths of simple bending and tension specimens in which ultimate capacity is controlled by tensile strength shows that for the same specimen the strength in uniform tension would be less than in simple bending, according to:

$$\sigma_T = \sigma_B[1/2(k + 1)^2]^{1/k} . \quad (20)$$

Or equivalently, to have a tensile strength equal to the bending strength, the volume of the specimens must be related by:

$$V_T = V_B/2(k + 1)^2 . \quad (21)$$

It will also be important for subsequent analysis and discussion to observe that if we assume a weakest-link model as an hypothesis, then the theory predicts that the relationship between strength and volume for geometrically similar specimens, similarly loaded, should be linear on a log–log plot. From Eq. 15.

$$S = \exp[-(x_{max}/x_o)^k \psi V] \quad (22)$$

and therefore

$$\ln(1/S) = (x_{max}/x_o)^k \psi V . \quad (23)$$

Accordingly

$$\log x_{max} = a - (1/k) \log V \quad (24)$$

where

$$a = (1/k) \log [\ln(1/S)] + \log x_o - (1/k) \log \psi . \quad (25)$$

The applicability of the weakest-link hypothesis can be assessed by studying the relationship between logarithm of failure stress and logarithm of specimen volume for geometrically similar structures with similar stress distributions. The slope of the relationship should be the inverse of the shape parameter. Therefore, working with a strength hypothesis, we are provided with information about the theoretical relationship between variables that otherwise could be deduced only by trial.

APPLICATIONS OF THE WEAKEST-LINK MODEL

Although the distribution presented by Weibull has found a great variety of applications, it does not appear to have been widely applied in studying wood mechanical behavior. In fact, the principal application appears in the study of size effects in bending by Bohannen (1966), who found that changes in strength of clear beams produced by changes in span and depth could be accounted for satisfactorily by a weakest-link model.

TENSILE STRENGTH PERPENDICULAR-TO-GRAIN

Design of structures that results in development of tensile stresses perpendicular-to-the-grain generally should be avoided (DIN 1969; Gower 1974). Tensile strength perpendicular-to-grain in all structural species is low, usually less than 1000 psi, even for small clear specimens. Additionally, the radial and tangential planes are natural cleavage planes in which natural cracks (checks) often develop because of initial drying or subsequent moisture content and temperature changes. Checks, once present, can propagate because of changing environments. Tensile strength perpendicular-to-grain characteristically exhibits a high variability, a fact recognized by design code requirements that mean tensile strength perpendicular-to-grain be reduced by a larger percentage than any other strength property in the calculation of allowable stress.

Significant tensile stresses perpendicular-to-grain can develop in curved beams, connections, and any structural element where applied loads are at an angle to the grain. In particular, the development of adequate radial tensile strength in pitched-tapered beams has received considerable attention recently. In general, the performance of pitched-tapered beams has been adequate, but a few beams have developed radial tension cracks in service at loads considerably below design-load levels. In the search for an explanation of these failures, a more exact linear-elastic stress analysis was developed (Foschi and Fox 1970) and experimentally verified (Fox 1970; Foschi 1971). More accurate stress analysis showed that the previously used design formula (Wilson 1939) could significantly underestimate the maximum radial stress in pitched-tapered beams, although the validity of the Wilson formula was verified for curved beams of constant depth. Independently, Thut (1970) and Gopu et al. (1972) have verified the findings of Foschi and Fox, but unfortunately, even using the improved stress analysis, some in-service failures cannot be explained. In addition to accurate stress analysis, successful design relies on a reliable knowledge of the ultimate load-carrying capacity of the material. If it is assumed that currently available stress analyses are adequately accurate, then it is logical to assume that our knowledge of ultimate strength is deficient. This conclusion is supported by the anomalies that exist in the literature.

At the present time in Canada, the allowable tensile stress perpendicular-to-grain is 65 psi for dry service conditions and normal load duration in glued-laminated Douglas-fir (CSA 1970). The maximum radial tensile stress allowed by the American Institute of Timber Construction for unreinforced members is currently 15 psi (AITC 1972). These allowable stresses are normally derived from tests of small, clear, green specimens according to ASTM procedures (ASTM 1971). The average strengths obtained from the tests must be modified to account for material variability, moisture content, duration of load, grade, and a factor of safety. The allowable tensile stress perpendicular-to-the-grain of 65 psi has been modified accordingly from an average air-dry strength of 444 psi for ASTM specimens (Kennedy 1965). This allowable stress is used for all curved and pitched-tapered beam designs independent of beam geometry and loading. Presumably, it has been tacitly assumed that the actual factor of safety against ultimate load is a constant, independent of other strength criteria. In subsequent sections, the effect of specimen volume on the tensile strength of clear and glued-laminated uniformly loaded specimens will be summarized. The applicable size-effect theories will be discussed and used to evaluate size effects in tension perpendicular-to-the-grain under static short-term loading conditions and constant environmental conditions.

SIZE EFFECTS IN TENSION PERPENDICULAR-TO-GRAIN

Markwardt and Youngquist (1956) reviewed the development of tensile test methods and presented some experimental information showing effects of specimen size and stress distribution on strength measured in tension. No attempt was made to explain differences in strength observed, however. Of particular interest to the present study is the information presented on test methods and results for tension perpendicular-to-grain. It appears, from the great number of different standard test specimens, that no universally accepted test method has been found. The results presented show that strengths obtained are specimen-dependent, which makes evaluation of material properties extremely difficult. For example, reducing the width of the ASTM specimen (Fig. 1a) from 2 inches to 1 inch was found to increase strength of Douglas-fir from 254 to 312 psi and from 395 to 398 psi for radial- and tangential-failure surfaces, respectively (Markwardt and Youngquist 1956).

Several authors have used uniformly loaded rectangular blocks (Fig. 1b) to study tensile strength of Douglas-fir (Fox 1974; Madsen 1972; Thut 1970; Schniewind and Lyon 1973; Peterson 1973). The major advantage of this type of specimen is that a more uniform distribution of stress is obtained, but problems are sometimes encountered in that failures may occur at or near the load-application points. Such failures also occur in necked specimens, however. Stieda[2] (1965) compared the strength of ASTM specimens with that obtained from

[2] Unpublished data, Western Forest Products Laboratory.

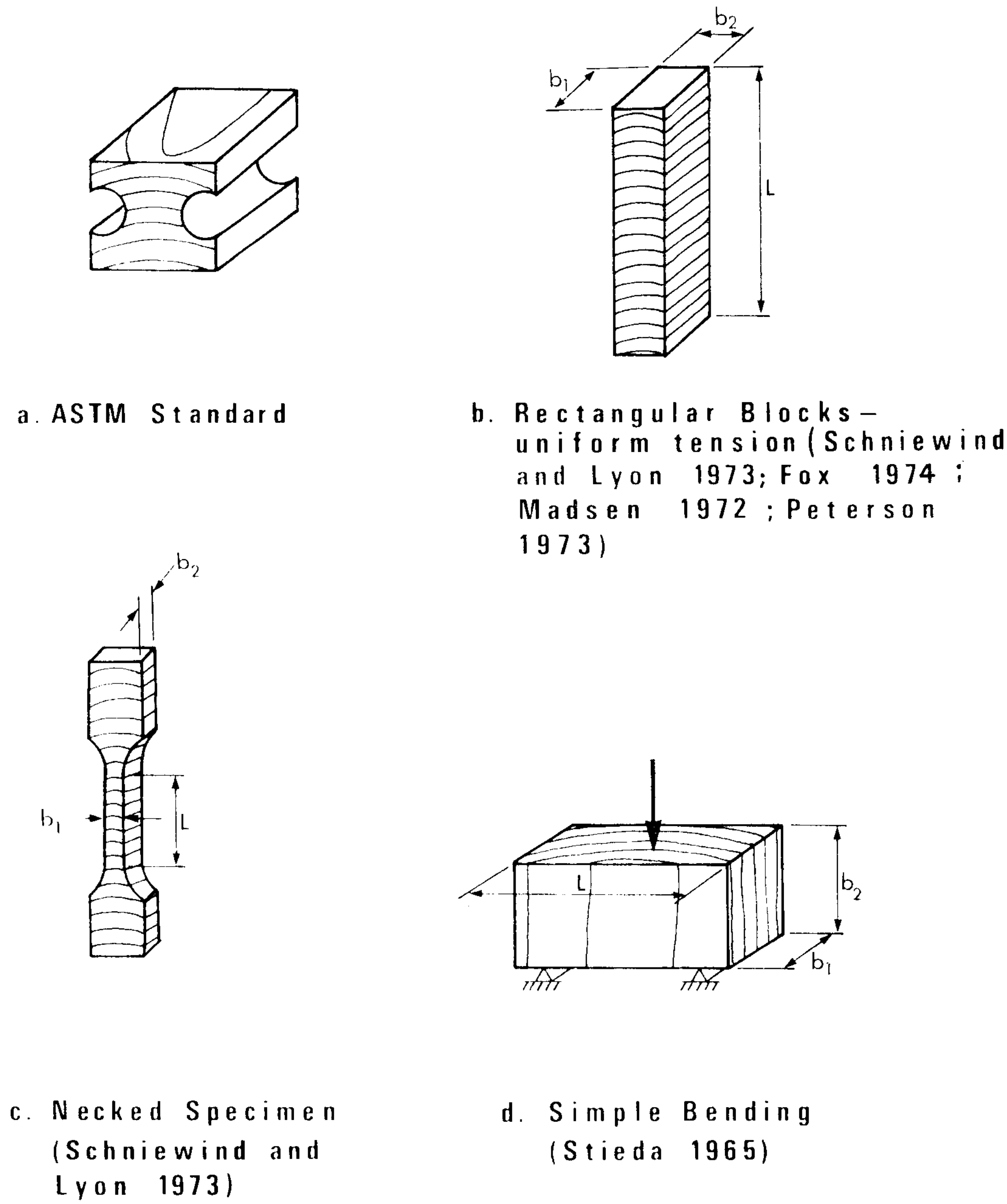

FIG. 1. Specimens used for measuring tensile strength perpendicular-to-the-grain cited in this study.

small, clear bending specimens of different sizes (Fig. 1d). Schniewind and Lyon (1973) also studied tensile strength of Douglas-fir using a small necked-down specimen (Fig. 1c) and a rectangular block. Although all authors recognized effects of specimen size, no attempts were made to relate the test results by applying size-effects theories.

Experimental studies of tensile strength perpendicular-to-the-grain in glued-laminated blocks studied by Thut (1970),

TABLE 1. *Tensile strength results for uniformly loaded glued-laminated Douglas-fir blocks of commercial material loaded perpendicular-to-the-grain*

Nominal* dimensions inches	mc %	n	Average volume inch³	Average strength psi	Standard deviation psi	Coefficient of variation	Source
4.5 x 4.5 x 13	12	22	263	128	42	.33	Thut (1970)
3 x 3 x 7	6-18	29	66	191	49	.26	Fox** (1974)
3 x 3 x 22		75	187	152	59	.39	
5 x 5 x 7		28	172	171	49	.29	
5 x 5 x 22		66	506	126	37	.30	
5.13 x 5.13 x 24	12	15	632	141	42	.30	Madsen (1972)
2.5 x 2.5 x 9	7-9	44	56	198	67	.34	Peterson (1973)

* All lamination thickness 1.5 inches except Peterson (1973) where thickness was 9/16 inch.

** Indicated as Fox (1974-1) on Fig. 2 and 3.

Madsen (1972), Peterson (1973), and Fox (1974) suggest that tensile strength of blocks is less than that obtained from ASTM specimens. Blocks studied were cut from glued-laminated beams of commercial material, except for one set of clear glued-laminated blocks studied by Madsen (1972). The dependence of strength on specimen geometry raised many questions, particularly for the assignment of allowable working stresses in tension perpendicular-to-grain for curved beams.

In a most detailed study of tensile strength perpendicular to grain, Fox (1974) used large glued-laminated blocks to study effects of testing speed, moisture content, specimen cross-section area, and length on strength of blocks obtained from four manufacturers. A statistical analysis showed that the effects of moisture content (6% vs. 18%) and testing speed (0.02 inch min^{-1} vs. 0.10 inch min^{-1}) were not significant. Changes in specimen length (7 inches vs. 22 inches) and cross section (3 inches × 3 inches vs. 5 inches × 5 inches) produced significant changes in strength. Results are summarized in Table 1 with results on uniformly loaded blocks obtained from the literature.

These data provided information that could be used to evaluate a size-effect theory. Since results of Fox (1974) showed that both the transverse and length dimensions of test specimens influenced strength, a volume effect was anticipated, and a plot of log strength vs. log volume for data of Table 1 appeared to support a hypothesis that a weakest-link failure mode was operative. On such a plot, the specimen volumes covered approximately one decade (56 $inches^3$ to 632 $inches^3$). Three additional tests were undertaken to extend the range of specimen sizes to cover approximately 2.5 decades (16 $inches^3$ to 3650 $inches^3$). The geometry of these additional specimens was chosen to test new cross-section areas and new lengths with volumes adjusted to provide the wide range of volumes required to evaluate the weakest-link hypothesis. As pointed out by Fox (1974), the material for the three additional tests was obtained and

TABLE 2. *Additional tensile test results for the glued-laminated Douglas-fir blocks Fox (1974)* used to assess weakest-link hypothesis*

Nominal dimensions inches	mc %	n	Average volume inch³	Average Strength inch³	Standard deviation psi	Coefficient of variation
2 x 2 x 4	12	30	16	355	100	0.28
2 x 2 x 20	12	30	80	181	71	0.39
10.75 x 10.75 x 34	12	22	3650	100	19	0.19

* Indicated as Fox (1974-2) on Fig. 2 and 3.

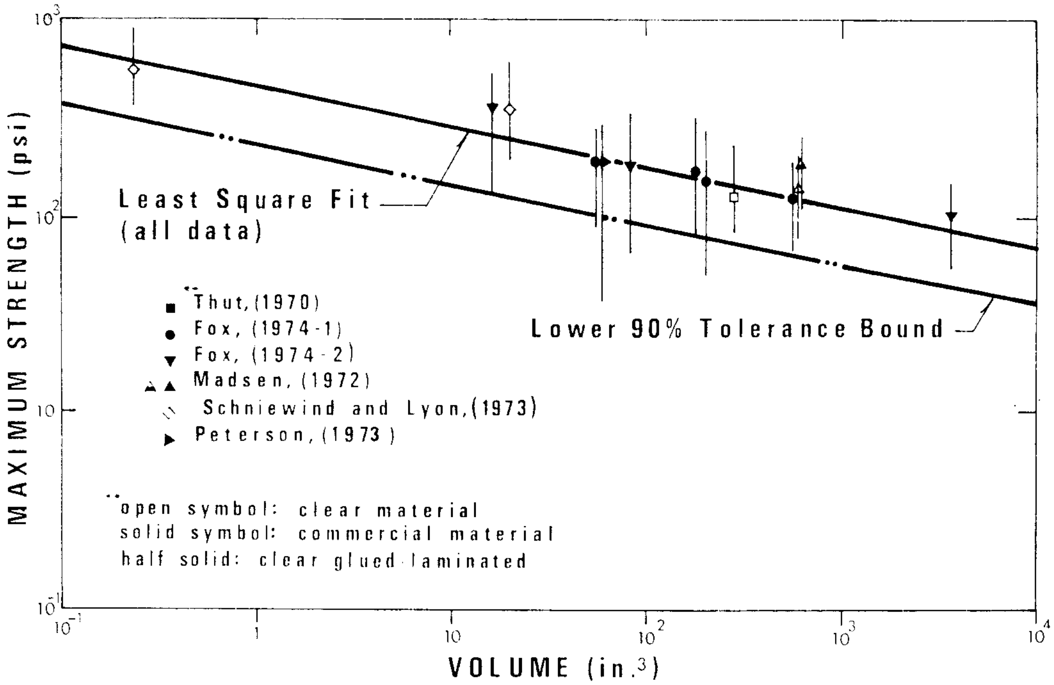

FIG. 2. Relationship between strength and volume for uniformly loaded Douglas-fir blocks.

tested approximately three years after the original experiment summarized in Table 1. Results for the three additional specimen sets are summarized in Table 2.

Tensile strength perpendicular-to-grain of small clear specimens is usually measured using ASTM specimens. However, Schniewind and Lyon (1973) measured tensile strength perpendicular-to-grain in Douglas-fir dimension lumber using a uniformly loaded block and a small necked-down specimen. Stieda (1965) studied the strength perpendicular-to-grain of small clear bending specimens in relation to ASTM tests. The results of these tests are summarized in Table 3.

TABLE 3. *Tensile strength results for clear Douglas-fir blocks loaded perpendicular-to-grain*

Specimen type	mc %	n	Nominal dimensions inches	Average volume inch^3	Average strength psi	Standard deviation psi	Coefficient of variation	Source
Uniform tension	12	11	5.13 x 5.13 x 24	632	191	40	0.21	Madsen (1972)
ASTM	12	24	-	-	317**	62	0.20	Stieda (1965)
Bending	12	23	1.5 x 1.5 x 2	4.5	911	111	0.12	
Bending	12	25	1.5 x 2 x 2	6	836	131	0.16	
Uniform tension	12	18	1.625 x 2 x 6	19.5	363	87	0.24	Schniewind & Lyon (1973)
Necked specimen	12	23	-	0.225*	564	123	0.21	
ASTM	12	374	-	-	443	154	0.35	WFPL (unpublished data)

* Assumed uniformly loaded volume was twice volume of the volume in the minimum cross section

** All failure surfaces in RL plane

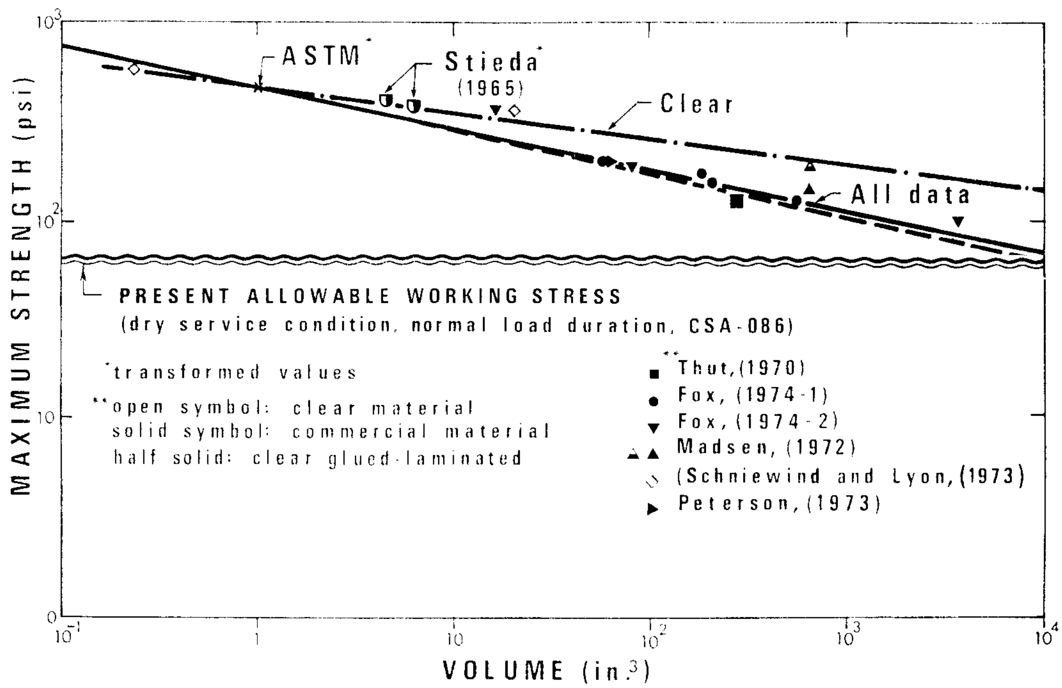

FIG. 3. Linear regression equations relating strength to volume for uniformly loaded blocks of commercial and clear Douglas-fir.

VERIFICATION OF THE WEAKEST-LINK MODEL

It is now possible to study the validity of the theoretical model using the results of tests on uniformly loaded specimens. The particular advantage of employing the model of behavior is that the theoretical analysis has shown an appropriate form for plotting of the data. Recall that Eq. 24 indicated that a log–log plot of stress vs. volume should be linear, if strength is controlled by the strength of the weakest volume element. All results cited are plotted accordingly in Fig. 2, with the range of experimental observations at specific volumes indicated by the vertical bars. The solid symbols represent data collected on glued-laminated blocks of commercial grades (1.5- and 0.56-inch lamination thickness, see Tables 1 and 2). The open symbols represent data derived from clear material and the partially solid

TABLE 4. *Summary of regression results obtained for clear and glued-laminated blocks loaded in uniform tension perpendicular-to-grain*

	Clear				Commercial				All data			
	a*	k*	R^2**	DF***	a	k	R^2	DF	a	k	R^2	DF
Individual data	2.673	7.68	0.76	50	2.675	4.584	0.35	359	2.656	4.884	0.54	411
Means only (no weighting)	2.676	7.369	0.97	1	2.659	4.857	0.83	8	2.656	5.182	0.85	11
Means only (weighting by sample size)	2.673	7.680	0.97	1	2.671	4.628	0.84	8	2.654	4.902	0.87	11

* Coefficients of regression eqn. $\log \sigma = a - \frac{1}{k} \log V$

** Coefficient of determination

*** Degrees of freedom

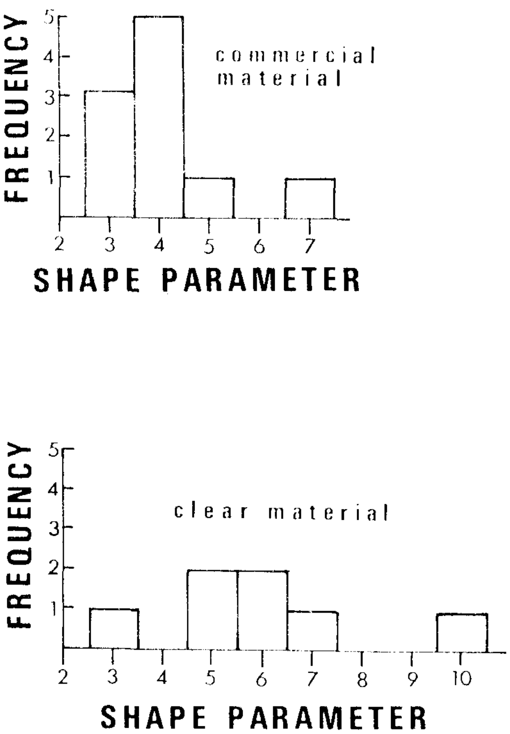

FIG. 4. Frequency distributions for computed shape parameters of the clear and commercial material.

symbols correspond to glued-laminated clear material. For the purposes of plotting, the effective uniformly loaded volume of the necked-down specimen (Schniewind and Lyon 1973) was chosen as 0.225 inch³. An analysis based on fitted-shape parameters will show this was an acceptable estimate.

Initially, linear-regression coefficients a and k of Eq. 24 were calculated using data from all tests, then the data for clear and commercial material separately. The results are presented as the first entry in Table 4. The coefficient of determination (R^2) gives a measure of goodness of fit. The R^2 values obtained using all data points are not high and would not support the hypothesis of a weakest-link strength concept with a high degree of confidence. The analysis does suggest that the clear material and commercial material behave differently with increases in volume. The reductions in strength with increasing volume in clear material do not appear to be as large as observed in the commercial material. It was then recognized that, as part of the development of the weakest-link theory, the variation in strength at a given volume was assumed to be explained, and consequently, a rational evaluation of the theory should be made by studying the change in mean strength with volume. Regression coefficients were then obtained for the three cases previously studied, by relating mean strength to mean volume first by using individual means and second by weighting the means by sample size. The results of these analyses are also presented in Table 4. The plotted regressions are shown in Fig. 3. As expected, the R^2 values improved and the lowest R^2 value is 0.83 for commercial material. The logarithm of strength of a unit volume predicted by the regression equation is essentially unchanged by method of analysis or material category ranging from 2.654 to 2.676. Weighting of the mean values improved the R^2 values and changed the shape parameters only slightly.

In order to plot the corrected ASTM strength values and the results of Stieda, values of the shape parameter are required. In the theoretical analysis it was shown that the value of the shape parameter obtained from individual cumulative-distribution functions should be the same as that obtained by fitting a linear-regression equation to the log-strength vs. log-volume plot. Cumulative-distribution functions were fitted to the data at individual volumes. For sample sizes of less than 50, the method of White (1969) without weights was used. For the larger sample sizes the methods of Miller and Freund (1965) and moments were used. Computed shape and scale parameters for the commercial and clear groups are presented in Table 5. Average shape parameters for clear material and commercial material were 6.35 and 4.04, respectively.

The frequency distributions for computed shape parameters for the clear and commercial material are given in Fig. 4. To test the significance of a given departure from a mean k value, a simulation was performed.

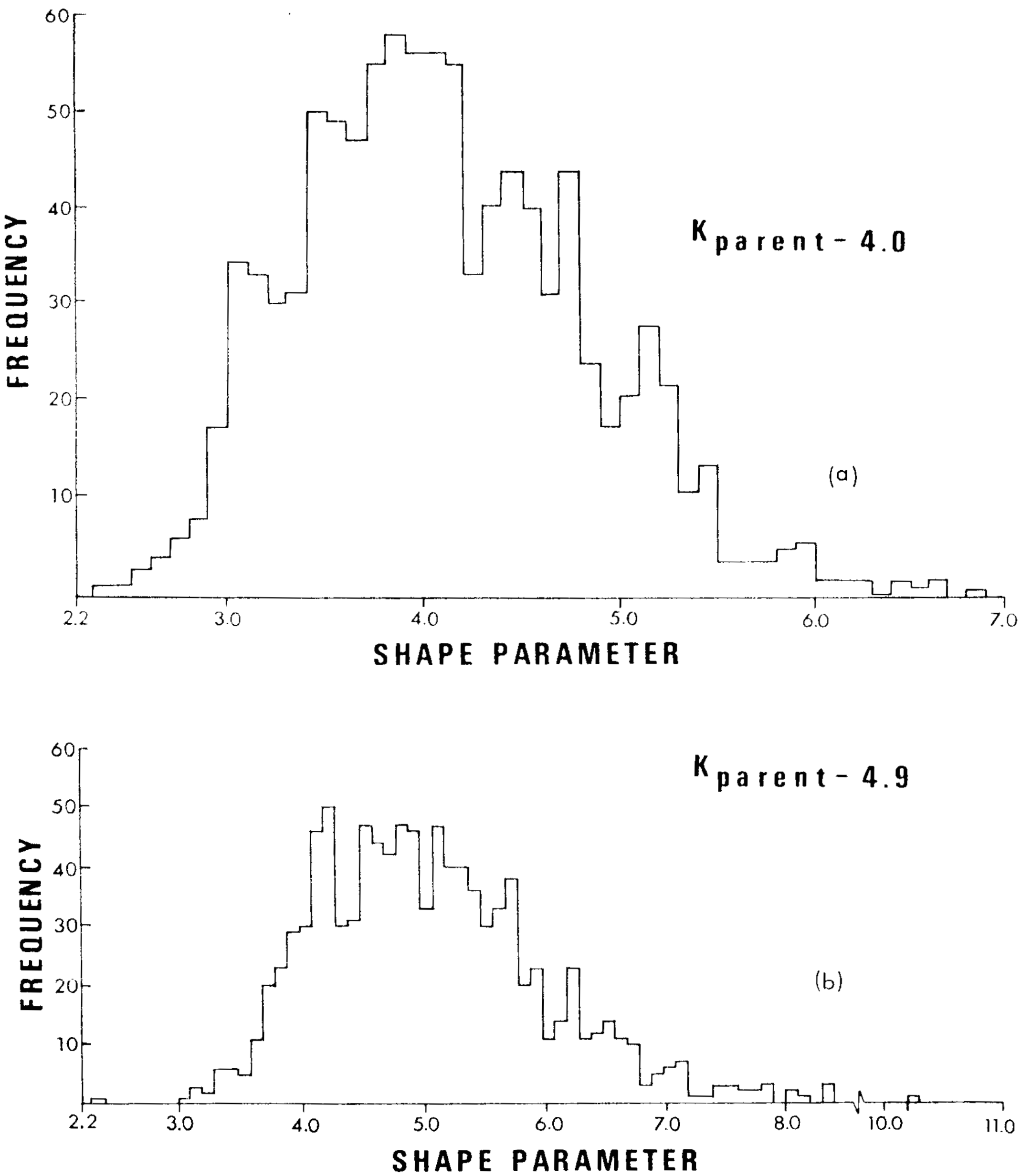

FIG. 5. Frequency distributions for k determined from a two-parameter Weibull distribution with specified shapes of (a) 4.0 and (b) 4.9 (sample size 30).

Samples of size 30 were selected at random from a known parent distribution. A two-parameter cumulative-distribution function was fitted and values of the estimated shape parameter k recorded. The frequency distributions for 1000 realizations of the simulation are presented in Fig. 5 for values of k = 4.0 and 4.9. The ranges of estimated-shape parameters for various intervals when the true values were 4.0 and 4.9 are given in Table 6. Based on the simulation, one could not reject the hypothesis that the shape

TABLE 5. *Parameters of cumulative distribution functions by specimen and source*

Specimen*	Average volume inch³	Sample size	Scale parameter psi	Shape parameter	Source
Commercial Material					
2 x 2 x 4	16	30	393.5	3.92	Fox (1974)
3 x 3 x 7	66	29	210.3	4.44	
2 x 2 x 20	80	30	202.6	2.91	
5 x 5 x 7	172	28	189.0	4.24	
3 x 3 x 22	187	75	171.6	2.74	
5 x 5 x 22	506	66	138.8	4.04	
10.75 x 10.75 x 30	3650	22	106.6	6.69	
4.5 x 4.5 x 13	263	22	138.0	4.84	Thut (1970)
5.13 x 5.13 x 24	632	15	157.4	3.72	Madsen (1972)
2.5 x 2.5 x 9	56	44	223.2	2.90	Peterson (1973)
			Mean	4.04**	
			s.d.	1.14	
Clear Material					
Necked specimen	0.225	23	606.1	6.21	Schniewind & Lyon (1973)
2 x 1.625 x 6	19.5	13	394.6	5.20	
1.5 x 1.5 x 2	4.5	23	956.2	10.45	Stieda (1965)
1.5 x 2 x 2	6	25	893.4	7.30	
ASTM	-	24	340.0	6.52	
5.13 x 5.13 x 24	-	11	208.9	5.18	Madsen (1972)
ASTM	-	374	489.8†	3.61	WFPL (unpublished)
			Mean	6.35**	
			s.d.	2.16	

* Nominal moisture content of all tests 12%

† Miller and Freund (1965)

** Mann-Whitney test between shape parameters of commercial and clear material attain the 2% level (2 tail)

parameters computed for individual tests were representative values that would be obtained from a parent distribution with the shape parameter equal to means given in Table 5. The shape parameters estimated by the average of individual test values (6.35 and 4.04) were lower than the least-square fit estimates determined by the plotting method, which were 7.7 and 4.7 for the clear and commercial groups.

For the purposes of subsequent analyses, it is important to determine whether a "best" estimate of the shape parameter for the commercial and the clear material can be obtained. The 95% confidence limits for the two estimates of the shape parameters were calculated using conventional techniques. The upper and lower bounds for the shape parameter $\hat{k}$ determined according to

$$\hat{k} = \frac{1}{n}\sum_{i=1}^{n} k_i \tag{26}$$

are

$$\hat{k} \pm t s_{\hat{k}} \tag{27}$$

where

$$s^2 = \sum_{i=1}^{n} (k_i - \hat{k})^2/n-1 \tag{28}$$

and

$$s_{\hat{k}} = \sqrt{s^2/n} \tag{29}$$

and t = the appropriate value of Student's t for $n-1$ degrees of freedom.

For the commercial material $3.19 \leqslant \hat{k} \leqslant 4.83$ and for the clear material $4.35 \leqslant \hat{k} \leqslant 8.35$. To compute the range of k estimated by the regression method, recall the regression coefficient $b = 1/k'$, where k' is the shape parameter estimated by regression methods. The 95% confidence limits for the slope b

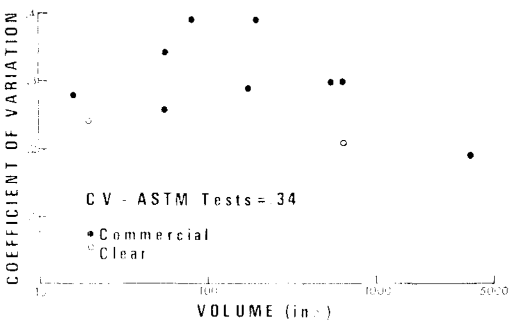

FIG. 6. Effect of volume on the coefficient of variation for uniformly loaded blocks.

can be computed according to

$$b \pm ts_b \tag{30}$$

where

$$s_b^2 = (s_{y \cdot x}^2)/\Sigma(x - \bar{x})^2 \tag{31}$$

where $s^2_{y \cdot x}$ = conditional variance of y given x, i.e. residual mean square after fit,

t = the appropriate value of Student's t for n – 2 degrees of freedom.

For the commercial material $-0.149 \leqslant b \leqslant -0.258$, or $3.87 \leqslant k' \leqslant 6.69$ and for the clear material $-0.437 \leqslant b \leqslant +0.177$, or $2.286 \leqslant k' \leqslant$ undefined. The upper bound on k was labelled undefined because it is predicted to be negative.

The confidence bounds predicted for $\dot{k}$ and k' for both the commercial and clear material have considerable overlap; therefore there does not appear to be any basis for accepting one particular estimate over another. Initially it was anticipated that the least-squares regression estimate (k') would be the "best," but the analysis has shown that the confidence limits are wider for k' than $\hat{k}$. However, for the purposes of subsequent analyses, the value of k determined from the regression of log strength vs. log volume by weighting mean specimen strengths by sample size for all data combined are used, i.e. k = 4.9.

Recall that Eq. 8 showed the relationship

TABLE 6. *Range of shape parameter corresponding to given central intervals obtained by simulation*

Central interval (%)	Range of shape parameter	
	true value 4.0	true value 4.9
50	3.57 - 4.60	4.38 - 5.63
90	3.02 - 5.39	3.78 - 6.76
95	2.94 - 5.77	3.61 - 7.18
99	2.59 - 6.47	3.20 - 8.19

between coefficient of variation and volume. If a weakest-link model was applicable, then the coefficient of variation should decrease if a nonzero lower limit on strength exists, or remain constant if $x_l = 0$. The experimentally determined coefficients of variation are plotted vs. volume in Fig. 6. These results suggest no distinct trend except that the coefficient of variation does not appear to be increasing with volume. Therefore, using this measure of fit the weakest-link hypothesis is supported.

Experimental results provided by Stieda, Schniewind and Lyon (1973), and the ASTM strength values can now be corrected for effects of the nonuniform stress distribution using the techniques presented earlier.

1. Bending tests

The tensile strength of a uniformly loaded volume $V_T = V_B$ can be computed from a bending test according to Eq. 20.

$$\sigma_T = \sigma_B[1/2(k+1)^2]^{1/k} \tag{32}$$

assuming k = 7.68

$$\sigma_T = 0.521\ \sigma_B.$$

Therefore the equivalent uniform tensile stresses are 474 and 435 psi for the 4.5

TABLE 7. *Effect of fitting technique on computed shape and scale parameters for 374 ASTM tension perpendicular-to-grain specimens*

Fitting technique	k	σ_0
Miller and Freund	3.6060	489.80
Moments	3.1508	494.85

inches^3 and 6 inches^3 specimens, respectively. These results are plotted in Fig. 3.

2. ASTM tests

The strength of an equivalent uniformly loaded unit volume is given by

$$\sigma_T = [\int_V \sigma^k dv]^{1/k} \qquad (33)$$

Because of the complex distribution of stresses perpendicular to the grain, the integration was performed numerically using stresses derived from a two-dimensional finite element analysis of the ASTM specimen. The results of the analysis can be expressed according to

$$\sigma_T = \beta\sigma_{max} \qquad (34)$$

where σ_{max} is the nominal stress at failure for the ASTM specimen and β is a parameter that incorporates the effects of the nonuniform stress distribution and volume change to unit volume. The dependence of β on the assumed shape parameter is given in Table 8. The transformed strength of the ASTM specimen computed using Eq. 3 is 467 psi for k = 7.68 and 482 psi for k = 4.63. These values agree closely with the strengths for a unit volume predicted from the regression equations for clear and commercial material, which were 470 psi and 460 psi, respectively.

3. Necked specimen

By replacing the curved portions at the ends of the specimen with straight lines joining the ends of the arcs, the following approximate analysis was obtained:

$$\sigma_T = \sigma(0.1538/V_T)^{1/k} . \qquad (35)$$

Transforming to a uniformly loaded volume of 0.225 inch^3, $\sigma_T = 0.95\ \sigma$ for $k = 7.68$. Therefore, the error in the assumed strength at a uniformly loaded volume of 0.225 inch^3 is less than 5%. For the purposes of this investigation, no additional refinement was considered necessary.

TABLE 8. *Factors used to correct for effects of volume and stress distribution in the ASTM tension block*

Shape parameter	β
3	1.1770
4	1.1046
5	1.0741
6	1.0602
7	1.0547
8	1.0533

DISCUSSION

On the basis of an analysis of tensile tests perpendicular-to-grain in Douglas-fir, a size effect has been identified and the weakest-link concept of failure has been applied to explain changes in mean strength with volume. The hypothesis that a weakest-link hypothesis applies was accepted on the basis of the high coefficients of determination ($R^2 \geqslant 0.85$) obtained by least-squares regressions relating log volume to log strength. The strength-volume relationships obtained suggest that, for specimens cut from commercial glued-laminated beams, the average tensile strength perpendicular-to-grain is reduced to the present allowable stress of 65 psi (dry service conditions and normal load duration; CSA 086) at a specimen volume of approximately 10,000 inches^3 (Fig. 3). From the limited data available, it appears that a reduction in strength with increasing volume for clear material may be considerably less. It is important to note that the reductions in strength observed experimentally were obtained in short-term tests under essentially constant environmental conditions. Effects of time and environmental change would be expected to further reduce these strength values and must be accounted for in the development of working stresses. Recent work by Madsen (1972) and Peterson (1973) suggests that the duration of load effect in tension perpendicular-to-the-grain may be considerably larger than previously anticipated.

It is important to realize that the weakest-link model provides a conservative estimate of the relationship between strength and volume, as it assumes that total failure occurs when the weakest element fails. There is no possibility of load transfer or load redistribution assumed, which may be possible in the real material. The analysis presented in this study suggests that, while the weakest-link assumption is conservative, it does in fact accurately model the behavior of wood loaded in tension perpendicular-to-grain for the range of specimen volumes studied.

The choice of a two-parameter Weibull distribution function (i.e. minimum strength assumed to be zero) simplified analysis. It also is considered acceptable, particularly for representing strength in tension perpendicular-to-grain. No minimum nonzero strength for wood, particularly in tension perpendicular-to-grain, can be justified theoretically without employing some selection process such as proof loading. Normally, in the process of specimen preparation, a selection process is operative that provides ultimately a truncated distribution, if only by virtue of the fact that specimens must have a finite strength to survive until a specimen reaches a testing machine. The two-parameter cumulative distribution functions, when plotted, show an acceptable fit to experimental data. There are cases, however, where the skewness suggested by the experimental data is not consistent with the skewness defined by the cumulative-distribution function. Larger sample sizes would be required accurately to characterize the skewness and the material cumulative-distribution functions, a fact dramatically demonstrated by the simulation results even with small samples.

Leicester (1973) defined a size parameter s according to

$$\sigma_f = A_1/L^s \qquad s \geq 0 \tag{36}$$

Leicester has shown that the theoretical size parameter s can be determined from the coefficient of variation of an assumed Weibull distribution according to

$$s = n(cv)^{1.085} \tag{37}$$

where cv = coefficient of variation and n = 1,2,3 depending on whether failure is dependent on length, area, or volume. Single-parameter estimates of this type can be useful in studying the fundamental aspects of underlying distribution, but considerable care must be exercised in any attempts to use such estimates to predict behavior. For example, Leicester (1973) suggests that the size effect s = 0.10 determined from test of geometrically similar, clear Douglas-fir beams would suggest an area effect and an area effect was not reported by Bohannen (1966) in similar tests. Although a cross-sectional area effect was not reported, Bohannen did retain Weibull assumptions to predict a size effect for geometrically similar beams loaded similarly according to

$$\sigma_1/\sigma_2 = (D_2/D_1)^{1/9} \tag{38}$$

where σ is the maximum strength in bending and D is the beam depth.

Using Eq. 14 and Eq. 36, the relationship between the size coefficient and the shape parameter can be determined, for similarly loaded beams of constant width, $s = 2/k$. The theoretical size parameter $s = 1/9$ ($k = 18$) agrees very closely with the value $s = 0.10$ ($k = 20$), which Leicester (1973) predicted from the data of Comben (1957).

The danger of relying on single-parameter estimates of size parameters (s or $1/k$) is emphasized by the variations in computed shape parameters obtained using different fitting methods. The shape parameters obtained for the 374 ASTM specimens using two different fitting methods are presented in Table 7. The variation of k with fitting method, even for large sample sizes in conjunction with the sampling variation to be expected with small sample sizes (see Fig. 5), suggests that great care must be taken in predicting size effects. Certainly tests at one volume, even with a large sample size, should not be used to estimate size effects. This is exemplified by the difference between k determined from the cumulative distribution function for the 374 ASTM specimens and the k computed from Fig. 3 for the clear or commercial material.

This research has demonstrated that a large size effect exists for Douglas-fir that has not previously been documented or quantified for tensile strength perpendicular-to-the-grain. Rational development of allowable working stresses must account for the size effect. This has been accomplished in development of allowable bending stress by reducing the basic working stress in accordance with Eq. 38. A similar technique could be applied to the development of working stresses for tension perpendicular-to-the-grain. The basic working stress is normally derived by reducing average strength to account for material variability. The regression equations plotted in Fig. 3 show the change in mean strength with volume and a tolerance band can be computed that will encompass a given percentage of the data points plotted. The lower bound of a 90% expectation tolerance interval cuts off an expected 5% of the population. This bound is plotted in Fig. 2 for all data cited. The use of such a simple size-effect model employed for bending would probably severely restrict design and it appears that a more refined analysis may be required to optimize design of structures such as pitched-tapered or curved beams. It is important to recognize that these size effects have been characterized for statically loaded specimens under essentially constant environmental conditions (Fox 1974). Results of recent studies on load duration effects in tension perpendicular-to-grain have accentuated the need for further study of load duration effects before final decisions are made on new working stresses for tension perpendicular-to-grain.

CONCLUSIONS

1. The tensile strength of Douglas-fir perpendicular-to-the-grain is strongly affected by the volume and stress distribution within a specimen.

2. The strength-volume relationship observed is described by a weakest-link strength concept based on a two-parameter Weibull cumulative-distribution function.

3. The experimental data suggest that a short-term test of a specimen with volume 10,000 inches^3 would have a mean strength of approximately 65 psi, which is the currently allowable working stress for dry service conditions and normal load duration.

4. The experimental data suggest that the magnitude of the size effect is dependent on the quality of the material in the specimen.

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